Yuri Mauergauz

Advanced Planning and Scheduling in Manufacturing and Supply Chains

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Preface to the English Edition

Despite the relatively large number of books related to production planning published in English, up to now information constituting the subject of Advanced Planning and Scheduling has not been gathered together. This situation inspired the author to present an English translation of his Russian-language book.

This book was conceived as a guide to modern methods of production planning, based on fairly new scientific achievements and various rules of thumb of practical planning. Most of the calculation methods are illustrated with numerical examples.

Attached to the English edition is a set of programs for calculating production schedules and an example of an ERP system operating in the cloud.

The author expresses his profound gratitude to Federica Corradi Dell'Acqua of Springer publishers. Her systematic support allowed this project to be implemented.

Moscow, Russia

Yuri Mauergauz

Preface to the Russian Edition

At the end of the last century, a large new field of knowledge developed. Nowadays, it is called “industrial engineering” and is a creative application of the methods and principles of various scientific disciplines to achieve and maintain a high level of productivity and profitability in modern industrial enterprises.

The application of industrial engineering is inextricably linked with the use of quantitative methods using information that circulates in the production system, and such methods often have complex mathematical justification. Historically, the concept of industrial engineering started to be used after wide application of methods known as “operations research”. Another name for these methods is “management science”, now more commonly called “industrial engineering”.

Since the foundation of industrial engineering is quite sophisticated mathematical techniques, its application possibilities are determined largely by the available computing power. Originally, computers were created to solve complex scientific problems. Subsequently, this equipment started to be used to develop automated control systems including production management systems.

The introduction of personal computers changed dramatically the possibilities and the main focus of application of computer technology. The main objective of computerization in the late twentieth century was automation of accounting of a variety of resources and operations with them, i.e. information storage. The automated control systems of enterprises were mainly designed to collect and integrate data referring to production and sales. Therefore, the development of industrial engineering at that time was mostly of a scientific and theoretical nature.

In the early twenty-first century, however, the situation changed dramatically. First of all, against the background of rising resource prices, the issue of production efficiency is becoming more and more important. In addition, it was found that, despite their great diversity, the number of accounting problems is limited and most problems had already been solved, while the increasing capabilities of computer technology allow more complex problems to be solved. As a result, researchers and production managers began to turn to the problems of enhancing production management.

There was a sharp increase in the number of articles in the field of industrial engineering and a rapid increase in the number of relevant scientific journals. Today, worldwide, there are at least 30 international English-language journals in which thousands of scientific articles on industrial engineering are published annually. In addition, there are a number of national engineering journals, and in some countries, such as Spain and Iran, they are published with simultaneous translation into English.

The field of industrial engineering includes management aspects such as the location of enterprises, determining the range of products, selection of necessary processes, organization of production divisions, etc. Many of these management objectives refer to pre-production, but not its realization. The effective implementation of production is only possible with organized and comprehensive sound planning, which is actually the final component of industrial engineering.

Until the end of the last century, production planning was mainly based on the knowledge and experience of the planners themselves who used quite elementary methods of calculation for different purposes. The use of computer technology, for the most part, was limited to calculation of the number of products and resources required.

Due to the complexity of the mathematical description of plans, their optimization appeared to be possible after the introduction of powerful personal computers at the beginning of this century. The relevant methods were used to create a number of new production control systems, known as APS and MES. In general, the new planning methods based on complex mathematical models are called Advanced Planning and Scheduling (AP&S). This book is intended for readers whose activities are related to production planning, though in different business areas.

First of all, the book is intended as a reference guide for operating production managers. As the workload of these specialists does not allow them to engage in a consistent and detailed study of the various methods, the book is designed so that almost every section, and sometimes even an individual paragraph, can be read independently of the other sections. At the same time, wherever possible when describing a method, reference is made to the preceding discussion of the method, to allow deeper examination of the material.

To make the presentation of each section independent of the previous text, most of the methods and examples use the same designations of variables and parameters, and these designations are listed in Appendix A. In those cases where the designation does not coincide or is not referred to in Appendix A, it is defined in the text. Each example is accompanied by the method reduced to a final calculation. The author hopes that this structure will be convenient for developers of production planning software as well as for production managers.

Not all planning methods described in this book are useful in practice. This applies to a number of problems and their solutions, which provide a scientific basis for comparison and a reference sample for other methods, which in turn may be used in practice.

On the other hand, the book is constructed to provide the opportunity to study the material consistently. The book is divided into two parts, the first of which is dedicated to detailed description of models of planning, and the second part describes the processes carried out on the basis of these models. Some of these models are quite complex, and at first acquaintance their study can be skipped. This construction is to facilitate learning by researchers, postgraduates, and students.

The challenge in writing this book was the selection of materials and the sequence of their presentation. An enormous number of different methods of production planning have been developed. In particular, G. Halevi's reference book on production planning methods dated 2001 describes 110 methods, which, of course, vary to a large extent in the degree of distribution and application. This book includes those models and planning processes which by the time of writing were in focus in the scientific literature. It was assumed that production planning itself is closely connected to the planning of inventory because the result of the manufacturing process is stock buildup.

The contents of the book, for the most part, are based on the results of scientific papers contained in a number of English-language guides, monographs, and articles written at the end of the twentieth and beginning of the twenty-first century. The author has also tried whenever possible to use the available, albeit few, modern Russian-language works. Materials relating to the period of development of computer systems in the Soviet Union in the 1970s and 1980s have also been used. The structure and nature of any presentation always depends largely on the author's position. In this case, when considering methods of production planning, special attention is paid to its regularity and dynamics, i.e. a periodic recurrence and at the same time the need to introduce various changes, including urgent ones.

Different scientific disciplines are used in the methods of production planning. Each discipline has its own set of traditional symbols. In this book, it was important to ensure consistent use of symbols, so one designation system was chosen as basic. Therefore, the nomenclature of symbols accepted in scheduling theory is used throughout; in other cases, some symbols may be different from the conventional ones.

The author is grateful to Professor A.L. Ryzhko whose comments and suggestions helped to improve the presentation significantly.

Moscow, Russia Yuri Mauergauz

Annotation

Advanced Planning and Scheduling (AP&S) in Production and Supply Chains

The book consists of two parts, the first of which considers construction of reference and mathematical planning models, production bottleneck models, and multi-criteria models; examples of such models are provided. The methods of forecasting and aggregate demand are discussed; background information about the storage and data processing methods for planning are provided.

The second part analyses various models of stocks planning and the rules for calculating safety stocks; it also describes the stocks dynamics in the supply chain. Various methods of batch sizing are detailed. Production planning is studied at several levels: planning of shipment to customers, calendar scheduling, and operational planning. Operational planning is considered separately for one-stage and multi-stage problems as well as for different multi-criteria problems. For some problems of multi-criteria, scheduling by the methods described in the book special software is developed.

The book can be used as a reference for modern planning methods as well as a teaching aid. It is intended for employees of planning and production services, specialists in enterprise information management systems, and researchers and graduate students involved in production planning. The book can be used by students at technical colleges as a guide when writing course papers and graduate theses.

A description of a collection of production schedule programs and an example of the ERP system operating in the cloud is included in the book.

Moscow, Russia

Yuri Mauergauz

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Part II Planning Processes

About the Author

Yuri Mauergauz is an Assistant Professor and a consultant of Sophus Group, Moscow, Russia. He gained his PhD from the St. Petersburg Navy Institute in 1970. He has worked at machine-building plants and research institutes and also taught at the Urals and Odessa technical universities. He has published around 80 research papers and 3 books dedicated to the application of computer engineering in production planning.

Part I

Modeling

Reference Model 1

1.1 Modelling of Business Process

Process is a sequence of operations combined to achieve a certain goal. The operations of the process are the elements of the chain that work together to create value necessary for some external customer. Each operation is defined by cost factors and performance. Ongoing operations influence each other since the efficiency characteristic of one operation is the cost factor for the subsequent operation.

The flow of operations passing from one person or company's unit to another is commonly called a business process. Managing the company's business in all its parts is actually related to the management of particular business processes. According to Ildemenov et al. (2009), generally, a list of 7–15 business processes covers the main activities of the majority of business enterprises.

Each business process must be identified. The identifiers may include: name, purpose, process action limits in space and time, and parameters of interaction with other processes. Besides these basic data, the process is defined by a number of values—attributes. These include, for example, the name of the owner (the person in charge) of a process, various characteristics of the process operations, process control methods, etc. Business processes are classified as follows: (a) processes that directly produce material and other values; (b) processes that produce the opportunity for operation of first group processes, e.g. management processes; (c) supporting processes necessitated by a variety of external requirements, such as tax accounting or staff advanced training.

The quality of any business process is evaluated mainly by three parameters: performance, efficiency, and flexibility. The first index assesses the business process adequacy in regard to needs and expectations of consumers. The second index measures how efficiently the business resources are used and ultimately how profitable the business is. Flexibility is defined as the capability of the business enterprise to adapt to the market changes, environment and development of technological processes, etc.

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Improvement in any process is impossible without its thorough analysis. In this regard, it is required to consider the entire chain of operations, their connection with each other and with other processes, i.e. to develop a structural model of the process. The most well-known and widely used methodology is SADT (Structured Analysis and Design Technique) based on which the standard of business process modelling IDEF0 was adopted in the USA in the 1990s.

In accordance with this standard, the basis for the modelling is a tree-structured functional model of the enterprise. Each branch of the tree (node) corresponds to a particular fragment of the model and can be represented graphically separately in the form of a diagram. Software such as BPwin, ARIS, and others are widely used as tooling for constructing such diagrams. For example, in BPwin system the works on the diagrams are shown as rectangles forming so-called functional blocks. These units are interconnected and connected to the outside world by the arrows that indicate the information flow, which provides work of the functional blocks. Figure 1.1 shows an example of such diagram constructed for modelling a production management business process.

The model in Fig. 1.1 presents the entire process of production management as a single functional block, to which the arrows are directed, carrying a variety of information. This block can be represented further during modelling as a tree root of subprocesses constituting production management. Here, for each branch of the tree it is necessary to build the corresponding diagram similar to that shown in Fig. 1.1. Construction of such set of diagrams is referred as decomposition.

The aim of constructing a model of a business process is to define underperforming areas of the process, to analyse the ways of possible improvements,

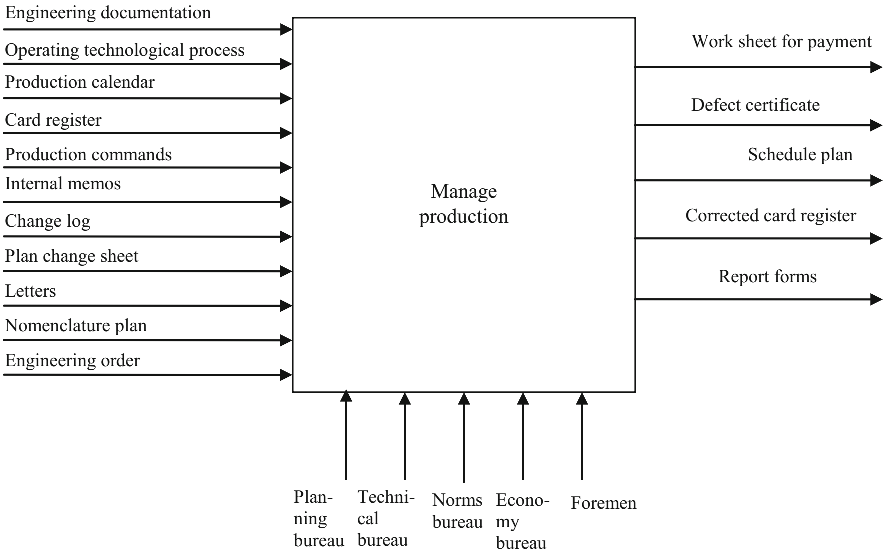


Fig. 1.1 Example of BPwin diagram of root model for production management

and to construct the model that is optimal in terms of model developers. The first stage of this work is to construct AS-IS model, i.e. model of the existing business processes. This model is built on the basis of study of available documentation and interviews with employees of the enterprise.

Sequential analysis of AS-IS model taking into account operating experience of existing business processes, its comparison with other similar examples, as well as with the available scientific and technical recommendations allows developing a new model that is currently optimal and real—TO-BE model. While constructing this model one should not be misled about the real possibilities of the enterprise and should not try to create a model that is ideal in terms of the customer or developer, but has no chance of implementation—so-called SHOULD-BE model.

The constructed model allows us to make assessment of the business process from different perspectives even before its start. The main requirements of the enterprise activities are made for its operation, management, efficiency, outcomes of activities, and customer satisfaction. Such analysis of business processes is called an audit of business processes. During the audit, Activity Based Costing analysis is made—measurement of costs and performance based on operations and costs of objects. Simulation allows revealing components of the business process with the highest cost and proposing improvements.

1.2 Concept of Reference Model

The analysis of the business process tree allows building the so-called reference model. The main difference of the reference model is that a business process is considered from several different viewpoints in it. Usually, there are two points of view, but there may be others. From the first point of view, the reference model is a certain standard of an efficient business process for an enterprise of a specific industry; from the other point of view, it is a set of logically interrelated processes containing references to the corresponding objects of the information system. The two main properties are reflected by the word "reference", which has two meanings—"standard" and "link to".

As a reference model, for example, the model of an existing business process, which has the highest efficiency in the industry and uses quite complete information system for its work, can be used. The reference models incorporating proven procedures and methods of control enable enterprises to start developing their own models on the basis of a ready set of business processes.

For this kind of development using the reference model, it is possible to divide the entire business process into its component parts and distribute tasks to elaborate an appropriate information system among the performers. In the course of work, the available reference model allows all participants to discuss achieved results in the framework of the terminology used in this model, which facilitates their interaction greatly. The reference model analysis allows elaborating quality indices, which will be discussed below. Finally, the availability of a detailed reference model allows evaluating of the acceptability of a particular information system suggested for implementation at the enterprise.

1.2.1 Reference Models in Supply Chains

The reference model concept started to be used first of all while analysing business processes in the supply chains created at the stages of manufacturing and sales.

Figure 1.2 shows a common-use example in the literature of the supplies by Proctor&Gamble of diapers for large retailer chain “Metro”.

As it can be seen from Fig. 1.2, in this case the production has three stages of supply and two stages of sales. Information is exchanged among all participants of this chain. As a result of this exchange the decisions on material flow are made. To enable all participants to discuss and coordinate their actions it became necessary to bring the data submission and interactions to some standard.

To describe, analyse, and evaluate the supply chain configuration of SCOR (Supply Chain Operation Reference) model was developed, which establishes the basic concepts of the relationship between supply chain participants. The model also contains a library of standard business functions and business processes for supply chain management, which correspond to the best examples in various fields of production and sales, but the model does not attempt to describe each business process in detail.

In principle, the SCOR model is based on three main ideas:

- Business process modelling
- Quantitative assessment of the processes quality
- Using the best (reference) examples

All business processes of this model can be divided into five groups.

- Planning that maintains certain balance between demand and supply. The necessary actions in manufacturing, procurement, and supply of various products are defined in the process of planning.
- Supplying that provides material flow to arrange focused effort of the supply chain.

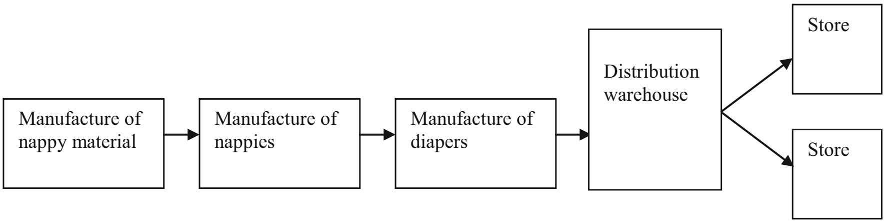


Fig. 1.2 Supply chain for manufacture and sales of diapers

- Manufacturing, which generates new products.
- Delivery of finished products and services to the distribution and consumption points.
- Return of the goods to the production or recycling sites. The return is possible for different reasons, e.g. expired shelf life, wearing out, or obsolescence.

Each business process of the supply chain is defined by the so-called functional attributes (Stadtler and Kilger 2008). For example, for the manufacture process the attributes (indices) describing the organization of this process, extent of operations repeatability, the presence of bottle necks, flexibility of working time use, setup cost, etc. are very important. Accordingly, the process of distribution is described by the scheme of products delivery, availability of vehicles and the limits of their load capacity, etc. The functional attributes can be grouped by categories either fully matching one of the processes or being part of the business process.

Stadtler and Kilger (2008) also specify that beside the functional attributes, the values of the so-called structural attributes are essential for the supply chain. These attributes are divided into two categories: topography and coordination. The first category includes network structure, degree of globalization, and position of the bottleneck in the network. The second category includes the following attributes: balance of power of the chain elements, type of information exchanged, and legal position of the entities in the supply chain.

1.2.2 Reference Modelling Methodology

Reference model according to the technique described in Hernández et al. (2008) is developed within six stages: information collection, evaluation, concept development, modelling, check, and recommendations drawing up.

At the first stage, the application field (industry, enterprise, supply chain, etc.) of the model is determined. The main entity objects of the described business process, which specify obtaining, processing, and transferring of information, are defined. At this point, the group of persons working on the model is determined and the main goals of modelling are described.

At this stage, the model developers using the meetings with the enterprise personnel and the documentation get familiar with the business-process objects to be described when modelling. For instance, when modelling production management, it is required to study issues of sales, supply, stock, production, and planning. During such study, it might appear insufficient to work within the enterprise but require studying the relations of the enterprise with other entities of the supply chains.

Within the second stage, the processes that are described in the enterprise's collected documentation are studied. Besides, the study may involve the materials from various systems connected with the modelling process. For example, when modelling the process of production management, the issues of the quality management, functionality of the applied information system, etc. may be studied as well. The stage is intended to determine required technical and software tools for the model development. When selecting these tools, one should consider not only their functional features but also their cost, requirement for licensing, training level of the developers, and possibilities to integrate with the existing information system. In consequence of this stage, it is reasonable to develop a special document describing the recommended tools for modelling.

At the stage of concept elaboration, the flows of products, information and decisions in the modelled business process are considered. For the product flow, the main production processes and existing constrains, e.g. capacities, are determined; the volumes of the existing demand and the extent of its satisfaction, predictions of the demand in future, average-in-time stocks and their fluctuations are defined. Production planning and management methods, as well as the production stability are evaluated; economic performance of the production and its profitability are studied. The results of the product flow study are suggested to be stored in a document named PFDO (product flow document).

The study of the information flow considers the following aspects: list of the main inputs of the business process and transformation of information in the business process. The flow of decisions include the mechanism of the decision making; objects, which are the matter of the decision; main actions of the personnel on the decisions implementation and their interaction. For the information flow and decision flow, it is recommended to draw up special documents named IFDO (information flow document) and DFDO (decision flow document).

At the fourth stage, the modelling itself is performed, i.e. based on the preselected tools and elaborated documents the model of the business process is made. It is suggested to name the relevant document as CMDO (conceptual model document). This document is checked at the fifth stage of development for compliance with the main objectives, objects, and flows of the existing business process. This stage finalizes construction of AS-IS-model.

The sixth stage of development is dedicated to the recommendation statement on transformation of the existing model to new TO-BE-model. These recommendations should describe the objects, which are necessary to be introduced or which must replace the existing objects of the business process and determine reasonable changes in relations of the process entities.

The described technique can be actually used to business processes of any type. Since in this book we consider mainly the issues of the production planning, the reference models described here refer to this process. The planning reference model pattern depends heavily on the production organization—"push" or "pull".

It is known that the "push" organization moves the material flow from one executor to each subsequent recipient strictly by the order (command) going from the management centre of local (work, shop, site) or general (enterprise) production. The "pull" production to the contrary provides determination of product volumes at each production stage according to the needs of the following stages solely.

1.3 Production Description

To build the reference model of planning the specific characteristics of production must be taken into account. It is often assumed that planning should be performed in different ways depending on whether the production is discrete or process. It is to be recalled that in the discrete production the product units are the pieces, and in process production—mass or volume. However, both of these types of production have a significant common property, which is that in all cases the final product is shipped to the consumer in the amount of one or more batches.

If we consider the process production, it is necessary to distinguish between two of its kind: periodic and continuous. In the periodic process production, all its stages are performed sequentially in a single apparatus, but in the continuous one—simultaneously in different apparatuses. Instead of term "apparatus" typical for process production hereinafter we shall always use more general term "machine". In the periodic production, of course, finished products are discharged from the machine after a processing period in the form of a batch. In the continuous production, despite the continuity of the process itself, the products can be extracted by parts as well, which also form a batch. Since the main objective of planning is to determine the batch size and the interval between them, the use of "batch-sized" property of the products allows unified planning for discrete and process production.

In terms of use of various methods of planning, the most important characteristics of the production are its technical structure, scale, and production strategy.

1.3.1 Basic Types of Production

The production possibilities are defined by its technical structure which must meet the number of requirements as follows:

- Compliance with the technological processes of finished product output;
- Compliance with the production scale;
- Possibility to simultaneously manufacture core and by-products;
- Provision of the external orders input into the system at its different points and correct flow of the orders inside the system.

We can provide the indicative list of the basic types of production based on these requirements and considering the possibility of concurrent use of machines with different capacities in the system. This list (Mauergauz 2012) is based on the analysis of a number of reported classifications compiled in terms of use of different methods of planning for different types of production.

Type 1. Single machine.

(a) It is fit to make only one type of product at a moment of time.
(b) It produces several types of products (core and by-products) simultaneously.
(c) It produces several orders as a single batch simultaneously.

Type 2. Parallel machines.

(a) Identical machines producing single type of product at a moment of time.
(b) Identical machines producing several kinds of products (core and by-products) simultaneously.
(c) Different unrelated machines that produce single type of products at a moment of time.
(d) Different unrelated machines that produce several kinds of products (core and by-products) simultaneously.

Type 3. Synchronized flow shop manufacturing with given cycle.

(a) Automated production line.
(b) Versatile transfer line, producing single type of products at a moment of time.
(c) Versatile transfer line, producing several kinds of products (core and by-products) simultaneously.
(d) Flexible assembly line.
(e) Complex multistage manufacturing that produces a set of core and by-products.

Type 4. Unsynchronized flow shop manufacturing (cell manufacturing).
Type 5. Job shop manufacturing.

(a) Set of individual machines.
(b) Set of technological sites (work centers).
(c) Set of individual workshops.

Type 6. Project manufacturing.

In every production type, there are raw materials and orders, which are input parameters; core and by-products are output parameters. In the flow manufacturing main production output comes out at the end of production line. By-products are made at intermediate stages. Complex multistage manufacturing represents set of serial and parallel lines with interdependent products. In single stage manufacturing all kinds of production may be of equal importance.

Figure 1.3 shows two schemes of single stage manufacturing. Four versions of flow shop manufacturing are shown in Figs. 1.4, 1.5 and 1.6 which demonstrate the schemes of job shop and project manufacturing accordingly. The examples of the complex multistage production will be presented below.

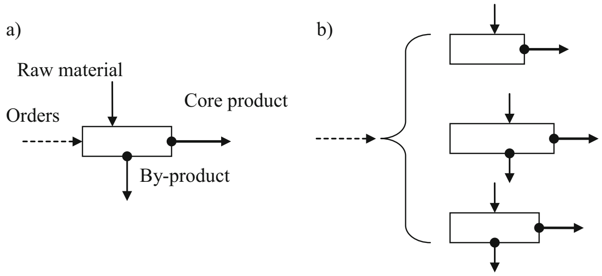


Fig. 1.3 Two types of one-stage production: (**a**) type 1b; (**b**) type 2d

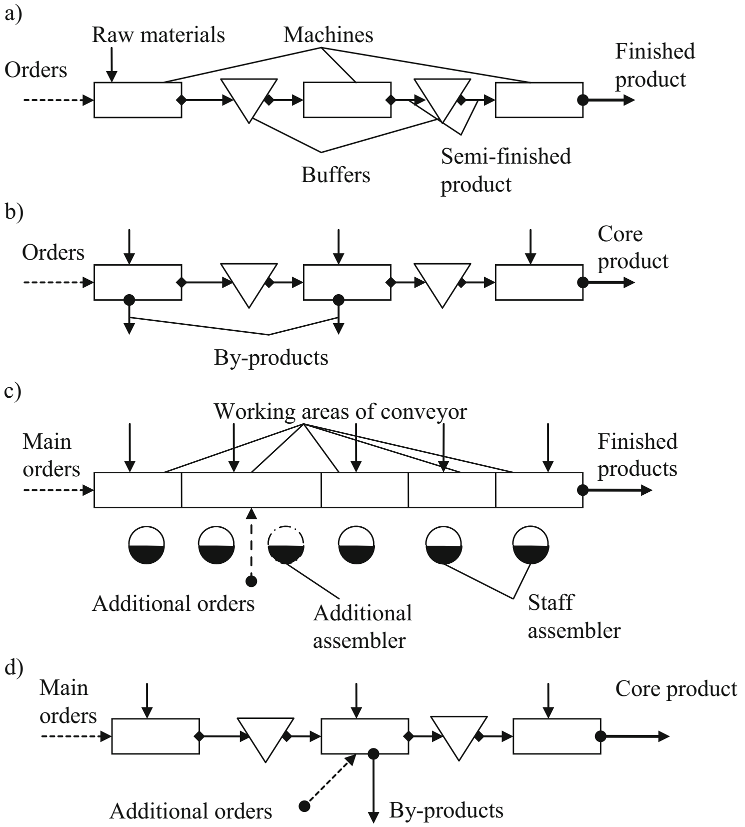


Fig. 1.4 Four types of flow-line production: (a) type 3a; (**b**) type 3b; (**c**) type 3c; (**d**) type 4

Solid arrows of three types shown in Fig. 1.3a designate raw material, core, and by-products accordingly, and the dashed arrow designates external orders. Brace in Fig. 1.3b means that external orders are to be fulfilled on any of the parallel machines. Similar designations are used in Figs. 1.4–1.6. From the perspective of

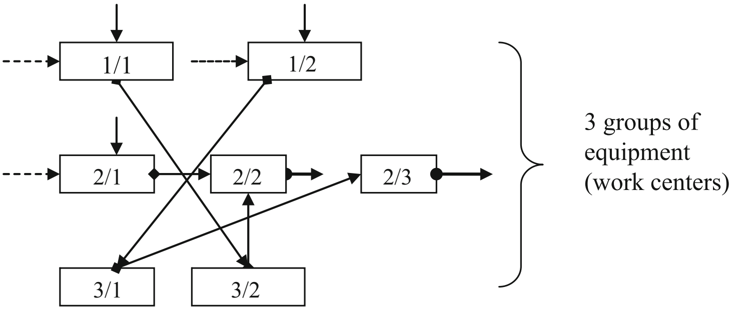


Fig. 1.5 Job shop manufacturing: type 5

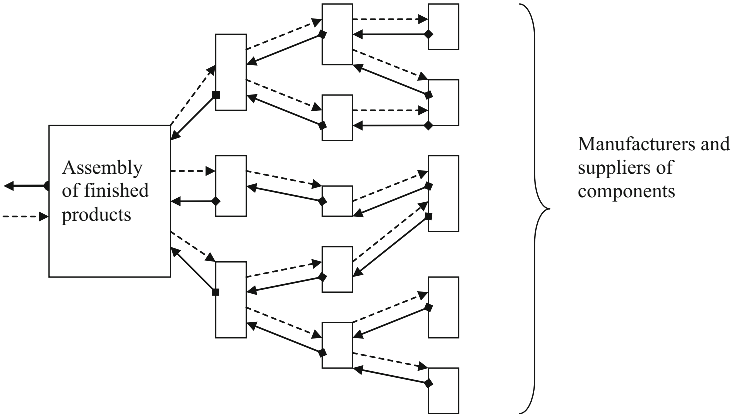

Assembly of finished products
Manufacturers and suppliers of components

Fig. 1.6 Project production: type 6

planning, the machines can mean not only a physical piece of equipment but a group of equipment (work centers), workshop, and even the whole enterprise. This allows drawing up plans uniformly, which is often used to prepare plans of different levels using MES-systems.

Having considered the diagrams in Figs. 1.3–1.6 one can make some conclusions about common properties of these technical structures.

- The supply of raw material is required at the points where the orders come into system though additionally the raw material can be supplied at any point.
- The main property differentiating the process production from discrete one, according to the diagrams, is possibility of concurrent output of core and by-products, both for a production line and for a single machine. Technically, such possibility for single machine is provided by the fact that despite its construction as a single unit, actually, inside it can have serial process operations, each of which produce a separate product. In this case, the core

Table 1.1 Correspondence of the basic types of production and classification by M. Pinedo

Types of production acc. to M.Pinedo	Basic types of production
Process production	
1a. Main processes of continuous production	Single machine—type 1b; unrelated parallel machines—type 2b, 2d; versatile transfer line—type 3c, 3e.
1b. Processes of preparation or final processing in continuous production	Single machine—type 1a; unrelated parallel machines—type 2a, 2c; versatile transfer line—type 3a, 3b; cell manufacturing—type 4.
Discrete production	
2a. Procuring processes in discrete production	Single machine—type 1a; unrelated parallel machines—type 2a, 2c; cell manufacturing—type 4.
2b. Main processes of processing in discrete production	Single machine—type 1a; unrelated parallel machines—type 2a, 2c; versatile transfer line—type 3a, 3b; cell manufacturing—type 4; multipurpose production—type 5a.
2c. Assembly processes in discrete production	Flexible assembly line—type 3d; cell manufacturing—type 4; project production—type 6

product shall be considered to be the product after the last operation, and the rest products are by-products.

- If orders are input directly at internal points of the production system, flow rate may be supported by additional workers in a kind of buffer zone. In the systems, where the synchronization of the machines in the flow is not available, entering additional orders at separate points improves the quality of planning.
- The job shop manufacturing exists only for discrete production. Figure 1.5 shows possible order flow in production for three groups of uniform equipment.
- The project manufacturing features full matching of order tree and product tree, the branch direction of which are opposite. Due to that feature, finished product is produced in the same place where the initial order was entered.

It should be noted that in the versatile transfer lines of types 3b and 3c the changes in the lines can occur not only during changeover of the whole line for other type of product but also during setups of individual machines for different process operations of the continuously produced product. The example of such line is presented below in Sect. 7.3.

Table 1.1 shows correspondence of the main types of production 1–5 and discrete and process production classification suggested by Pinedo (2005). As we can see from this table, the above listed types of production indeed provide description of both discrete and process production.

1.3.2 Production Scale and Strategy

Practical selection of a particular technical structure of production is closely associated with its scale (Table 1.2).

Table 1.2 Association of production type with its scale

Main types of production	Production scale
1. Single machine	Any possible scale of production
2. Unrelated parallel machines	Mass, large serial
3. Synchronized flow shop manufacturing	Mass with modifications or large serial
4. Cell manufacturing	Serial, small serial
5. Job shop manufacturing	Small serial, individual
6. Project production	Individual

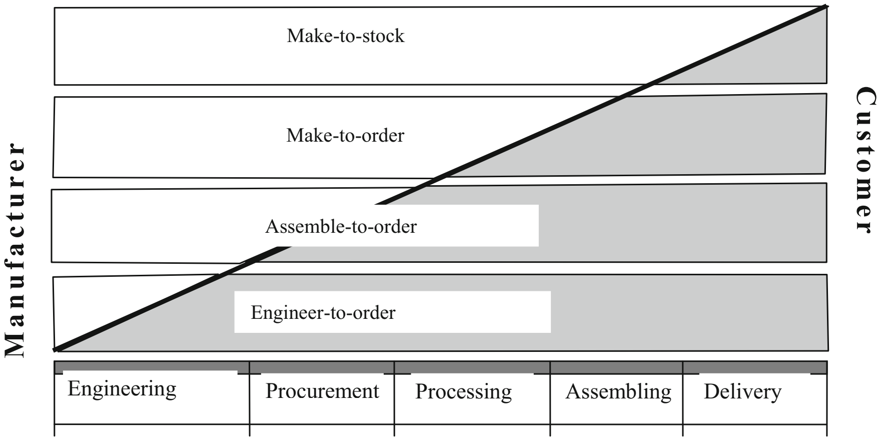


Fig. 1.7 Depth of decoupling for different production strategies

The use of a single machine is the most versatile in terms of scale. More complex technical structures tend either to large- or to small-scale production. Intermediate example is the case of nonsynchronized flow production (cell manufacturing) where the line method (typical for large-scale production) is combined with some features of the job shop manufacturing.

The production strategy describes the products readiness to meet consumer demand. This readiness determines the speed of response of the production enterprise to the received orders and influences the status of the enterprise on the market. There are four strategies for production readiness:

- Make-to-stock, MTS
- Make-to-order, MTO
- Assemble-to-order, ATO
- Engineer-to-order, ETO

Difference in production strategy is actually in selection of the depth of the decoupling in the production activities. This connection is shown in Fig. 1.7.

In Fig. 1.7, we can see that the decoupling point, corresponding to the border of the grey area, moves into the depth of the production activity so far as the strategy approaches to the tracing of the external orders.

Selection of the production strategy is influenced by the duration of the production cycle, the admissible waiting time for order fulfilment in competitive environment, need for adjustment of the product to the customer's requirements, availability of enough current assets, etc.

1.4 Advanced Planning in IT Systems

The purpose of the application of any reference model is to improve the enterprise management. As the efficient management in the present market economy is impossible without use of modern methods of planning, an adequate reference model of the planning process needs to be built because it is a necessary step in the management upgrading. Reference model of planning is the foundation on which it is possible to use different mathematical models, recommendations for decision-making and other methods that are currently unified by concept "advanced" planning.

The main difference of methods Advanced Planning and Scheduling, AP&S, is not so much the use of mathematical methods for finding optimal solutions as the idea of planning as a dynamic process. This method of planning usually uses a concept of "sliding" horizon, which means that at every moment of planning the plan decision is prepared for a certain upcoming interval of time, which does not necessarily have to be permanent.

The second feature of the Advanced Planning and Scheduling is a compulsory evaluation of available information to support the decisions to be made. Thus, as a rule, quality criteria of decisions are set and the methods of achieving high values of these criteria are selected.

The third essential feature of this approach is consideration of various constraints, especially regarding capacity, immediately in the planning process.

1.4.1 Planning in IT Systems

Since the nature of the planning decision generally depends on the interval duration, i.e. on the planning horizon, in the information systems that support production planning, there are several levels of planning. Currently, there are four types of information systems which provide the planning of production and supply at different levels:

- Enterprise Resource Planning, ERP
- Manufacturing Execution System, MES
- Advanced Planning System, APS
- Supply Chain Management, SCM

One should distinguish AP&S approach from APS-systems as such. Certainly, that the specially developed APS-systems use AP&S approach, however, the latter can be used in various other systems including ERP-systems. Table 1.3 presents planning possibilities at different levels for the above listed systems.

Table 1.3 shows five possible levels of planning. As seen from the table, none of the above types of information systems involve the use of each of these levels. The most common (and early developed) ERP-systems usually do not have the level of strategic (business) planning, but have a complete set of functions for production planning including scheduling at the workshop level. ERP-systems belong to the class of so-called transaction systems, where each business transaction is associated with initial data accumulation or its processing to get reports on the data.

In new MES-systems and certainly in special APS-systems, optimization models are used, which function on the basis of a dedicated database. This database is created in the framework of the reference model and required for decision-making.

When the first APS-systems were under development, it was supposed that their application field will be very wide—from the corporate to the workshop level. Moreover, all these levels were to be provided by three types of planning—long-term (strategic), medium-term (tactical), and short-term (operational) (Stadtler and Kilger 2008; Shapiro 2001). Since APS-systems do not have initial data on products, equipment, personnel, etc., their operation is only possible in combination with ERP-systems. Besides, to perform the tasks it is necessary to reload the data into the APS-system, add the data required for decisions and missing in ERP-system into this system, and return the obtain results to the ERP-system for their application.

Application of such complex data processing mechanism has led to rather limited capabilities of APS-systems in practice. Currently, these systems seem reasonable to be used at the level of a strategic plan for the corporation in general and the individual enterprise as well as at the level of preparation of master production schedule. Short-term planning using these systems typically is reasonable for a large-scale process production with a relatively large (for a week or more) planning horizon. Table 1.4 lists the main modules that make up the structure of some APS-systems.

APS-systems, listed in Table 1.4 are intended for a large range of scheduled tasks in various industries, and, respectively, are provided with a number of powerful planning modules. These systems, for the most part, are developed by the same company that developed ERP-systems and are in fact superstructures over the latter. Such APS-systems are quite expensive and their use is rather difficult.

There are also specialized simpler and cheaper APS-systems that perform particular planning tasks. However, to use these systems effectively, adequate matching of their capabilities and features of a specific task is required. A successful example can be the use of Lawson M3 Supply Chain Planner for the task of optimal planning of seed stocks and their distribution among farmers (Andersson and Rudberg 2007). These systems usually have units that provide the possibility of their cooperation with various ERP-systems, which allows selecting the best

Table 1.3 Covering of the planning levels by information systems

Administration level	ERP-systems	APS-systems	MES-systems	SCM-systems	Plan’s level of detail
Corporation (supply chain)	–	Long-term (strategic) planning	–	Coordination of supply chain	Aggregative
Enterprise administration	–		–	–	Aggregative
Economic planning office	Plan for sales and operations	Medium-term (tactical) planning			Aggregative
Production scheduling office	Master production plan		–	–	Partially aggregative
	Nomenclature plan (plan for physical resources)		Planning of workshops cooperation		Detailed by composition
Workshop	Operative plan	Short-term planning	Planning of workshop operation	–	Detailed by operations

Table 1.4 Some APS-systems and their composition

System type	Module name	Module purpose
AspenOne	Strategic Planning	Economic tasks of long-term planning.
	Collaborative Demand Management	Forecasting and development of master plan
	Supply Planner	Medium-term planning of procurement
	Plant Scheduler	Short-term planning of production
	Distribution Scheduler	Transport objectives
	Inventory Planner	Planning of stocks
SAP Advanced Planner and Optimizer (APO)	Demand Planning	Planning of demand
	Supply Network Planning	Planning of supply in the chain
	Global ATP	Orders reservation
	Production Planning and Detailed Scheduling (PP/DS)	Development of master plan and short-term planning of production
	Deployment and Transport Load Builder	Stocks distribution to the destination place and transport load ratio
	Transportation Planning and Vehicle Scheduling (TP/VS)	Transport objectives
Oracle JDEdwards APS	Strategic Network Optimization	Strategic planning
	Production & Distribution Planning	Development of master plan and product distribution objectives
	Demand Planning	Forecasting of demand
	Production Scheduling	Short-term planning of production
	Order Promising	Reservation and ordering

combination of commercially available small APS-systems with ERP-system existing at the enterprise.

As seen from Table 1.3, the planning modules for different planning levels also exist in ERP-, MES-, and SCM-systems. These modules are widely presented in the ERP-systems, but in them advanced scheduling techniques are practically unusable.

A completely different situation occurred in the MES-systems. The systems of this type were developed much later than ERP-systems, and the planning techniques used in MES-systems (developed by that time) were greatly influenced by advanced planning methods. In fact, we can assume that all production schedules developed in the most available MES-systems use advanced planning methodology in some way.

As for SCM-systems, they are usually intended to perform transactions (Information Operations) in the supply chain, manage relationship between chain entities, and control the current business processes in the supply chain, i.e. coordination of work in the chain. However, practice shows that in many cases there are attempts to use such systems to optimize some planning tasks as to the allocation of stocks between the parties of the supply chain. As an example, the paper of Kim (2005) describes the experience of using SCM-system to optimize stocks and reduce the cost of storage of medicines. In this case, the supply chain consists of many pharmaceutical companies, distribution centres, and hospitals.

Unlike the above-described application of APS-system in the supply chain (Andersson and Rudberg 2007), application of SCM-system in this case has much broader purpose than optimal stocks planning. Here, in addition to determining the amounts of stocks and their timely delivery, the system monitors the implementation of plans and their updating, defining responsibilities of entities of the business process, etc.

1.4.2 Popularity and Effects of Advanced Planning

The results below are based on private data, which the author have got during International scientific and practical conference "Effective technologies of production management" held in Moscow in October 2009.

The certain totality of the companies may be divided into three categories: high level of use of information technologies in production activities (Category A); with an average level (Category B); and enterprises, which are slow with the introduction of information technology (Category C). Table 1.5 presents the data characterizing some aspects of work quality for these groups of enterprise.

Although the duration of the production cycle for the enterprises of category A is much lower than for category C, it certainly does not mean that such a decrease is directly related to the intensive implementation of information technologies. If anything, companies with relatively short production cycle, mass production, and strong competition in the market are forced to use modern information systems, including systems planning, to increase their competitiveness. The enterprises, whose products have a long production cycle, generally have a small number of customers and, at the same time, a strong position in the market. Therefore, they can afford not to hurry with the introduction of modern information systems.

Careful examination of Table 1.5 shows that with more intensive use of information systems—successive transition from category C to category A—the use of relatively old ERP-systems is gradually reduced, and the use of newer MES- and APS-systems increases. These changes are connected with the transfer of planning

Table 1.5 Information categories of enterprises and their performance

Performance	Category A	Category B	Category C
Percentage of totality	20	50	30
Timely fulfilled orders, %	97	87	75
Stock of finished products in days of discharge	4	14	32
Duration of production cycle in days	9	33	75
Usage of ERP-systems, %	65	69	69
Usage of MES-systems, %	24	22	20
Usage of APS-systems, %	27	20	17

functions of ERP-systems to MES- and APS-systems providing new planning technique.

In general, of course, the use of new information technologies enhances the quality of order fulfilment and production performance. The main value of information systems use is to increase the flexibility of production, which allows responding to changes in demand. Improving the efficiency of production is the second important objective of new information systems.

Figure 1.8 shows (by arrows) linking between different information systems operating in businesses and in the supply chain, as well as AP&S approach. This figure reflects only the modules of information systems, the use of which has been widely spread. For this reason, short-term planning modules of APS- and ERP-systems, modules of chain management in APS-systems, and planning in SCM-system are not illustrated.

Let us consider now the question of the place of advanced planning in information systems set forth in this paragraph. On this, there are different and sometimes conflicting opinions in the references. The paper by Frolov and Zagydullin (2008) argues that APS-system should be applied at the level of long-term and medium-term planning, and MES-system—for short-term planning, which is close to the current practice. At the same time, in some books on MES-systems, e.g. in Meyer

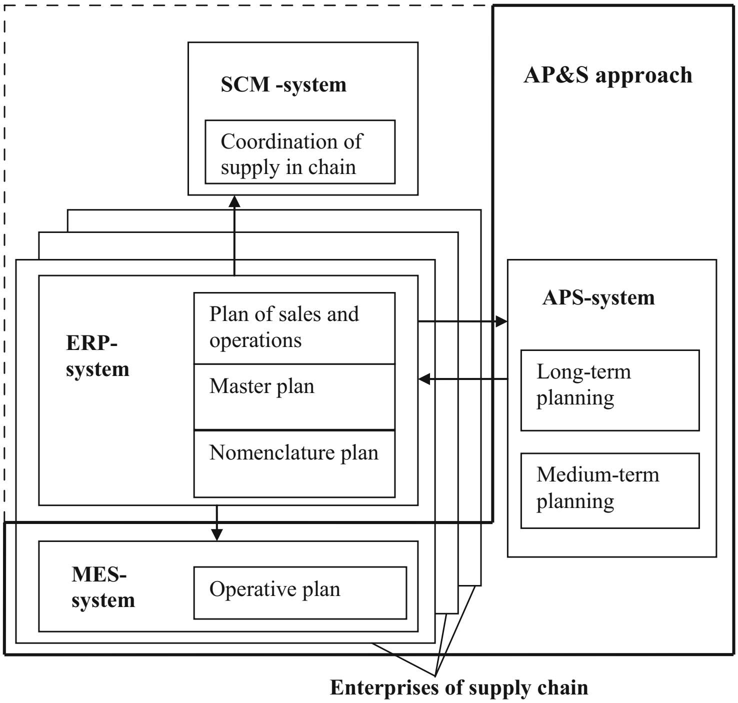


Fig. 1.8 AP&S approach and planning modules of information systems

et al. (2009), it is suggested that APS-system should be included directly into the MES-system.

Most probably, the last statement comes due to the fact that the book (Meyer et al. 2009) identified the notion of APS-system and the AP&S approach. Indeed, developers of MES-systems planning modules usually have to use the above basic principles of AP&S approach from the very beginning, namely—responsiveness, optimization, and constrains consideration as otherwise the estimated plans will be either unrealistic or have low quality. At the same time inside the MES-system, typically no particular APS-system exists.

Figure 1.8 shows ERP- and MES-systems as integral parts of the production enterprise, and SCM-system as an object that is used to manage the chain of enterprises and, accordingly, located over these enterprises. At the same time, the presence of APS-system as a separate system is not necessary or even desirable for the enterprise operation, and that is why APS-system is shown outside the enterprise.

Currently, ideas and techniques of AP&S approach are implemented for planning using APS- and MES-systems circled in Fig. 1.8 by the bold line. However, it is quite possible that during development of new ERP- and SCM-systems advanced planning methods will be included in them, and the need for special APS-systems will gradually decrease. The dashed line in Fig. 1.8 shows occurring in this case additional field of application of AP&S approach.

1.5 IT System Interaction Standards

Large variety of information systems in the market has required a certain ordering, allowing users, firstly, to know their way around in this set of systems, and, secondly, to assess the possibility of their cooperation. Simultaneously, within the last two decades, the developers of information systems became aware of practicability to standardize some aspects related to the objectives and interaction of the systems.

Establishment of many public associations of developers and users of information systems has been essential for elaboration of standardization issues. The most famous associations are APICS (American Production and Inventory Control Society), SCC (Supply Chain Council), MESA (Manufacturing Enterprise Solutions Association), ISA (Instrumentation, Systems and Automation Society), OPC Foundation, and others. As a rule, these associations elaborate not the standard but some recommendations, which in case their wide-spreading are somehow legalized by International Organizations for Standardization.

Standardization in information systems is performed mainly in three directions: system structure and report documents; data; and information exchange methods. In this Chapter, only the first direction is analysed and the others will be analysed in Chap. 5.

The most famous example of the system structure standardization is standard MRP2. Historically, first, association APICS developed a number of recommendations on composition of industrial information systems using MRP2 technique. Later, these recommendations were presented in the form of standard ISO/IEC 2384-24: 1995, issued by the International Organization for Standardization (ISO) in cooperation with the International Electrotechnical Commission (IEC).

In accordance with this standard the systems of MRP2 class must perform the following planning tasks without limitation:

- Forecasting
- Master Production Scheduling
- Materials Requirement Planning
- Finite Scheduling
- Production Activity Control

In the course of the years after the adoption of this standard, the application of ERP-systems has led to the fact that the objectives set forth by this standard are transformed into the development of plans of four levels shown in Table 1.3.

Another famous example of standardization is SCOR model described in Sect. 1.2.1 and developed by SCC Association. This model provides:

- Standard description of the processes of supply chain management
- Standardization of business processes relationships
- Standard metrics that allow measuring and comparing efficiency performance (productivity) of the processes

Recently, new standards have become increasingly popular. They are developed by ISA—ISA-88 and ISA-95, which were subsequently approved by the American National Standards Institute (ANSI) as official public standards.

The first section of standard ISA-88 was developed in 1995 and regulated the batch products management in process manufacturing. The standard classified rules describing the technical structure of the enterprise and the structure of the production process. At each stage of the process for each machine, the values of the process parameters must be set in accordance with the standard. Using this standard allowed for the possibility to unify similar processes at different enterprises. Subsequent (2–4) sections of this standard, developed in 2001–2006, define data structures and methods of its analysis and storage.

From the viewpoint of advanced planning, the most interesting thing seems to be ISA-95 standard. This standard considers ERP-system as the basis of information support of business processes and establishes the terminology and rules of integration of ERP-system with other systems at the enterprise. The standard provides five levels of information processing, reflecting the production activities of the enterprise. Table 1.6 provides description of these levels.

The lower (zero) information level is represented by data collection elements (sensors), the first level—devices with programme control (e.g. CNC controllers). On the second level, there are automated control systems SCADA (Supervisory

Table 1.6 Specification of information levels according to ISA-95 standard

Level	System type and its composition	Main functions	Time range
4	ERP	Technical-and-economic planning, distribution of resources and accounting	Days, weeks, months
3	MES	Workshop planning and management	Minutes, hours
2	SCADA, LIMS	Process studies and management	Seconds, minutes
1	Basic control systems (programme devices, controllers)	Setting of work programme	Milliseconds, seconds
0	Sensors	Collection of information	Continuous scale of time

Control and Data Acquisition) interacting with the hardware. Furthermore, the second level also includes the so-called laboratory information systems LIMS (Laboratory Information Management Systems) providing control and analysis of current processes in real time.

SCADA and LIMS provide information MES-systems located on the third level; MES solutions in their turn provide aggregated information for ERP-systems of the top management.

The lower the level of data processing is, the faster the appropriate reaction must be. As shown in Table 1.6, the range of responses in the production information systems is very large: at the lower level, the response is often to be almost instantaneous, and the top-level decision-making process can take several months.

Besides the levels of information processing, ISA-95 standard sets four routine tasks for production management:

- Production result (products)
- Resources for obtaining results
- Schedule of required subsequent works
- Work performance control by production indices

Each routine task corresponds with its information flow. Analysis of the information transmitted by these flows allows the management to perform process control. In ISA-95 standard, it is considered that there are three types of production that require different approaches to solving problems of management—continuous, periodic, and discrete. In Sect. 1.3 the definitions of continuous and periodic production were given, as well as detailed description of the differences between these types and discrete manufacturing in terms of planning.

Figure 1.9 shows an example of the management structure at the third level of information processing (MES-System) developed by Siemens under the provisions of ISA-95. Four rectangles at the top of the figure describe the relevant tasks assigned by ISA-95 standard. Each oval area is a set of actions performed at a production site, and each action must match the function module of MES-system.

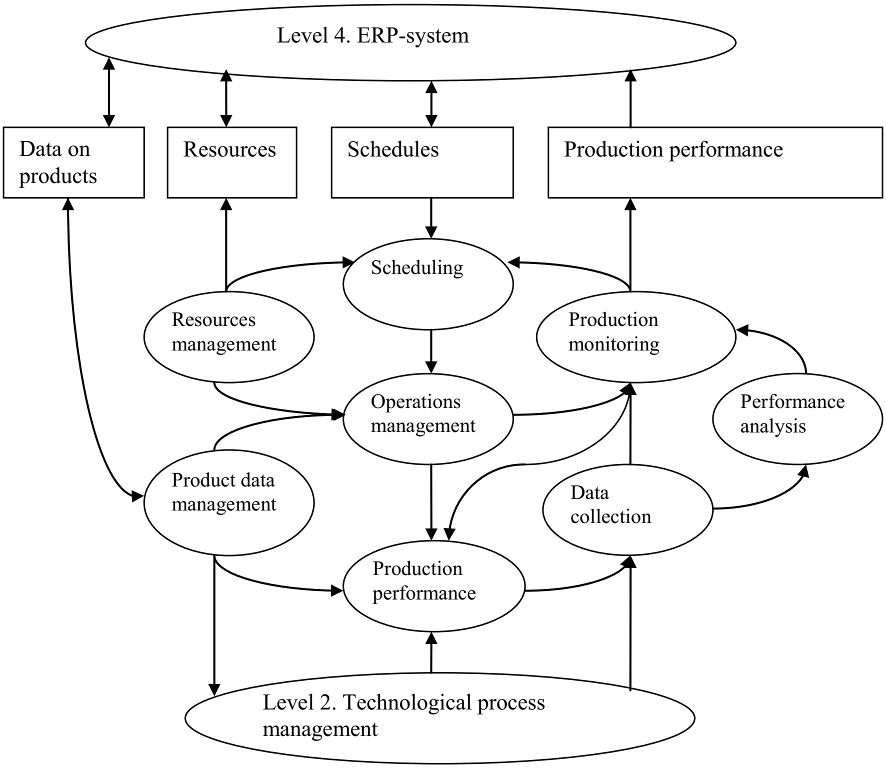


Fig. 1.9 Management structure based on ISA-95 standard (based on www.siemens.com/simatic-it)

The lower and upper parts of Fig. 1.9 have the second and fourth levels, the arrows show the direction of information interchange.

Besides the above standards, the so-called OPC-standard has been widely used recently for collecting and processing of the primary information received from the equipment. Purpose and content of this standard are discussed below in the description of data obtaining for planning in Chap. 5.

1.6 Quality Parameters in Supply Chains

The reference model SCOR, which was discussed in Sect. 1.2.1 provides four categories of quality assessment of each supply chain:

- Customer service level
- Economic efficiency
- Flexibility in meeting the demand
- Ability to develop

Since any supply chain is designed to work in some market, the assessment of the chain quality chain can only be based on the characteristics of this market.

1.6.1 Markets and Their Main Properties

The main indicator of the market is a proportion between the buyer's needs and seller's capabilities—between supply and demand. A widespread market model addresses four types of markets with different combinations of values of supply and demand. These together form the so-called quadrants of the market (Fig. 1.10).

In the first quadrant on Fig. 1.10, the market is characterized by low and unpredictable values of both supply and demand. They are usually either geographically new markets or markets of the newly created products. In addition, such a market can be opened due to the emergence of a new layer of customers who gained, for some reason, opportunity to purchase. The cost of goods in this market is high and stocks are low.

In the second quadrant—a growing market—the value of demand is large and the possibility of its satisfaction is not enough. Management actions in these conditions are aimed at increasing the production and delivery of products in accordance with customer demand, although this may not always be accomplished. In such a market, special price markups are possible for urgency and guaranteed timely delivery. In these circumstances, it is advisable to create some reserves of goods, as their price may eventually grow.

Stable market values of both demand and supply are quite high and relatively predictable. The main objectives of management are relevant to cost savings at all levels of the supply chain. The price of stably manufactured products may gradually go down due to competition; inventories are maintained at approximately constant and fairly significant level.

Fig. 1.10 Quadrants of market

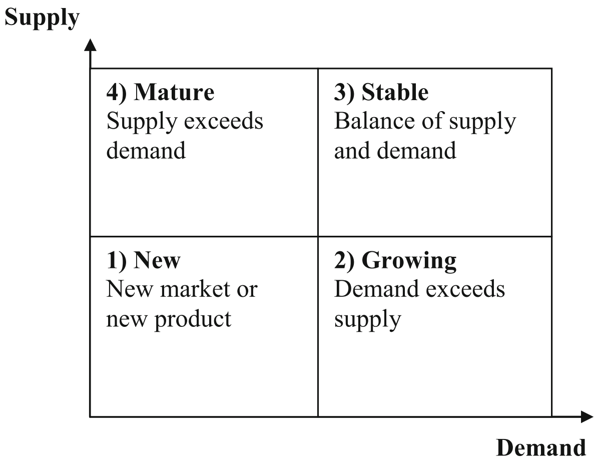


Table 1.7 Quality categories of supply chains for different markets

Market	Demand	Supply	Customer service level	Economic efficiency	Flexibility in meeting the demand	Ability to develop
New	Low	Low	+			+
Growing	High	Low	+			
Stable	High	High	+	+		
Mature	Low	High	+	+	+	

The fourth quadrant relates to the market where demand for the sale product goes down gradually. The reasons for such a situation are usually associated with product obsolescence due to entry of new, more promising competitive products or due to changes in customers' mind—for example, fashion change. This, however, does not exclude the possibility of increase in demand, although it is difficult to predict. In such a market, the entire supply chain's ability of flexible response to demand fluctuations comes to the forefront.

In addition, correct pricing policy for discounts and surcharges is very important for rapid fulfilment of orders, size of the order, etc. The stocks should be small here, since products may simply be outdated.

For each of the markets in Fig. 1.10, importance of each of the above quality categories is significantly different. Usually all four quality categories are not used simultaneously in one market and the order of their importance varies. Table 1.7 shows the quality categories corresponding to each of the markets.

According to this table, the category of customer service level is important in all markets and, consequently, all the stages of life of a particular type of goods, and in the case of a growing market in general it is the only importance. Economic efficiency is essential for the late phase of the existence of marketable appearance—for stable and mature markets. The ability to develop should express mostly in the new market during development and initial promotion of the goods.

1.6.2 Quality Parameters and Different Supply Chain Levels

Reference model SCOR distinguishes three levels of quality assessment criteria. The above four quality categories refer to the first (top) level and each of the categories includes several criteria. The set of these criteria for the service level category depends on the production strategy—"make-to-stock" or "make-to-order". Table 1.8 presents the composition of the first level criteria for each quality category.

The meaning of the above criteria is mostly clear from their names, so we will enlarge just upon some of them. In the criteria relating to the categories of service levels, the order item is a specific name of the ordered goods from a list in the order. Since an order can be fulfilled only partially from the list of the declared items the percentage of fulfilment of the totality of order items is used as one of the criteria.

Table 1.8 Quality criteria composition at the first level of SCOR model

Category	Strategy	Criteria
Service level	Make-to-stock	Percentage of completion of orders and the percentage of completion of order items; Percentage of timely delivered orders; Cost and number of unfulfilled orders; Frequency and duration of the delay of orders; Percentage of returned items of orders.
	Make-to-order	Response time to order and the percentage of timely fulfilled orders; Percentage of timely delivered goods; Quantity and value of overdue orders; Percentage and amount of delay of overdue orders; Number of claims and repairs.

Economic efficiency	–	Cost of stocks;
		Stock turnover;
		Cost of returned goods;
		Duration of the payment cycle.

Flexibility in meeting the demand	–	Duration of changeover of the supply chain to a new product or a new entity of the chain; Flexibility in the quantity of supplied goods; Flexibility in additional order items.
Ability to develop	–	Percentage of quantity of new product types in annual sales; Percentage of the cost of new products; Average duration of development cycle and introduction of a new product.

The criteria of economic efficiency of the supply chain at the first level do not include indicators related to the efficiency of an individual enterprise—its income, profit, etc. Therefore, when considering the quality of planning for every enterprise, it is also necessary to take into account indicators of this particular enterprise beside the criteria for the supply chain in general. These criteria belong to the second level quality assessment.

Duration *D* of payment cycle in Table 1.8 is calculated as follows:

$$D = Z + S - P, \tag{1.1}$$

where *Z* is average number of days of goods' staying at the warehouse after receipt from the supplier and to the moment of its demand; *S* is average time for payment receipt from the buyer in days; *P* is average time of payment for the received goods in days.

Flexibility in meeting the demand in Table 1.8 is divided into flexibility in delivery quantities of previously ordered products and flexibility in the delivery of goods not covered by the current order. The first of these criteria is evaluated as percentage of the potential increase in the scope of the delivery. The second criterion can be determined by the average percentage of substitution of previously ordered goods by other goods.

The degree of application of the criteria from the list to evaluate the performance of the supply chain is different depending on which of the five groups described in Sect. 1.2.1 the considered business process belong to. For example, for business processes from planning and procurement groups (Hugo 2006) criteria for the first three categories in Table 1.8 are commonly used (in various combinations), and for the processes of the group of manufacturing—practically all of the above criteria.

Standard SCOR states that except the first strategic level, the processes in the supply chain should be evaluated on the second and third (tactical and operational) levels as well. As mentioned above, the second level of evaluation includes (above all) economic indicators defined in the business process of planning. The production process is defined by the criteria of production cycle duration, level of product quality, and timely execution of orders. In general, a set of criteria for the second level should be sufficient for a complete and impartial assessment of the quality of the current business processes.

The objective of the third level of evaluation is the so-called diagnostics of business processes. The measured or calculated indices of this level should allow managers identifying lower-level deficiencies of the ongoing process, deciding on correction of production situation, and transmitting the required information to their superiors. All figures in the third level are divided into three groups:

- Process complexity indices
- Characteristics of supply chains configurations
- Current parameters of the process

For example, for business process from the planning group the complexity indicators are percentage of order changes, the number of goods items, volume of production, and the cost of inventories. The configuration of the supply network for the same group, i.e. for planning processes, is characterized by delivery scope in one supply chain, the number of chains, and the placement of the supply chain.

To assess the current status of planning processes, it is recommended to use parameters such as the duration of the planning cycle, forecasting accuracy, and the percentage of available and obsolete inventories.

More detailed composition of criteria of the second and third levels of SCOR standard is described in the book of Hugo (2006) and on the website www.supply-chain.org. In SCOR standard, starting with version 8.0, the terminology is unified with the terminology described above in Sect. 1.5 of ISA-95 standard.

1.6.3 Balanced Scorecard

Balanced Scorecard (BSC) is a concept for achieving company's strategic goals by purposeful actions at various levels of management. Figure 1.11 shows the rations of main causes and effects defining the result of company's performance.

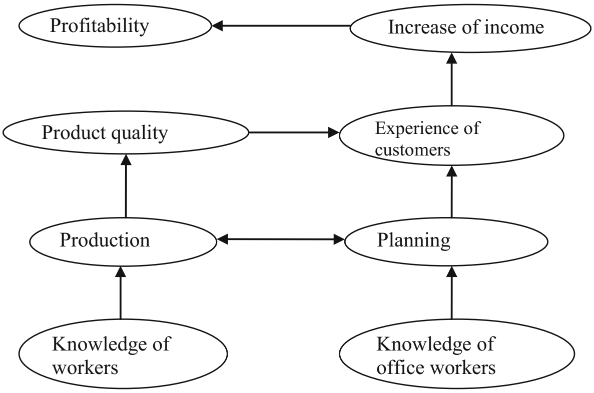


Fig. 1.11 Relations of main causes and effects [based on Chernikov (2011)]

As seen in Fig. 1.11, all the knowledge and actions of the company's employees must be combined to achieve one goal—high profitability. As the stated objectives are achieved due to the directional action of all employees, a special system of personnel motivation, methods, and means of measuring the degree of approximation to the desired result, i.e. mechanism of sequential communication of the company's strategic goals to each employee and his/her involvement in the relevant business processes are needed. To assess the correctness of these actions, sets of financial and nonfinancial criteria of all the business processes are developed for the enterprise as a whole and for its departments and employees. These criteria are called Key Performance Indicators (KPI) and they are largely connected with the system of motivation.

All the system covers four perspectives (directions), being the main groups of strategic goals, the achievement of which is evaluated by key performance indicators:

- Finance
- Customers
- Business processes
- Training and development

In the system of balanced scorecard, strategic goals of the enterprise are presented on the strategy map in the form of goal decomposition. The strategy map is a description of the strategy by cause–effect relationships at every level of the company's management. Table 1.9 shows an example of such a map.

The main difference between the balanced scorecards of effectiveness and arbitrary set of indicators lies in the fact that all KPIs, included in a balanced system, firstly, focus on the strategic goals of the enterprise and, secondly, are interrelated and grouped according to certain criteria. The KPI should be determined based on the critical factors of company's success. They can be either absolute (total revenue) or relative (profitability), and for many of the indicators

Table 1.9 Example of strategy map [based on Chernikov (2011)]

Perspectives	Motivation	Performance	Goals	Initiatives
Finance	Business profit gain	Profit	20 % growth	Relevant programme
		Annual sales	12 % growth	
Customers	Product quality, associated with its trademark	Amount of product return	Decrease by 50 % annually	Quality management programme
		Increase in number of customers	60 %	Customer loyalty improvement programme
		Increase of sales per customers	20 %	
Business-processes	Quality improvement of the manufactured products	Percentage of sold products from the produced products	70 %	Production development programme
		Stock reserve in comparison to the schedules one	85 %	
Training and development	Training of personnel	Percentage of trained personnel	1 year—50 % 2 year—75 % 3 year—90 %	

the absolute value is not so much important as their dynamics (for example, for the volume of overdue receivables).

In capacity of KPI it is reasonable to use the most mutually independent parameters, since it is their combination will describe the system as a whole best of all. Among the KPI there should be no directive settable parameters. For example, the payroll budget should not be used as a KPI since this figure is directive. At the same time, the ratio of sales to the salary paid is suitable as KPI since with this ration it can be determined whether the marketing is running effectively.

A wide variety of examples of KPI sets for the various business processes and methods of motivation is reported. For example, for the process of developing master plan the following combination of efficiency indices can be recommended (Mauergauz 2007):

- Production output
- Margin
- Current assets
- Capacity utilization percent
- Sum of penalties due to nonfulfilment of contracts
- Sum of lost profit
- Work-in-process
- Average scope of production order
- Average time for contract fulfilment
- Current planning horizon

For a specific subdivision within the BSC/KPI a special card is developed, which contains not only a list of key performance indicators but also the so-called operating indicators. Operating indicators are subject to direct measurement and characterize the quality of the ongoing process. These indicators directly influence on key performance indicators, although not always this connection can be established mathematically. In fact, the key indicators represent some aggregated set of operating indicators. As an example (Table 1.10) a set of indicators for material warehouse can be presented.

Key performance indicators are often used as a basis for development of employee incentive programme. In addition, for each employee a specific set of KPI is established and assessment is done periodically, which directly affects on his/her salary. Detailed consideration of such a system is beyond the scope of this book.

1.7 Utility of Quality Parameters

In the previous paragraph the different ways of creating a set of criteria to assess the quality in the supply chain were discussed. However, despite the fact that the balanced scorecard, even according to its name, must have some possibilities of such balancing, this issue is not resolved as such. Indeed, creation of a reasonable balance of performance indicators is difficult because, firstly, they have very different dimensions, and, secondly, they can be contradictive.

In some cases, it is possible to reduce different rates of the same dimension—often to the time or cost. For example, in Jahn's paper (2007), which deals with the interaction of several enterprises in the supply chain, each of the quality criteria of business processes is associated with a penalty function in value terms. This approach allows aggregating penalties by all indicators and make appropriate analysis.

However, as a rule, reducing all the criteria to cost value cannot adequately reflect the different aspects of the business process being evaluated. Only the property of any criteria that always exists and can match the reality is its utility. In the academic literature, the meaning of "utility" and "value" is differentiated, but in this book we assume that these concepts are equivalent.

1.7.1 Concept of Utility

The routine practice of decisions made by a person, who is in a certain system of relations with the environment, technology, and society, shows that the nature of any decision depends essentially on the opportunities that may be obtained based on the decisions made, as well as on the initial state at the time of decision making.

As an illustration, we present two examples of the game, the result of which is the outcome of tossing a coin. In this case, the probability of any part is obviously the same. Let us suppose that before the game the player has \$10 and during the

Table 1.10 Indicators of BSC/KPI system for warehouse [based on Bubnov (2010)]

Business-process and its holder	Regulation (subprocess)	Key indicators	Operational indicators
Contractual work with suppliers; executive director	Preliminary work with suppliers	Accuracy of preliminary evaluation of suppliers; Efficiency of preliminary work with suppliers	Supplier's response rate, hour; Commercial evaluation of supplier, point; Production logistic evaluation of supplier, point; Final evaluation of supplier, point; Enforceability level of protocol of intent, %.
	Contractual work with suppliers	Legitimacy and efficiency of supply contracts (% of contract clauses in version of Customer/Contractor); Full time for contracting, days	Time for contract correction, hour; Time for elaboration of commercial offer, hour; Time of supply deviations analysis, hour; Accuracy of supply schedule, hour; Estimation time, hour; Time for contract sending, min; Supplier's response rate, hour; Time for amendments comparison, min; Time for correspondence, days; Time for transferring documents to supplier, min.

Safety stock replenishment; Executive director	Drawing up orders for suppliers	Safety stock level, days of work
		Accuracy in time of order fulfilment, days
			Stocktaking, min;
			Estimation time, min;
			Accuracy in time of order fulfilment, days

Acceptance of goods for storage; Warehouse manager	Acceptance of goods for storage	Shortage at delivery, %; Goods misgrading, %; Costs for grading of the batch; Rejected deliveries, %; Average time for transport processing, min.	Downtime when waiting for unloading; Quantity of unloaded pallets, pcs./h per person; Time for transferring documents, min; Quantity of defect pallets, pcs.; Losses due to rejects; Accuracy of delivery lot identification, %; Shortage at delivery, %; Goods misgrading, %; Observance of rule "First in—first out", %; Time for grouping and moving the pallets to the storage area, min; Time for processing the shipment documents, min; Processed documents with mistakes, %; Time for loading tare into the vehicle, min.

(continued)

Table 1.10 (continued)

Business-process and its holder	Regulation (subprocess)	Key indicators	Operational indicators
	Placement of goods in the warehouse	Observance of rule "First in—first out", %; Expired goods batches in the storage area; Loss due to goods returned; Loss due to incorrect placement; Average time for moving one pallet, min.	Incorrect placement of pallets, %; Time for moving faulty goods to the rejected area, min.; Accuracy in moving the pallets, %; Average time for moving one pallet to storage area, min; Time for execution of certificate of unserviceability, min; Write-off of goods due to return and natural loss, %; Time for moving from the rejected area to waste container, min.

game in case of winning this amount is increased by the same amount and in loss it is reduced to zero.

In the first example, the player is going to spend the available money for food, which may be purchased for $10 as well (though in smaller amounts). In the second example, assume that a player is full and prefers to spend the available money for a movie ticket, which costs exactly $20.

It is obvious that in the first case the game is a serious threat to the player, as in case of loss, he risks being without any money for food. In the second case, the game is quite reasonable, because losing changes very little in the player's life and the possible gain allows having a good time. It appears that the difference in the initial situation leads to different purposes in behaviour and different attitude to the winning or losing, even if these values are the same in different cases.

In each of the examples two results are possible—winning I_1 and losing I_2, each of which, as we have seen, has a different value in different cases. The measure of this value is called "utility". It is known that under some rather general conditions each expected result of possible action I corresponds to a certain number $u(I)$ called as utility of action I. Set of numbers $u(I)$ forms a utility function characterized by two important properties.

- First property of utility function.

  $u(I_1) > u(I_2)$ then and only then, when a person making a decision prefers I_1, but not I_2.
- Second property of utility function.

  If I is an expected result of possible actions, moreover which with probability P this expected result will be I_1, with probability $1 - \mathrm{P}$ it will be I_2, then

$$u(I) = \mathrm{P}u(I_1) + (1 - \mathrm{P})u(I_2). \tag{1.2}$$

1.7.2 Typical Utility Functions

Figure 1.12 shows a diagram of the utility function, which Chernoff and Moses (1959) call as function of Campbell. Along the horizontal axis in this figure the value of capital in some units is plotted. The capital is available for either Campbell or may be formed by the actions of the latter. Ordinates of the utility value of this capital are determined according to the representations of Campbell depending on its current amount, and the combination of these ordinates forms the diagram of utility function. The diagram in Fig. 1.12 makes it possible to analyse different cases of actions of a decision maker in different situations and assess their reasonability.

Suppose that in the initial state Campbell has a capital of 8 units and this capital corresponds to utility equal to 1. We consider two variants of the game with different conditions. In the first case, the variant in terms of which with probability 0.5 you can win 2 units and just as likely to lose 1 unit. Apparently, this game can be favourable for Campbell. Indeed, since the amount of capital in the event of winning will be equal to 10 units, the utility according to the diagram in Fig. 1.12 will be equal to 1.18. In case of loss, the capital decreases to 7 units and the utility will be equal to 0.86. Using formula (1.2), we obtain

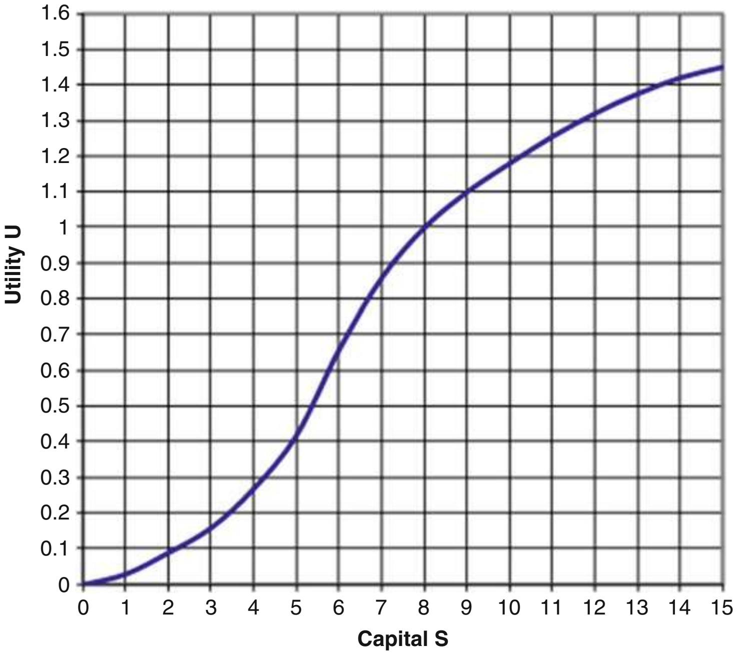


Fig. 1.12 Utility function of Campbell

$$u(I) = 0.5 \times 0.86 + 0.5 \times 1.18 = 1.02,$$

which means increase of utility from the initial value equal to 1 by 0.02 and confirms the assumption of reasonability of this variant.

In the above variant, the game was clearly unfair because with the same probability of winning and losing the winning amount was much bigger. Consider now the case of the so-called fair game. Suppose that three equally possible variants are possible in this game, wherein the gain is possible in only in one of them. The magnitude of the gain is 2 units and the amount of loss in either of the two variants is 1 unit. This game is valid in the sense that the expectation E equals to

$$E = 1/3 \times 2 + 2/3 \times (-1) = 0.$$

By determining the utility of this game using Campbell diagram

$$u(I) = 1/3 \times 1.18 + 2/3 \times 0.86 = 0.97,$$

which shows obvious inexpedience of the game as the utility decreases from the initial value 1 to 0.97.

For graphic illustration of the decision in the case of the first variant of the game, in Fig. 1.13 chord of Campbell curve is laid between points A and B, the corresponding capital values 7 and 10. The decision itself is displayed by point 1 on chord AB and has a utility of 1.02, which is greater than the initial value of 1. In the second variant of the game the value of capital in winning and losing is the

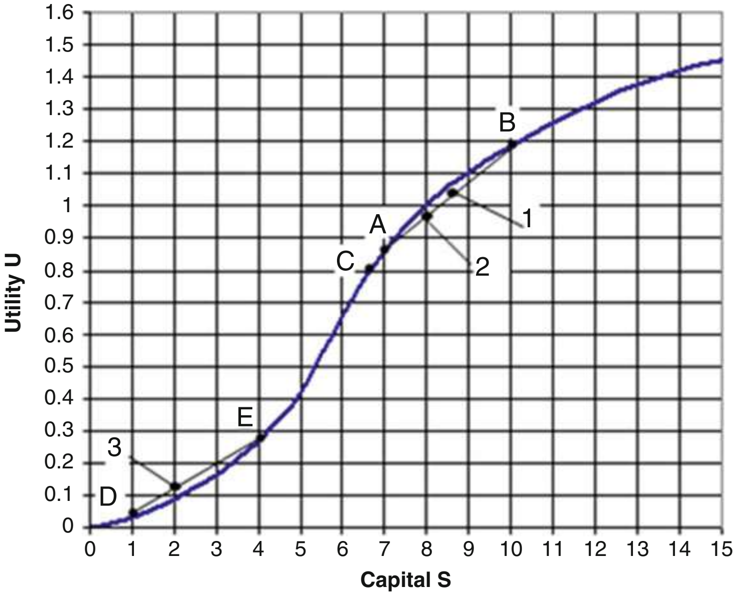


Fig. 1.13 Variants of the game for the Campbell curve

same as in the first one, so it uses the same chord AB. However, in this case the decision is displayed by point 2 with a utility of 0.97, which is less than 1.

The inexpedience of the fair game in the second of the considered cases is due to the nature of the utility function on part CAB. As seen from the diagram, in this part of curve, the slope of the tangential of the curve with respect to the abscissa axis and, accordingly, the first derivative of the utility function gradually decreases. Such curve is known to be called convex and its second derivative is negative. The sign of the second derivative is opposite to the so-called aversion to risk. This means that on the convex part of the Campbell curve the decision maker cares more about saving his capital than for its increase, i.e. in this situation, the person is averse to risk.

Let us consider now the third variant of the game, when its conditions match the conditions of the second variant, but the player has capital of not 8 units but only 2 units, and the utility of such capital is 0.09 according to the Campbell curve. In this case, the possible limit values of the capital are 1 and 4 units. These values correspond to points D and E in Fig. 1.13, between which the chord is laid. The value of the utility function is equal to

$$u(I) = 1/3 \times 0.28 + 2/3 \times 0.04 = 0.12,$$

which confirms the obvious reasonability of the game in this variant. This result is due to the fact that within part DEC the utility function is concave, the second derivative is positive, and risk aversion is negative, respectively.

Thus, using these examples we can conclude that point C, being the inflection point, separates the whole set of possible decisions by two areas where the player behaves in a different way: on the convex part caution and risk aversion dominate and on the concave one the risk is quite admissible. The Campbell utility function will be used significantly in Chap. 2 in construction of some mathematical models.

Figure 1.14 shows some of the possible utility functions for different criteria in the supply chains and production (Mauergaz 2007).

The curve in Fig. 1.14a is typical for indicators such as revenue and profit, the utility of which grow with the growth in production, but the rate of this growth decreases gradually. The curve in Fig. 1.14b belongs, for example, to the disutility

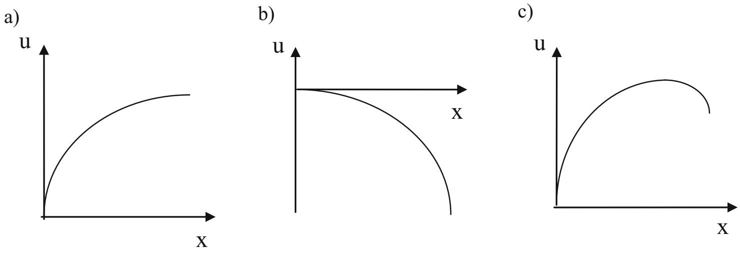


Fig. 1.14 Possible utility functions of criteria

of necessary current assets, which with increasing volumes cannot just grow, but grow with increasing speed.

Both of these curves are monotonically changing; alternatively to them, the curve on Fig. 1.14c is not monotonic, i.e. its utility increases to a certain value of the abscissa and then starts decreasing. This situation is typical for equipment load criteria, which usually has the best value not at full load but at a lesser value. Note that all the curves in Fig. 1.14 have a negative second derivative and, accordingly, a positive risk aversion.

1.7.3 Utility Functions in Business Process Quality Evaluations

Each of the utility functions, presented in Sect. 1.7.2, describes the change in the utility of one variable—profit, amounts of used current assets, machine utilization, etc. These utility functions are called one dimensional. At the same time, as it follows from the balanced scorecard described in Sect. 1.6.3, the objectives of the enterprise are possible to achieve only with sufficiently high values of a set of indicators, given in the strategic map, for example—Table 1.9. Although the balanced scorecard allows determination of the set, but it does not define the level of importance of an indicator or take into account the possible interaction between the indicators.

To determine the joint effect of indicators set on the quality of the process under consideration, it is obviously necessary:

- Construct the utility function for each indicator.
- With this function define the indicator’s utility by its current value.
- Using any method, establish the total utility of the existing values of the indicators in this set.

The latter is possible, generally speaking, by any of the two methods. In the first case, the utility of each indicator can be considered as an independent criterion and for assessing their joint effect can use methods for solving multiobjective problems. This method will be discussed further in Chap. 4. The second method is to construct a multidimensional utility function, which is a certain combination of several one-dimensional functions. An example using this method is given below.

The article of Youngblood and Collins (2003) describes the approach of the joint study of the balanced scorecard (BSC) and Multi-Attribute Utility Theory (MAUT), which includes four stages of the business process modelling. On the first stage, BSC map is developed and measurable criteria are defined for each of the four directions of BSC, as described above.

On the second stage, a description of one-dimensional utility functions for each criterion is made. Besides, the lowest value of utility is deemed to be 0 and the highest one is 1. On the utility curve, the lower and upper limits of admissible values of utility are set. The convexity or concavity of the curve of utility, as described above, determines the sign of risk aversion. The utility function having the form of a straight line has zero risk aversion.

On the third stage, the criteria are ranked in order of importance and the weight of each criterion among the criteria in this direction is set as a percentage. The sum of these weights should be equal to 100 %. The range of possible values for each criterion, with which this criterion is independent of other criteria, is determined. Utility functions of several independent criteria can be combined into one group considering the weight of each factor, forming the so-called additive utility function. If the criteria are dependent on each other, their utility functions are multiplied together to form the so-called multiplicative function.

On the last, fourth stage of the study the BSC map is reviewed for compliance of the numerical description of the business process utility with the ideas of a decision maker, which play different scenarios of changes in the business process. If necessary, some adjustments are performed on utility functions and maps in general. The resulting map is called Utility-based Balanced Scorecard (UBSC).

In the paper of Youngblood et al. (2004), the mentioned approach is illustrated by a practical example. This study examines the production activities of one of the branches of a large company involved in the wholesale trade all over the world with the nomenclature of approximately 500,000 items. With this nomenclature, this department performs nearly 25 million transactions a year.

Table 1.11 shows UBSC map, including data on the criteria utility, their impact in all directions, and the importance of these directions. Department management believes that business processes are the most important direction, so for this group of criteria the greatest weighting factor is set equal to 33 %. The financial direction with a weighting factor of 27 % is the second in importance. In both groups, the weighting factor is the same and equals to 20 % in each group. For the first three directions, all indicators and their parameters are listed in Table 1.11 and for the latter group only general data of the group as a whole is given. The meaning of each of the indicators is generally clear from their names; the quantity of the current value was determined at the time of the study.

The values of the entries in columns 3–7 of Table 1.11 reflect the character of relevant utility function, where the value of index “insufficient” corresponds to utility value equal to 0. When the index values equal to “below average”, “average”, “above average”, the utility takes values of 0.25, 0.5 and 0.75, respectively. Utility is equal to 1 for the “best” values of the index. The current value of the utility matches the current value of the index. Figure 1.15 presents two examples of utility functions for indicators of Table 1.11.

Each of the utility curves in Fig. 1.15 has three parts: a part with 0 utility, a part with utility equal to 1, and the actual utility curve between these two parts.

The utility of the average price deviation from the basic values (Fig. 1.15a) increases linearly with decreasing prices and reaches one in the case price decrease by 10 %. Point K marks the current state of the system, the utility of which is close

Table 1.11 UBSC card for the studied department of a company [based on Youngblood et al. (2004)]

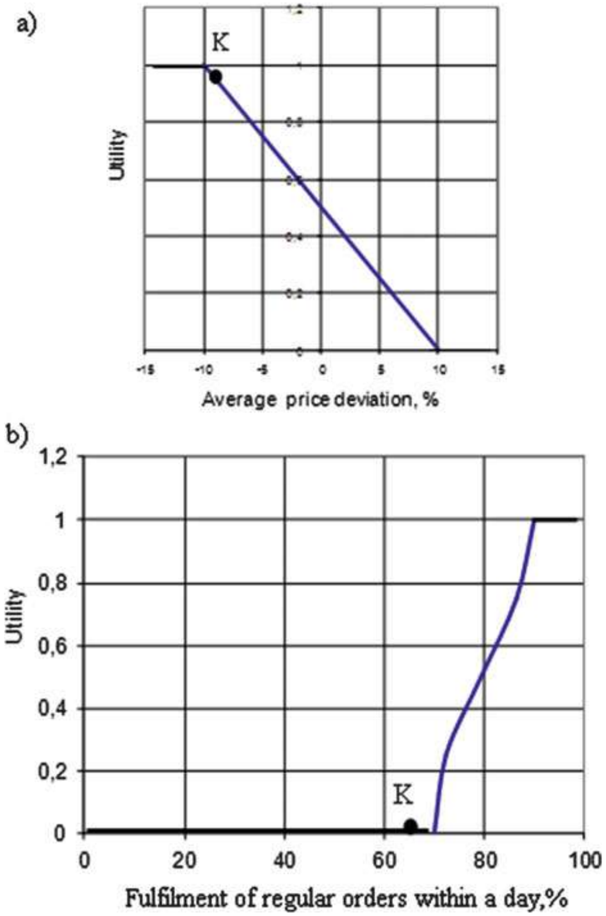
Indicator	Current value	Insufficient	Below average	Average	Above average	Best	Current utility	Weight in group	Weighted utility
Finance—weight factor of direction 27 %									
Fulfilment of requests of the department from the company's budget, %	100	92	93	96.5	99	100	1.0	12.5	0.125
Deviation from the value of supply forecast, %	70	10	7	4	2	0	0.0	12.5	0.0
Average deviation of price, %	-9.2	10	5	0	-5	-10	0.96	25	0.24
Percentage of implemented projects for 3 years	125	75	90	100	110	125	1.0	25	0.25
Percentage of business improvements, completed in time	100	90	95	99	99.5	100	1.0	12.5	0.125
Percentage of solved problems on audit for a year	100	90	95	100	100	100	0.5	12.5	0.06
Total in direction	0.8								
Business-processes—weight factor of direction 33 %									
Percentage of clients requests with regular urgency	90	80	85	90	95.5	98	0.5	20	0.10
Fulfilment of urgent orders within 1 day, %	75	80	85	90	95.5	98	0.0	20	0.0
Fulfilment of regular orders within 1 day, %	65	70	72.5	79.5	86.5	90	0.0	20	0.0
Average number of accounting units per transaction	298.4	250	262.5	288	313	325	0.6	20 %	0.12
Quality of making investments in business, %	100	88	89.5	93.5	97.5	100	1.0	20	0.2
Total in direction	0.42								

(continued)

Table 1.11 (continued)

Indicator	Current value	Insufficient	Below average	Average	Above average	Best	Current utility	Weight in group	Weighted utility
Clients—weight factor of direction 20 %									
Index of meeting the clients' requirements, %	77	75	77	82	87	90	0.25	16.7	0.042
Actual response to the clients' requests, %	77	75	77	82	87	90	0.25	16.7	0.042
Timely fulfilment of contractual obligations, %	82	80	82	87	92	95	0.25	33.3	0.083
Requests where accidental changes can occur, %	65	50	65	85	95	100	0.25	16.7	0.042
Percentage of accidental changes requiring immediate response	83.3	70	83.3	90	96.7	100	0.25	16.7	0.042
Total in direction	0.25								
Training and development—weight factor of direction 20 %									
Total in direction	0.24								
Total in all directions	0.45								

Fig. 1.15 Examples of utility curves for the indicators in Table 1.11



to 1, i.e. to the most desirable value. Figure 1.15b shows a diagram of utility change of indicator "Fulfilment of regular orders within 1 day, %". As seen from this diagram, the actual fulfilment of 65 % is unsatisfactory value with utility equalling to 0.

Each *j*th indicator K_{ij} in direction (group) *i* has its own weight within group ν_{ij} specified in the ninth column. The sum of all weights in one direction is 100 %. The last column contains the product of the current utility value per its weight in the direction (group). Total utility of the studied system is determined by summing the amount of utility n_i in each of the four directions taken with consideration of weight w_i of the direction, meaning

$$u = \sum_{i=1}^{4} w_i \sum_{j=1}^{n_i} \nu_{ij} K_{ij}. \tag{1.3}$$

In this example, the total utility of the system equals to 0.45, which is quite a moderate value in comparison with the maximum possible value 1. It is believed that the system quality is "average".

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Mathematical Models 2

2.1 Simplest Planning Models

Generally, production and supply chain planning shall provide answers to the following three main questions:

- What products will we need?
- How much of each product will we need?
- When will we need this?

As a general matter, this is quite difficult to give answers to these questions in the current market environment. Actually, all methods, techniques, and solutions described herein have been developed exactly for this purpose. However, they do not provide a fully exhaustive and comprehensive solution to this challenge. Complexity and sometimes the whole possibility of a planning task depend on the complexity of a certain production situation. Therefore, it is methodologically practical to consider various task cases in the ascending order of complexity. So, we shall start with the simplest mathematical model (described in Sect. 2.1.1).

2.1.1 Classical Supply Management Model

Let us consider a warehouse where a certain product is accepted, stored, and gradually issued to consumers. Figure 2.1 shows the time plot of the product quantity changes.

According to this plot, the product in the quantity Q is delivered to the warehouse at a constant time frequency T which exactly equals the time of consumption of the accepted product batch, i.e.

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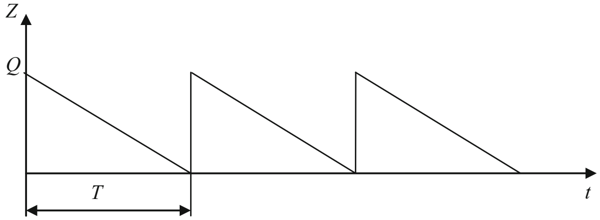


Fig. 2.1 Product availability at warehouse

$$T = Q/D, \tag{2.1}$$

where D is the quantity of product consumed during a certain time unit, e.g. per day.

The plot shown in Fig. 2.1 presupposes a number of assumptions as follows:

- Demand D is constant and continuous
- Delivery batch quantity Q and delivery time frequency T are constant values
- Lack of stocks (stock shortage) is not acceptable.

Besides, the smooth warehouse operation according to this plot (Fig. 2.1) is possible provided only the delivery order lead time is known and constant. In such a case, we can make a delivery order at a certain time point which will ensure that the delivery will take place exactly by the time of stock depletion at a warehouse.

It is evident that as we have only one product there is no need to find an answer to the first question, i.e. "What products will we need?". Moreover, the correlation expressed as a formula above (2.1) leads us to the conclusion that the planning task requires a solution to one question only, i.e. "What quantity the delivery batch shall be?" (and, consequently, the quantity of a respective delivery order).

This task is solved under Harris-Wilson EOQ (Economic Order Quantity) model. According to this model, the order quantity shall cover the minimum possible costs of the warehouse owner for a certain period, i.e. year, month, etc. A certain component c of such costs depends on the batch quantity and includes two components as follows: the ordering cost c_o and holding cost c_h. Note that such costs do not change in time, i.e.

$$c = \frac{Q}{2} c_h + \frac{D}{Q} c_o. \tag{2.2}$$

The first component in the above formula (2.2) describes the cost of storage for an average product quantity $Q/2$ during the storage period. The second component describes the cost of orders made during this period.

By calculating the derivative of the cost c using the order quantity Q and equating this to zero, we will obtain the equation which will allow calculating the order quantity with minimum possible costs:

$$\frac{dC}{dQ} = \frac{C_h}{2} - \frac{D}{Q^2}C_o = 0. \tag{2.3}$$

Let us use this formula to derive the Economic Order Quantity:

$$Q^* = \sqrt{\frac{2C_oD}{C_h}} \tag{2.4}$$

And costs associated with such order quantity:

$$C^* = \sqrt{2DC_oC_h.} \tag{2.5}$$

Let us take an example where the demand D is 300 units per month, the cost of one order c_o is \$2000, and the storage cost c_h is \$200 per month. Here, the Economic Order Quantity will be as follows:

$$Q^* = \sqrt{\frac{2 \times 300 \times 2000}{200}} = 77.5.$$

In practice, this is often the case that the quantity of ordered product must be, firstly, an integer quantity and, secondly, multiple of the product quantity in one package, etc. In such a case, we need to find out to which extent costs will increase as a result of order quantity rounding. Such a change is called model sensitivity. In order to estimate the model sensitivity, we need to make a relation of the cost under formula (2.2) to the minimum cost value (formula 2.5).

$$\frac{C}{C^*} = \frac{Q}{2}\sqrt{\frac{C_h}{2DC_o}} + \frac{1}{2Q}\sqrt{\frac{2DC_o}{C_h}} = \frac{1}{2}\left(\frac{Q}{Q^*} + \frac{Q^*}{Q}\right). \tag{2.6}$$

Figure 2.2 shows the relation of the costs according to formula (2.6) to the order quantity variation. According to this diagram, in the neighbourhood of the optimal solution, i.e. $Q/Q^* = 1$, the cost value has rather a weak relation to the order quantity deviation from the optimal value. For instance, if the order quantity increases by 20 % as compared to the optimal order quantity, the related costs during the calculation period will increase only by 1.7 %. Such a low sensitivity of costs to the order quantity allows broad deviations from the optimal quantity without any material losses.

2.1.2 Continuous Linear Optimization Model

The task solution described in the above paragraph is based on a non-linear relation of the cost to the order quantity. Such a relation allows purely analytical

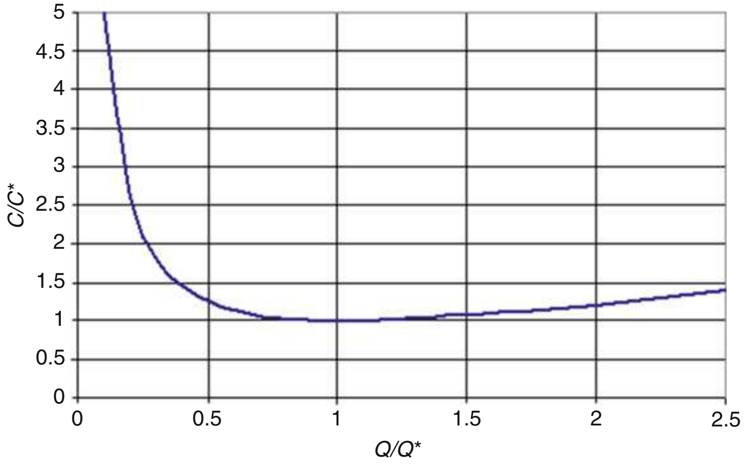


Fig. 2.2 Relation of relative costs to the ratio of order quantity to Economic Order Quantity

calculations of the minimum position of the target optimization function, i.e. costs, without any constraints imposed on the main variable, i.e. order quantity. Unfortunately, the above solution is practically the only planning task that would allow this. In the vast majority of cases, it is impossible to find a purely analytical solution to a planning task and we have to find computer-assisted numerical solutions.

The simplest case is a mathematical model where a target optimization function often expressed by the cost has a linear relation to independent variables. In order to find an optimal solution to a planning task, we will have to impose certain constraints on such variables. If such constraints are linear, the model is linear as well and the optimization task can be solved by applying the linear programming. In the process of optimization, independent variables can change continuously or discretely. Note that in the latter case it is much difficult to find a solution. In this section, we will consider the continuous linear optimization only.

Let us consider the task under which we need to develop a production plan to produce three sorts of paint and to determine the optimal production output for each paint sort during the nearest month. The objective function of a mathematical model is a potential gain from the sales of all produced paints. Note that different sorts of paint yield different gain.

From the point of view of a maximum gain, it would seem reasonable to produce paint sorts that yield the maximum gain and, preferably, as much as possible. However, a host of factors confines this natural motivation of a producer. First of all, manufacturing capacities allow producing a limited quantity of paint. Let us assume, for example, that such limit is 1000 tons of all paint sorts per month. Besides, production of each sort of paint requires three types of raw stock and the availability of each is limited for a planned month. And last but not the least, the production output for each sort of paint depends on the conditions of sales and

Table 2.1 Product parameters and constraints

Parameter	Paint no. 1	Paint no. 2	Paint no. 3
Gain, dollars per ton	200	280	250
Quantity of raw stock 1, kg per ton	150	220	170
Quantity of raw stock 2, kg per ton	80	70	100
Quantity of raw stock 3, kg per ton	100	80	90
Minimum number of tons per month	400	–	–
Maximum number of tons per month	–	200	–

Table 2.2 Raw stock constraints

Parameter	Value
The maximum possible cumulative production output for all paint sorts, tons per month	1000
The maximum possible consumption of raw stock 1, tons per month	160
The maximum possible consumption of raw stock 2, tons per month	70
The maximum possible consumption of raw stock 3, tons per month	70

marketing. Generally, only small quantities of expensive paint are sold, while cheaper paints can be pre-ordered, etc.

The product parameters and constraints are described in Table 2.1, and raw stock constraints are shown in Table 2.2. The quantity of a water-based is paint assumed unlimited.

Using Tables 2.1 and 2.2, we can create a mathematical model which shall contain three basic components as follows:

- Variables whose values need to be calculated, i.e. find solutions to the task
- Objective function whose value depends on the values of the variables (indicating the solution objective, i.e. minimum or maximum)
- Task constraints expressed as mathematical inequalities (or equalities).

Let us set x_1, x_2, and x_3 as variables that determine monthly production outputs for paint sort No. 1, No. 2, and No. 3, respectively. Using gain values from Table 2.1, we will obtain the expression for the objective function, i.e. cumulative gain.

$$f(x) = 200x_1 + 280x_2 + 250x_3, \tag{2.7}$$

which shall be maximized.

According to Tables 2.1 and 2.2, the mathematical model provides for a host of constraints as follows:

Production capacity constraint:

$$x_1 + x_2 + x_3 \leq 1000. \tag{2.8}$$

Marketing constraints:

$$x_1 \geq 400; \; x_2 \leq 200. \tag{2.9}$$

Raw stock consumption constraints:

$$\begin{aligned} &0.15x_1 + 0.22x_2 + 0.17x_3 \leq 160; \\ &0.08x_1 + 0.07x_2 + 0.1x_3 \leq 70; \\ &0.1x_1 + 0.08x_2 + 0.09x_3 \leq 70. \end{aligned} \tag{2.10}$$

Another implicit constraint is that variables of paint production outputs shall be non-negative, i.e.:

$$x_1 \geq 0; \quad x_2 \geq 0; \quad x_3 \geq 0. \tag{2.11}$$

The first of these conditions (2.11) is actually redundant as it is overlapped by a stronger condition (2.9). Therefore, only two conditions from (2.11) remain in force:

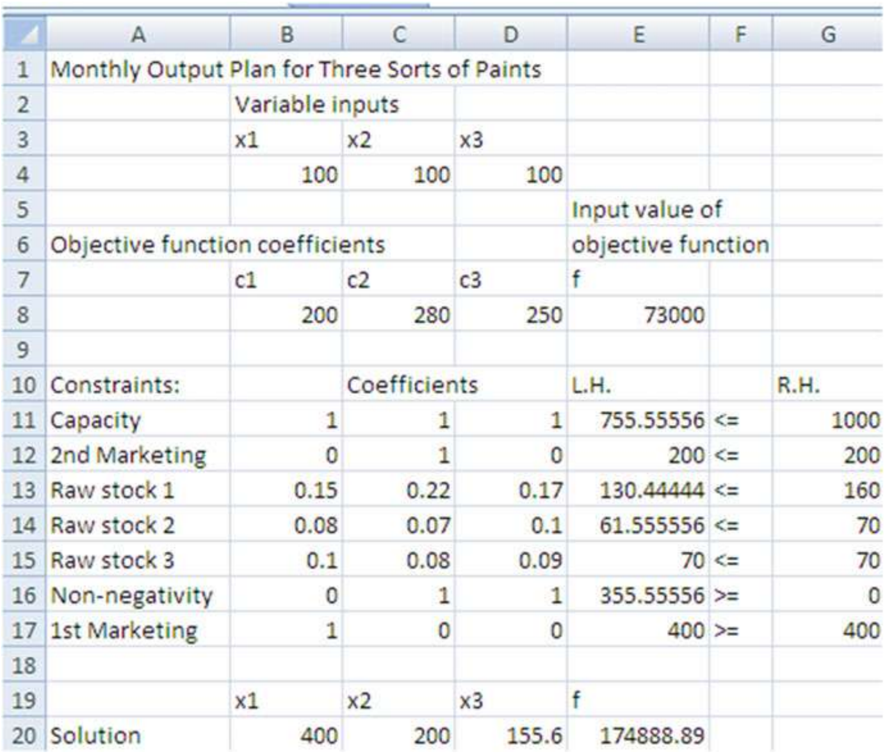
$$x_2 \geq 0; \quad x_3 \geq 0. \tag{2.12}$$

The resulting mathematical model is somewhat more complicated than the model described in Sect. 2.1.1 above as it takes account of planning for multiple products rather than one product. However, here as well, types of produced products are pre-set rather than determined during planning. And the most important thing is that this mathematical model does not determine the production sequence for such types. Similarly to the model described in Sect. 2.1.1, this model takes into account only the quantities of products planned for production in a respective planned period and disregards the production details. Therefore, this task can also be deemed as simplest.

Expressions (2.7)–(2.10) and (2.12) describing the created mathematical model show that each expression contain first-degree variables, i.e. the mathematical model is linear to the variables. As mentioned above, a solution is found using any method of linear programming. It is practically feasible to use MS Excel which has respective built-in methods. Figure 2.3 shows the solution to the example task.

The upper part of MS Excel table shown in Fig. 2.3 contains randomly selected input values of task variables. They are followed by coefficients of the objective function; then, the input value of the objective function is calculated based on expression (2.7).

Lines 11–17 list all the constraints imposed on the variables according to inequalities (2.8)–(2.10) and (2.12). Such constraints are grouped by the correlations of the left-hand and right-hand sides of the inequalities. Line 20 shows variable values and the resulting value of gain for an optimal solution. The solution is found by initiating the Solver add-in for a continuous linear optimization case.



	A	B	C	D	E	F	G
1	Monthly Output Plan for Three Sorts of Paints						
2		Variable inputs					
3		x1	x2	x3			
4		100	100	100			
5					Input value of		
6	Objective function coefficients				objective function		
7		c1	c2	c3	f		
8		200	280	250	73000		
9							
10	Constraints:		Coefficients		L.H.		R.H.
11	Capacity	1	1	1	755.55556	<=	1000
12	2nd Marketing	0	1	0	200	<=	200
13	Raw stock 1	0.15	0.22	0.17	130.44444	<=	160
14	Raw stock 2	0.08	0.07	0.1	61.555556	<=	70
15	Raw stock 3	0.1	0.08	0.09	70	<=	70
16	Non-negativity	0	1	1	355.55556	>=	0
17	1st Marketing	1	0	0	400	>=	400
18							
19		x1	x2	x3	f		
20	Solution	400	200	155.6	174888.89		

Fig. 2.3 Solving a continuous linear optimization task using MS Excel

If we compare the resulting gain value shown in E20 cell with the input value in E8 cell, we will see that the optimal solution provides for a substantial gain increase. However, this solution is calculated under a host of imposed constraints and this poses a question to which extent the optimal solution depends on the values of such constraints. First of all, we need to determine which constraints are material for this task.

Constraints shall be deemed material or coupled if their left-hand values equal to right-hand values under an optimal solution. In Fig. 2.3, there are three coupled constraints, i.e. two marketing constraints and one raw stock 3 constraint. Production capacity constraint is not coupled and this actually means under-utilization of available production capacities. Naturally, in order to evaluate the quality of a found solution and the viability of any changes, we need to analyse sensitivity of such solution (somewhat similar to sensitivity analysis as described in Sect. 2.1.1).

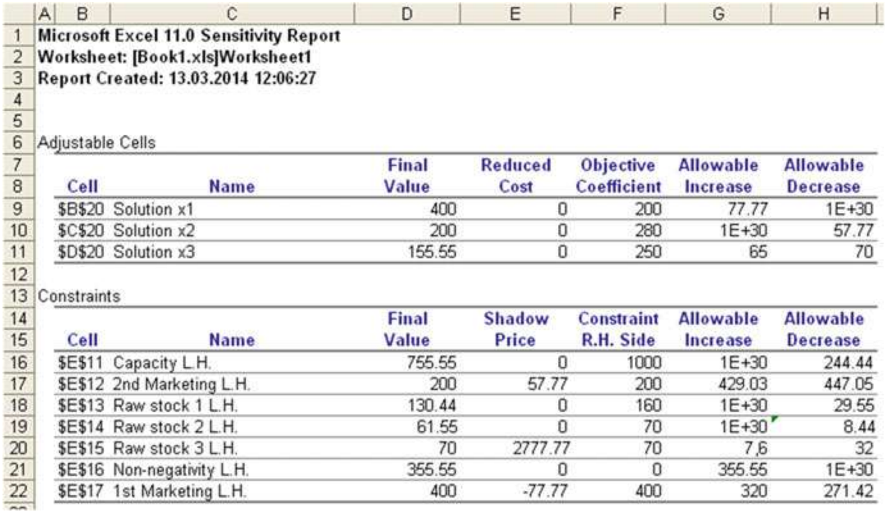
A more detailed sensitivity analysis may provide answers to the following questions:

- To what extent can model parameters change without any material changes to the solution?
- Which constraints are coupled (and under what conditions) and which are not?
- How will changes in right-hand sides of coupled constraints affect the task solution?
- If the value of any variable resulted from optimization equals to zero, under what conditions such value can exceed zero?

For the purpose of solution review and potential planning improvement, we can use a so-called sensitivity analysis report created in MS Excel after the optimal solution has been found (Fig. 2.4).

This report includes two tables. The upper table lists variable cells and respective solving results. The Reduced Cost parameter is not equal to zero only if the resulting optimal solution for a respective variable is at its possible value limit, for instance, is equal to zero. The Reduced Cost value other than zero shows to which extent the objective coefficient will deteriorate if the variable value is assumed as non-optimal and equalling to 1 rather than optimal and equalling to zero. The Allowable Increase and Allowable Decrease columns show to which extent a respective objective coefficient may change provided that the optimal values of variables remain unchanged.

The lower table contains constraint cells and their respective values for a resulting solution. For each coupled constraint, the table shows the so-called Shadow Price which indicates to which extent the objective coefficient will change if the right-hand side of a respective constraint increases by 1. The Allowable Increase and Allowable Decrease columns show the change limits for right-hand side of a respective constraint within which the Shadow Price value applies.



Microsoft Excel 11.0 Sensitivity Report
Worksheet: [Book1.xls]Worksheet1
Report Created: 13.03.2014 12:06:27

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$20	Solution x1	400	0	200	77.77	1E+30
$C$20	Solution x2	200	0	280	1E+30	57.77
$D$20	Solution x3	155.55	0	250	65	70

Constraints

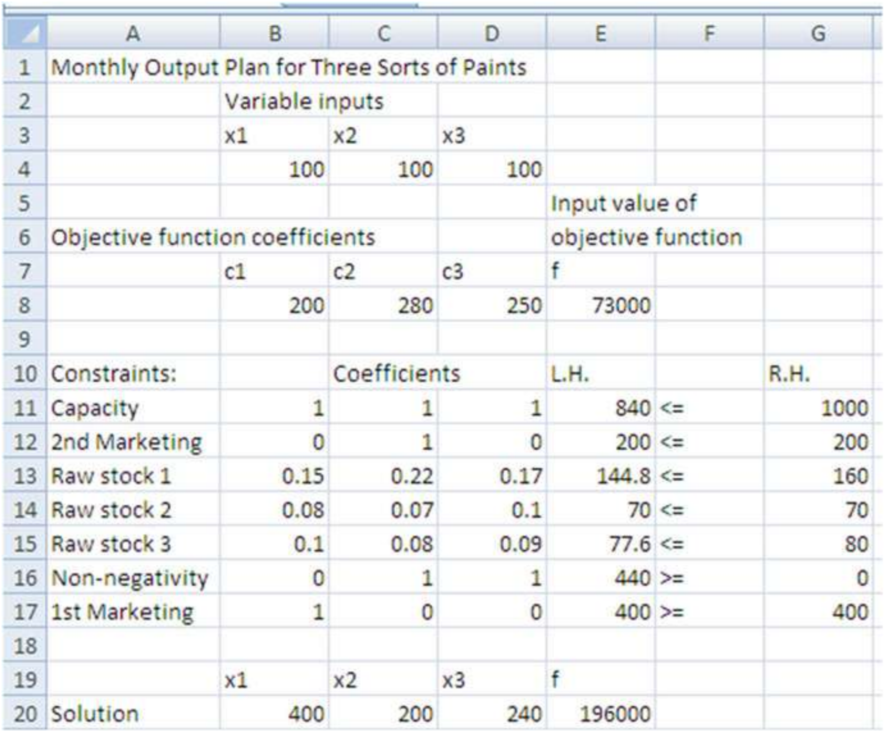
Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$E$11	Capacity L.H.	755.55	0	1000	1E+30	244.44
$E$12	2nd Marketing L.H.	200	57.77	200	429.03	447.05
$E$13	Raw stock 1 L.H.	130.44	0	160	1E+30	29.55
$E$14	Raw stock 2 L.H.	61.55	0	70	1E+30	8.44
$E$15	Raw stock 3 L.H.	70	2777.77	70	7,6	32
$E$16	Non-negativity L.H.	355.55	0	0	355.55	1E+30
$E$17	1st Marketing L.H.	400	-77.77	400	320	271.42

Fig. 2.4 Optimization results sensitivity analysis

If we use the sensitivity analysis report shown in Fig. 2.4 to answer the above questions on solution sensitivity, the upper table will provide possible variations of objective coefficients for each variable under which the solution will not change. According to the lower table, constraints shown in lines 17, 20, and 22 are coupled. As under the optimal solution none of the variables equals to zero, the last sensitivity question does not matter in this example.

Possible changes of the right-hand sides of constraints are of principal interest as they could improve the solution. The sensitivity analysis report shown in Fig. 2.4 shows that the Line 20 constraint has the biggest Shadow Price. This constraint represents the maximum possible consumption of raw stock 3. It is evident that even a small increase in consumption of this raw stock may substantially improve plan targets. For example, let us assume that the upper limit of possible consumption of raw stock 3 can be increased from 70 to 80 tons. Now, let us recalculate the plan (Fig. 2.5).

If we compare solutions shown in Figs. 2.3 and 2.5, we will see an evident plan improvement as increased possible consumption of raw stock 3 resulted in increase in the production output of paint No. 3 from 155.6 tons to 240 tons, increase in production capacity utilization from 755.5 tons to 840 tons, and increase in resulting gain from $174,888.89 to $196,000. Note that in a new situation, raw



	A	B	C	D	E	F	G
1	Monthly Output Plan for Three Sorts of Paints						
2		Variable inputs					
3		x1	x2	x3			
4		100	100	100			
5					Input value of		
6	Objective function coefficients				objective function		
7		c1	c2	c3	f		
8		200	280	250	73000		
9							
10	Constraints:		Coefficients		L.H.		R.H.
11	Capacity	1	1	1	840	<=	1000
12	2nd Marketing	0	1	0	200	<=	200
13	Raw stock 1	0.15	0.22	0.17	144.8	<=	160
14	Raw stock 2	0.08	0.07	0.1	70	<=	70
15	Raw stock 3	0.1	0.08	0.09	77.6	<=	80
16	Non-negativity	0	1	1	440	>=	0
17	1st Marketing	1	0	0	400	>=	400
18							
19		x1	x2	x3	f		
20	Solution	400	200	240	196000		

Fig. 2.5 Improved solution for optimization task

stock 3 constraint is not coupled anymore and does not limit further production while raw stock 2 constraint becomes coupled. Naturally, for further production plan improvements we should increase consumption of raw stock 2 instead of further consumption increase for raw stock 3.

2.2 Correlations Between Mathematical and Reference Models

The example described in Sect. 2.1.2 demonstrated how it was possible to solve a planning task with a preset objective function (gain) and known constraints on resources and marketing. It is evident that when creating any other mathematical planning model we need, first of all, to determine what plan parameter (criterion) should be taken as an objective function. Other parameters can be taken as constraints.

2.2.1 Main Criteria and Constraints

According to Kiran and Smith (1984), all criteria that can be used as an objective function may be classified as follows based on the following planning parameters:

1. Process parameters including:
   (a) Planned work completion time
   (b) Work-in-process
   (c) Capacity load
2. Order completion dates as contractually established
3. Production costs

On the other hand, as mentioned in Sect. 1.6.1, according to SCOR model, planning quality criteria shall belong to one of the main categories, i.e. consumer service level or cost-effectiveness. Under this approach, we can assume that criteria based on such parameters as due dates and plan completion dates ensure a high service level, while other criteria determine cost-effectiveness of production.

Below is a non-exhaustive list of potential optimization criteria and main constraints (Table 2.3).

First two criteria shown in Table 2.3 evaluate, directly or indirectly, the level of customer service; last three parameters are constraints, while others represent various factors of cost-effectiveness. In addition to three main constraints as shown in Table 2.3, certain task may impose on the mathematical model other constraining conditions that can be related, for instance, with batch quantities, batch sequence, and possible breaks during processing, etc.

Table 2.3 Main criteria and constraints

Criteria and constraints	Code	Area of improvement
1. Customer service level	C1	Minimum tardiness
2. Stock reserve maintenance	C2	Timely stock replenishment
3. Direct costs	K1	Minimized work content
4. Effective usage of raw and materials	K2	Optimal product outputs and minimum waste
5. Manufacturing throughput time	K3	Order lead time decrease
6. Equipment utilization and staff charge	K4	Optimal average load, minimum variations, and minimum interruptions
7. Consumption dynamics of raw materials and components	K5	Consumption uniformity
8. Saving of production resources	K6	Reduced power consumption, saving of equipment service life, etc.
9. Production capacity	O1	Optimal value
10. Balance between core products, by-products, and raw stock	O2	Calculation of proper proportions
11. Storage capacity	O3	Reasonable storage capacity

2.2.2 Standard Classification of Planning Optimization Models

Recent two decades saw the numerous publications on production planning optimization using operation research methods. For instance, the survey (Allahverdi et al. 2008) dedicated exclusively to publications on optimization of production planning with setup times (since 2000) mentions over 350 articles. Classification offered in Graham et al. (1979) contributed a lot to harmonization of optimization tasks. This classification is as follows:

$$\alpha \mid \beta \mid \gamma. \tag{2.13}$$

Under this classification, equipment of any type is called “machine” and total activities required to produce a batch of any product are called a “job”. Evaluation fields represent three areas as follows: type of production (type of machine in use), type of jobs and various constraints, and type of objective function. This classification ensures quite a detailed description of main characteristics almost for any optimization task. However, the more the complex tasks are, the more parameters will have to be included into evaluation fields for each of three areas.

The initial classification version provided for five types of production is as follows:

- Single machine: this type is used not only for job planning for a single machine but also serves as a basic type to develop planning algorithms for more complex cases.
- Parallel machines: they can be absolutely the same or similar by their parameters or absolutely different by parameters but used for one and the same purpose.

- Flowshop: machines are arranged according to the process sequence and all operations of each job are completed in the same sequence. It is acceptable that different machines and different jobs may have different processing times.
- Jobshop: different jobs can be arranged in any process sequence.
- Openshop: different operations of each job can be completed in any sequence.

The number of machines for a respective type of production is indicated in field α.

Field β contains constraints of different types and nature. First of all, there are various types of constraints imposed on job start and end dates. This field can also specify codes of constraints on job priority, possible breaks during processing, required batch processing, activity sequence, etc. Refer to Appendix C for the detailed list of constraint codes.

An objective function indicated in field γ of the standard classification generally represents one of work time indicators as follows:

- The most popular criterion is the total time of completion of all planned jobs $C_{\max}$ (makespan). $C_{\max}$ equals to the time interval from the job start (0 point) according to the plan till the completion of the last job indicated in such plan. It is clear that:

$$C_{\max} = \max(C_i), \tag{2.14}$$

where C_i is the time of completion of i-job.

- Flow time $F_{\max}$ determines the maximum length of i-job

$$F_{\max} = \max(F_i) \text{ at } F_i = C_i - r_i, \tag{2.15}$$

where r_i (release date) is the time of start of i-job.

- $T_{\max}$ represents the maximum tardiness in completion of i-job, i.e.:

$$T_{\max} = \max(T_i) \text{ at } T_i = \max(0, C_i - d_i), \tag{2.16}$$

where d_i (due date) is the time of work completion as specified by the customer.

- Lateness $L_{\max}$ represents the maximum deviation of i-job from the due date, i.e.:

$$L_{\max} = \max(|L_i|) \text{ at } L_i = C_i - d_i. \tag{2.17}$$

Other planning criteria and their combinations can also be used as an objective function. For instance, we can use the sum of tardiness on each job T_i with weight w_i for all n jobs:

$$T^w = \sum_i^n w_i T_i. \tag{2.18}$$

- Cost criterion f can apply either to gain or to costs. In the first case, the criterion will be $f_{\max}$, and in the latter case it is $f_{\min}$. When the weighted cost of tardiness is used as a criterion, the following value will be determined:

$$f^w = \sum_{i=1}^{n} w_i T_i c_i, \tag{2.19}$$

where c_i is the cost of one day of tardiness.

Refer to Appendix C for some other criteria.

2.2.3 Production Scale and Plan Hierarchy in Classification

Any existing production features all factors shown in Table 2.3; however, extents of their influence may be quite different. Therefore, different types of production as described in Sect. 1.3.1 and different levels of planning will have different criteria priorities (Mauergauz 2012).

The nature of production has different effects on the planning methods at different levels of planning. The least influence is applied to the Plan of Sales and Operations where the main quality criterion is a possible gain value. The production type and planning strategy, i.e. "Make-to-stock" (MTS) or "Make-to-order" (MTO), play the critical role for other plans.

You might think that the planning strategy is rigidly bound to the production scale; however, this is not always the case. The planning strategy MTS can be used even for small-scale production provided that there is a relatively stable demand for products. If the composition of products can be customized for orders, the MTS strategy will be modified into the "Assemble-to-orders" (ATO) strategy, but the bulk of components will be "Make-to-stock", anyway. The MTO planning strategy is often used for large-scale production, especially at the lower level of planning.

Tables 2.4, 2.5, and 2.6 show the structure of planning in accordance with the terminology used in ERP systems and respective indicators for various production types as described in Sect. 1.3.1. Refer to Table 2.3 for codes of criteria and constraints.

Tables 2.4–2.6 lead us to a number of conclusions as follows:

- Any production type and any planning strategy require a master and short-term plans. In some cases, for instance, in case of single machine bulk manufacturing or automated production line these plans can be combined. Generally, the smaller the scale of production is, the more plans will be required to ensure such production.
- Most frequently, the main criterion for a master plan is minimized direct costs (K1) that include setup and storage costs, order delay penalties, personnel hiring/firing costs, overtime payments, outsourcing and subcontracting costs, etc. In

Table 2.4 Planning structure and indicators: one-stage production

Planning level	Production strategy	Main objective	Additional criteria	Constraints
Type 1 (single machine)—bulk and large-scale manufacturing				
Master production scheduling	MTS	K1	K2	O1, C2
	MTO		C1	O1, K2
Shop floor scheduling	MTS	K4	C2	O1, O3
	MTO	C1	K4	
Type 1 (single machine)—batch and small-scale manufacturing				
Master production scheduling	MTS	K1	C2	O1, K2
	MTO	C1	K1	
Shop floor scheduling	MTS	K3	K1	O1, C2, K6, O3
	MTO	C1		O1, K6
Type 2 (parallel machines)				
Master production scheduling	MTS	K1	K4	O1, C2
	MTO			O1, C1
Shop floor scheduling	MTS	K4	C2	O1, O3
	MTO	C1	K4	

Table 2.5 Planning structure and indicators: synchronized flowshop production

Planning level	Production strategy	Main objective	Additional criteria	Constraints
Type 3a (automated production line)				
Master production scheduling (alias shop floor scheduling)	MTS	–	–	O1, C2
Type 3b (versatile transfer line with output of core product only)—discrete manufacturing				
Master production scheduling	MTS	K1	–	O1, C2
	MTO		–	O1, C1
Shop floor scheduling	MTS	C2	K1	O1
	MTO	C1		
Type 3c (versatile transfer line with output of core product and by-products)—process manufacturing				
Master production scheduling	MTS	K2	K1	O1, O2, C2
	MTO	C1		O1, O2, K2
Shop floor scheduling	MTS	K1	C2	O1, K2, O3
	MTO	C1	K1	O1
Type 3d (flexible assembly line)				
Master production scheduling	MTS	K5	K1	O1, C2
	MTO			O1, C1
Shop floor scheduling	MTS	C2	–	O1
	MTO	C1	–	

Table 2.6 Planning structure and indicators: batch, small-scale, and unique manufacturing

Planning level	Production strategy	Main objective	Additional criteria	Constraints
Type 4 (cellular manufacturing)				
Master production scheduling	MTS	K1	K4	O1, C2
	MTO			O1, C1
Material requirements planning	MTS	K3	K4	O1, C2
	MTO			O1, C1
Shop floor scheduling	MTS	K3	C2	O1, K4
	MTO	C1	K3	
Type 5 (job-shop manufacturing)				
Master production scheduling	MTS	K1	C2	O1, K4
	MTO		C1	
Material requirements planning	MTS	K3	C2	O1, K1
	MTO	C1	K3	
Shop floor scheduling	MTS	C1	K6	O1, K4
	MTO			
Type 6 (project manufacturing)				
Master production scheduling	MTO	C1	K1	O1
Shop floor scheduling			K4	

some cases, the master plan optimization will require indirect costs as a main criterion. For instance, effective use of raw stock (K2) will be the most important criterion for process manufacturing (type 3c), while for assembly line (type 3d) the main criterion will be even consumption of materials and components (K5). For discrete manufacturing, the smaller scale of production is the more important master plan optimization constraints will be. In some cases, such constraints shall be considered as additional criteria rather than constraints.

- A batch and small-scale production will require material requirements planning if products include at least two components. However, simplest types of products are manufactured in quite a large scale only. The main criterion of material requirements planning is minimized throughput time (K3). Since material requirement planning is based on a master production planning, optimization generally concerns only the batch quantity. Generally, the need for timely production is the constraint of material requirements planning. In some cases, such need may become top priority; in this case, minimized throughput time will be used as an additional criterion.
- The smaller the scale of production is, the greater need for timely completion will be. This factor will gradually shift from constraints to criteria. It will become the main criterion for short-term plans. In case of job-shop manufacturing, the discipline of completion of orders transferred from the master plan and material requirements plan will become the main indicator of performance of production shops no matter which production strategy is adopted.

- The smaller scale of production is and the more detailed planning is, the more criteria and constraints shall be taken account for discrete manufacturing planning.

In addition to the above main criteria and constraints, in practice, the industry-specific production planning system shall take account of various additional industry-specific parameters. For instance, the paper (Zagidullin 2010) lists 30 parameters specific for the machine building industry.

2.3 Priority Rules

As mentioned in Sect. 2.2.2, single machine schedule algorithms often serve as a basis for scheduling in more complex cases. Therefore, it is extremely important to establish rules to determine single machine job sequence. There are quite a number of such rules that are called priority rules. For instance, the publication (Panwalkar and Iskander 1977) describes over 100 priority rules divided into 5 groups as follows:

- Simple rules that are based on information pertaining to a certain job, i.e. established due date, processing time, number of operations, etc.
- Simple rule combinations under which the job queue is divided into parts to which certain simple rules apply.
- Weighted priorities under which simple rules apply to each job given its weighted index.
- More complex rules that use various combinations of simple rules and take account of time-based changes in characteristics of certain jobs.
- Other rules used under specific circumstances.

Let us consider some simple and weighted priority rules.

2.3.1 Simple Rules

The first of simple rules is a classic queuing rule according to which i-job that appeared first in a queue shall be performed first, i.e. whose start time r_i is earlier. This rule is called FIFO (First In First Out). Despite the usual "fairness" of this rule, sometimes an opposite LIFO (Last In First Out) rule can be applied according to which a job that has just arrived shall be performed first. This is the case, for instance, when a rejected item must be corrected as soon as possible, etc. Another simple rule that is less common though absolutely natural is the rule of first-priority completion of a job with the nearest due date d_i. This rule is called EDD (Earliest Due Date). The above simple rules are actually based on a time parameter, i.e. job start or end.

There are a number of simple rules that are based on processing time. Two simplest rules of this kind include the SPT rule (Shortest Processing Time) and the LPT rule (Longest Processing Time). Despite their opposite purposes, both rules have certain scopes of application.

To illustrate the practicality of SPT rule, let us consider the case where three jobs are completed on a single machine with the processing time parameters being $p_1 = 1, p_2 = 5$, and $p_3 = 8$ h, respectively. It is clear that if we take the makespan $C_{\max}$ as a process optimization criterion, the sequence of planned jobs does not matter; in any case, $C_{\max} = 14$ h. However, if we take the mean flow time $\overline{F}$ as an optimization criterion, its value will greatly depend on the job sequence. For instance, at {3, 2, 1} order the production cycle of job 1 will be 8 h, the mean flow time of job 2 will be 13 h, and that of job 3 will be 14 h. In such a case, the mean flow time for three jobs will be:

$$\overline{F} = (8 + 13 + 14)/3 = 11.67 \text{ h.}$$

However, if the order is {1, 2, 3}, the mean flow time will be just:

$$\overline{F} = (1 + 6 + 14)/3 = 7 \text{ h.}$$

The LPT rule is useful, for example, to distribute jobs between several parallel machines in order to reduce the makespan $C_{\max}$. In this situation $C_{\max}$ greatly depends on the job sequence as even load applied on all parallel machines ensures the minimum $C_{\max}$ value. It is much easier to achieve even load if each of the parallel machines will first be used for the longest jobs and then perform less prolonged jobs at vacating machines.

The SPT rule has a popular modification, i.e. the Weighted Shortest Processing Time rule (WSPT). According to this rule, jobs are completed in the descending order for w_i/p_i which represents the ratio of priority factor w_i of i-job to its processing time p_i.

According to the terminology introduced in Pinedo (2005), all the above rules are static as they do not directly depend on the time point where the plan has to be performed. Simple rules also include some time-dependant dynamic rules, for example MST rule (Minimum Slack Time).

The slack time value for each i-job at the planning time t is calculated as $\max(d_i - p_i - t, 0)$, i.e. slack time exists if $d_i - p_i - t \geq 0$; otherwise, it does not exist. In this situation production shall start with the job whose slack time is the closest to zero. If there are several jobs with zero slack time (this actually means past due date), there is no way to determine the sequence of such jobs.

Another dynamic rule is the so-called Critical Ratio (CR). According to this rule, production shall start with a job which has the minimum value of $(d_i - t)/p_i$. Similar to the previous rule, CR equals to zero if $d_i - t \leq 0$.

Let us take an example of applying several simple priority rules to jobs shown in Table 2.7.

Table 2.7 Jobs to be planned

Job number, i	Processing time (days), p	Due date, d	Tardiness weight coefficient, w
1	20	42	1
2	25	31	2
3	4	4	1
4	18	26	2

The planning objective function is minimum T^w that represents the sum of weighted tardiness on each job (formula 2.18).

First, let us apply the EDD rule. According to this rule, the job sequence shall be {3, 4, 2, 1}. Now let us describe this sequence using the methodology proposed in Sule (2007). Let us write sequence parameters into four lines. The first line will show the job start time, its processing time, and completion time. For each job, the start time is followed by the brackets containing the job number slash processing time. The end time of one job is the start time of the following job. The second line shows the job number in brackets followed by the due date. The third line calculates tardiness on the planned completion (with the ± sign). The last line is used to calculate the objective function.

Job sequence	0	(3/4)	4	(4/18)	22	(2/25)	47	(1/20)	67
Due date		(3)	4	(4)	26	(2)	31	(1)	42
Tardiness			0		−4		16		25
Objective function value	$T^w = 1 \times 0 + 2 \times 0 + 2 \times 16 +; 1 \times 25 = 57$								

Please remember that in case of negative tardiness T_i equals to 0.

Now let us consider the Shortest Processing Time according to which the job sequence shall be {3, 4, 1, 2}. Here:

Job sequence	0	(3/4)	4	(4/18)	22	(1/20)	42	(2/25)	67
Due date		(3)	4	(4)	26	(1)	42	(2)	31
Tardiness			0		−4		0		36
Objective function value	$T^w = 1 \times 0 + 2 \times 0 + 1 \times 0 + 2 \times 36 = 72$								

Now let us calculate the objective function using the Critical Ratio rule. Unlike two above options, it is impossible to determine the sequence for all jobs at the start of planning. We can determine only the first of planned jobs; the next job is chosen after the previous one is completed. In this case, the solution includes multiple iterations as follows.

Iteration 1: Planning start time $t = 0$. For each job, we get the following values:

$$CR_1 = (d_1 - t)/p_1 = (42 - 0)/20 = 2.1;$$

$$CR_2 = (31 - 0)/25 = 1.24;$$

$$CR_3 = (4 - 0)/4 = 1;$$

$$CR_4 = (26 - 0)/18 = 1.44.$$

According to the CR rule, the job to be done first shall be job 3. At the second iteration, the planning start time will be end time of job 3, i.e. $t = 4$.

Iteration 2: $t = 4$.

$$CR_1 = (42 - 4)/20 = 1.9;$$

$$CR_2 = (31 - 4)/25 = 1.08;$$

$$CR_4 = (26 - 4)/18 = 1.22.$$

According to Iteration 2, the second job to be done shall be job 2 that will end at $t = 4 + 25 = 29$. Let us apply the same procedure to Iteration 3.

Iteration 3: $t = 29$.

$$CR_1 = (42 - 29)/20 = 0.65;$$

$$CR_4 = \max(0, (26 - 29)/18) = 0.$$

It is clear that the third job to be done shall be job 4. The resulting job sequence will be $\{3, 2, 4, 1\}$. Now let us calculate the objective function similarly to EDD and SPT rule procedures.

Job sequence	0	(3/4)	4	(2/25)	29	(4/18)	47	(1/20)	67
Due date		(3)	4	(2)	31	(4)	26	(1)	42
Tardiness			0		−2		21		25
Objective function value	$T^w = 1 \times 0 + 2 \times 0 + 2 \times 21 + 1 \times 25 = 67$								

If we compare planning results for these three rules, we can see that in this situation the EDD rule yielded the best results.

2.3.2 Some Useful Theorems

In some situations we can choose the best priority rule using any of the following theorems, theorem proving is provided, e.g., in Pinedo (1995).

- Theorem 1: The least mean flow time $\overline{F}$ is achieved if the SPT rule is applied. This rule also ensures the least value of total flow time of the job package.
- Theorem 2: The least value of the maximum lateness for the whole job package $L_{\max}$ is achieved if the EDD rule is applied.
- Theorem 3: The least mean lateness $\overline{L}$ is achieved if the SPT rule is applied.
- Theorem 4: If all jobs have a common due date, the least value of mean tardiness $\overline{T}$ is achieved if the SPT rule is applied.
- Theorem 5: The least value of maximum tardiness $T_{\max}$ in the job package is achieved if the EDD rule is applied.

2.3.3 Combined Priority Rules

An example of a combined priority rule is the combination of WSPT and MST rules proposed in Vepsalinen and Morton (1987). According to this combined rule, priority is given to a job with the greatest value of the so-called Apparent Tardiness Cost (ATC). This value is calculated using the following formula:

$$I_i(t) = \frac{w_i}{p_i} \exp\left(-\frac{\max(d_i - p_i - t, 0)}{k\overline{p}} \right), \tag{2.20}$$

where k is the so-called scale parameter and

$\overline{p}$ is the mean processing time of all planned jobs.

Depending on k value, this combined rule tends closer either to the WSPT rule or to the MST rule. Like the MST and CR rules described above, the ATC rule is a dynamic one, i.e. a priority job is determined every time a machine gets vacant. Naturally, t value changes giving rise to the need to re-calculate $I_i(t)$ for each i-job. The greater the k is, the smaller influence the time factor has, i.e. the ATC rule gets closer to the WSPT rule.

According to Lee et al. (1997), k depends on the following parameter:

$$\theta = \frac{d_{\max} - d_{\min}}{C_{\max}}, \tag{2.21}$$

where the numerator represents the difference between the greatest and the nearest due dates established for jobs, and the denominator is equal to makespan.

This relation is as follows:

$$\begin{gathered} k = 4.5 + \theta \quad \text{at} \quad \theta \le 0.5 \\ \text{and} \\ k = 6 - 2\theta \quad \text{at} \quad \theta \ge 0.5 \end{gathered} \tag{2.22}$$

Now Let's consider this combined rule for the example shown in Table 2.7. In this situation, the mean process time will be:

$$\overline{p} = (20 + 25 + 4 + 18)/4 = 16.75.$$

The total duration of all jobs (makespan) will be:

$$C_{\max} = \sum_{1}^{4} p_i = 67.$$

The parameter θ will be:

$$\theta = (42 - 4)/67 = 0.56$$

And the scale parameter:

$$k = 6 - 2 \times 0.56 = 4.88$$

As this priority rule is dynamic, i.e. depends on the moment of plan execution, further calculations shall take place in iterations.

Iteration 1: In order to determine the first job at the start of planning let us assume that $t = 0$. Thus, for each job we will get the following results:

$$I_1(t) = \frac{w_1}{p_1}\exp\left(-\frac{\max(d_1 - p_1 - t, 0)}{k\overline{p}}\right) = \frac{1}{20}\exp\left(-\frac{\max(42 - 20 - 0.0)}{4.88 \times 16.75}\right) = 0.05 \times 0.76 = 0.038.$$

$$I_2(t) = \frac{2}{25}\exp\left(-\frac{\max(31 - 25 - 0.0)}{4.88 \times 16.75}\right) = 0.08 \times 0.93 = 0.074.$$

$$I_3(t) = \frac{1}{4}\exp\left(-\frac{\max(4 - 4 - 0.0)}{4.88 \times 16.75}\right) = 0.25 \times 1 = 0.25.$$

$$I_4(t) = \frac{2}{18}\exp\left(-\frac{\max(26 - 18 - 0.0)}{4.88 \times 16.75}\right) = 0.11 \times 0.9 = 0.1.$$

According to the ATC rule, the first job to be done shall be job 3 with the greatest $I_i(t)$. At the second iteration, the planning start time will be the completion time of job 3; thus, $t = 4$.

Iteration 2: $t = 4$.

$$I_1(t) = \frac{1}{20}\exp\left(-\frac{\max(42 - 20 - 4.0)}{4.88 \times 16.75}\right) = 0.05 \times 0.8 = 0.04.$$

$$I_2(t) = \frac{2}{25}\exp\left(-\frac{\max(31 - 25 - 4.0)}{4.88 \times 16.75}\right) = 0.08 \times 0.97 = 0.078.$$

$$I_4(t) = \frac{2}{18}\exp\left(-\frac{\max(26 - 18 - 4.0)}{4.88 \times 16.75}\right) = 0.11 \times 0.95 = 0.1.$$

According to Iteration 2, the second job to be done shall be job 4 that will end at $t = 4 + 18 = 22$. Let us apply the same procedure to Iteration 3.

Iteration 3: $t = 22$.

$$I_1(t) = \frac{1}{20}\exp\left(-\frac{\max(42 - 20 - 22.0)}{4.88 \times 16.75}\right) = 0.05 \times 1 = 0.05.$$

$$I_2(t) = \frac{2}{25}\exp\left(-\frac{\max(31 - 25 - 22.0)}{4.88 \times 16.75}\right) = 0.08 \times 1 = 0.08.$$

As we can see, the third job to be done shall be job 2. The resulting job sequence will be $\{3, 4, 2, 1\}$ and this coincides with the job sequence calculated above using the EDD rule.

2.4 Production Intensity and Utility of Orders

The key purpose of the above priority rules is to consequently assign vacant machines to items ready for processing exactly at such machines. Technically, this concept is absolutely true as any planning engineer will take this into account when planning a shift task. However, it is clear that the practical worthiness of such algorithms will depend on the fact whether a correct criterion has been selected to determine the sequence of processing of competing batches under given circumstances.

Due to difficulties associated with the selection of criteria to determine the processing sequence, instead of IT systems it is a user who has to decide and select from a vast range of potential choices (up to 100). Such an approach does not seem reasonable.

In addition to difficult selection of criteria to determine the sequence of competing batches, there is another problem that makes shift task planning even more challenging. This is a substantial uncertainty about the batch readiness for processing. In practice, when planning a shift task a planning engineer never has complete information on the status of all manufactured batches. Besides, planning takes place and the next shift starts at different time points; this makes uncertainty about batch readiness for processing even more acute.

2.4.1 Production Intensity

When criteria for an optimal plan to be performed are determined, in addition to process or economic factors, we also should take account of personal relations between responsible persons. According to Afonichkin and Mikhailenko (2009), there are two key courses of decision-making, i.e. rational and psychological. As for the rational course, decisions are based on mathematically proven results, while psychological course of decision-making is substantially based on intuition and psychology of employees. A good production plan should be well balanced between needs and wishes of all people involved in the production process.

People involved in the production process are all part of an extensive network of relations that can be considered as a certain psychological field. Each field, e.g. magnetic or electric, has a quantitative characteristic that is called “intensity”. Similarly, a psychological field arising during production process can also be described quantitatively; this quantitative description is called the production intensity H (Mauergauz 1999). However, unlike physical fields, production intensity cannot be measured physically; it has no physical dimensions, i.e. it is a dimensionless value.

Such dimensionless intensity of a psychological field (whether large or small) can only be estimated by comparison with another field or with another state of this field, e.g., at any other point of time. We face a natural question how to calculate production intensity.

For the purpose of intensity quantification, let us note that the situation in a production shop is mainly characterized by two factors, i.e. total time required for job completion and slack time. Please remember the so-called dynamic priority rules described above that take account of these two factors. These rules include the Minimum Slack Time (MST), Critical Ratio (CR), and the Apparent Tardiness Cost (ATC) rules.

However, criteria that are calculated under any of these rules have substantial deficiencies. First of all, there is no way to determine the values of such criteria if the job due date has already expired by the start of planning. However, the most important thing is that the cumulative machine and personnel load resulting from different jobs cannot be determined using such criteria.

Unlike the above dynamic priority rules, we can determine production intensity values not only for each job but for each machine and for the production shop on the whole. Production intensity for a certain machine will be equal to the sum of intensities for jobs awaiting processing on such machine, while production intensity for a production shop on the whole equals to the sum of intensities for all jobs or, which is the same, for all machines.

The total time required to perform "handed down" plan targets is made of two values, i.e. the production processing time and setup/job transfer time. Slack time can be a positive or negative value or be equal to zero. Therefore, calculation relations used to determine production intensity shall be different at different signs of slack time values.

Thus, for one i-job:

$$\begin{gathered} H_i = (T_{1i} + T_{2i})/z_{1i} \text{ at } d_i - t \geq 0 \\ \text{and} \\ H_i = (T_{1i} + T_{2i}) \times z_{2i} \text{ at } d_i - t \leq 0, \end{gathered} \tag{2.23}$$

where T_{1i} is the time duration determined by processing time pending at the time of planning, T_{2i} is the time component arising from the need to transfer jobs for further processing, z_{1i} is the production slack time in relation to plan targets, and z_{2i} is the tardiness from plan targets.

T_{1i} can be determined based on the following relation:

$$T_{1i} = \frac{1}{G}\sum_{j=k_i}^{n_i} \frac{p_{ij}\left(1 - \eta_{ij}/100\right)}{m_{ij}}, \tag{2.24}$$

where k_i is the number of the first unfinished j-operation for i-job, n_i is the total number of operations required to perform i-job, p_{ij} is the processing time of each remaining operation in hours or days, η_{ij} is the coefficient of readiness of j-operation (%), m_{ij} is the number of simultaneously processed items of i-job during j-operation, and G is the average number of work hours or days in a planning period.

η_{ij} can be other than zero only if $j = k_i$, i.e. when the first unfinished operation is in process. This coefficient is used to take account of intensity of a job that is in process. If a planned job is to produce a batch of items or assembly parts, a certain operation allows simultaneous processing of multiple items or parts, e.g., on parallel machines. If each machine processes one item only, m_{ij} will be equal to the number of machines involved in parallel processing of one planned job. G, the denominator, is used in formula (2.24) to ensure that H_{1i} is dimensionless.

T_{2i} is calculated using the following relation:

$$T_{2i} = \frac{1}{G}\sum_{j=k_i}^{n_i} s_{ij}, \tag{2.25}$$

where s_{ij} is the duration of required transportation and machine setup associated with j-operation for i-job.

Let us assume the following relation for dimensionless slack time available to a planning engineer:

$$z_{1i} = \frac{d_i - t}{\alpha G} + 1, \tag{2.26}$$

where α is the "psychological" coefficient of a given enterprise representing the level of "complacency" at available slack times or the level of "nervousness" if production due dates are failed to meet.

And, finally:

$$z_{2i} = \frac{t - d_i}{\alpha G} + 1. \tag{2.27}$$

Similarly to the WSPT or ATC rules described above, intensity of a certain job can include a respective weight coefficient of priority w_i. In particular, in the simplest case (where all readiness coefficients $\eta_{ij} = 0$, the number of simultaneously processed items under one job $m_{ij} = 1$, planning takes place for the first operation and all $k_i = 1$, while transportation and setup time s_{ij} can be ignored) intensity can be determined using the following formulas:

$$\begin{gathered} H_i = \frac{w_i p_i}{G} \frac{1}{(d_i - t)/\alpha G + 1} \text{ at } d_i - t \geq 0 \\ \text{and} \\ H_i = \frac{w_i p_i}{G}((t - d_i)/\alpha G + 1) \quad \text{at} \quad d_i - t \leq 0. \end{gathered} \tag{2.28}$$

Now let us consider the relation between intensity and available slack time or deadline tardiness (Fig. 2.6).

The slack time (difference between the due date for each job d_i and the current time point t) is plotted on the X-line. On the positive part, i.e. where $d_i > t$, intensity values decrease hyperbolically as far as slack time increases. If slack time is negative, i.e. there are tardiness on plan targets $d_i < t$, intensity values increase linearly as far as tardiness increases. In expressions (2.26) and (2.27) if $d_i = t$, i.e. at the point of transfer from one relation to another, $z_1 = z_2 = 1$. Intensity values calculated using formulas (2.23) or (2.28) also coincide at this point. Besides, this point also sees the match of values for the following derivative:

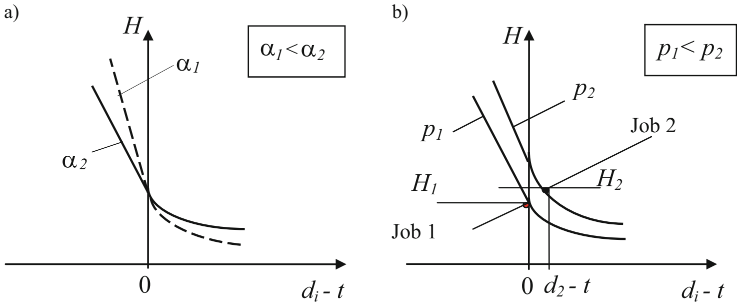


Fig. 2.6 Relation between production intensity and slack time (**a**) at different psychological coefficient α; (**b**) at different job processing time p_i

$$\frac{dH}{d(d_i - t)},$$

which have been calculated been using both formulas. Thus, the straight line in the left-hand part of the chart is tangent at the transition point to the hyperbolic curve in the right-hand part of the chart.

Figure 2.6a shows two curves for H changes; these curves differ by the value of "psychological" coefficient α; note that $\alpha_2 > \alpha_1$. As we can see, the greater the α is, the less intensity is associated with tardiness; however, the level of "relaxation" is also lower when slack time is available. Therefore, we can consider α as a factor of balance for relations at production.

Figure 2.6b shows two charts of intensity dependency on $d_i - t$ at different p_i values. The charts display the position of two jobs with different values of p_i and $d_i - t$. In this case, as $d_i = t$ for job 1, then, assuming that $w_1 = 1$, Eq. (2.28) yields that $H_1 = \frac{p_1}{G}$. As $d_i - t \geq 0$ for job 2, intensity will be:

$$H_2 = \frac{p_2}{G} \frac{1}{(d_2 - t)/\alpha G + 1}.$$

As we can see from Fig. 2.6b, even if $d_i - t = 0$ (as true for job 1), it is still possible that the intensity associated with another job where $d_i > t$ will be greater, i.e. $H_2 > H_1$, and job 2 will have priority over job 1. This feature makes the intensity-based priority rule substantially different from, for example, the Critical Ratio Rule.

Let us consider how to use the intensity parameter to determine the sequence of jobs shown in Table 2.8. As intensity is a dynamic criterion as its value depends on time t, calculations will include multiple iterations similar to the procedure applied under the CR, ATC, and other rules. The planning period G is assumed 30 days and the "psychological" coefficient $\alpha = 0.1$.

Iteration 1: To determine the first job to be done, at the start of planning it is assumed that $t = 0$. As for all jobs in Table 2.7 $d_i > t$, we use the first formula from (2.28).

$$\begin{aligned} H_1 &= \frac{w_1 p_1}{G} \frac{1}{(d_1 - t)/\alpha G + 1} = \frac{1 \times 20}{30} \frac{1}{(42 - 0)/0.1/30 + 1} \\ &= 0.66 \times 0.07 = 0.044. \end{aligned}$$

Table 2.8 Planning inputs

Items	Capacity, items per day		Cost per item	Minimum daily output
	Type 1	Type 2		
A_1	20	15	6	1510
A_2	35	30	4	4500

$$H_2 = \frac{2 \times 25}{30} \frac{1}{(31 - 0)/0.1/30 + 1} = 1.66 \times 0.09 = 0.14.$$

$$H_3 = \frac{1 \times 4}{30} \frac{1}{(4 - 0)/0.1/30 + 1} = 0.13 \times 0.42 = 0.06.$$

$$H_4 = \frac{2 \times 18}{30} \frac{1}{(26 - 0)/0.1/30 + 1} = 1.2 \times 0.1 = 0.12.$$

The first job to be done shall be the job with the greatest intensity, i.e. job 2. This job will be released at $t = 25$. At this point $d_i > t$ is still true for jobs 1 and 4; however, $d_i < t$ for job 3. Accordingly, we still use the first formula from (2.28) to determine intensity for jobs 1 and 4, while job 3 intensity should be calculated based on the second formula from (2.28).

Iteration 2: $t = 25$.

$$H_1 = \frac{1 \times 20}{30} \frac{1}{(42 - 25)/0.1/30 + 1} = 0.66 \times 0.15 = 0.1.$$

$$H_3 = \frac{w_3 p_3}{G}((t - d_3)/\alpha G + 1) = \frac{1 \times 4}{30}((25 - 4)/0.1/30 + 1) = 0.13 \times 8$$
$$= 1.06.$$

$$H_4 = \frac{2 \times 18}{30} \frac{1}{(26 - 25)/0.1/30 + 1} = 1.2 \times 0.75 = 0.9.$$

Thereafter, job 3 will have the greatest intensity. Job 3 will be start at $t = 25 + 4 = 29$. By this time, job 4 will become past due, and we should use the second formula from (2.28) to determine its intensity.

Iteration 3: $t = 29$.

$$H_1 = \frac{1 \times 20}{30} \frac{1}{(42 - 29)/0.1/30 + 1} = 0.66 \times 0.18 = 0.12.$$

$$H_4 = \frac{2 \times 18}{30}((29 - 26)/0.1/30 + 1) = 1.2 \times 2 = 2.4.$$

Thereafter, job 4 will have the greatest intensity. The resulting job sequence will be {2, 3, 4, 1}. Now let us calculate the objective function T^w using the same procedure as for other rules described above.

Job sequence	0	(2/25)	25	(3/4)	29	(4/18)	47	(1/20)	67
Due date		(2)	31	(3)	4	(4)	26	(1)	42
Tardiness			-6		25		21		25
Objective function value	$T^w = 2 \times 0 + 1 \times 25 + 2 \times 21 + 1 \times 25 = 92$								

2.4.2 Dynamic Utility Function of Orders

As we can see from the previous example, the intensity-based priority rule yields substantially worse values of the objective function T^w as compared with, for example, the EDD or SPT rules (refer to Sect. 3.2.1). This is inevitable as according to Theorem 5 (Sect. 2.3.2) the least tardiness results from the EDD rule. However, intensity-based priority rule gives much better results if we use other criteria as the objective function, for example, the weighted sum of tardiness costs f^w (2.19). The procedure of applying the intensity-based rule will be further described for such a case.

As the production intensity has the additive property, when we use intensity to describe statuses of jobs during planning, we can get a wide range of evaluations for the production situation, i.e. we can compare intensities for different machines, production shops and floors, orders, etc. In this paragraph, we will describe how to use the production intensity to determine dynamic utility of orders.

Basically, we can use functions of the dynamic utility of orders V to evaluate whether it is possible and desirable to ensure a high level of customer service C1 (Table 2.3). In order to identify evaluation rules, let us consider how to apply typical utility functions described in Sect. 1.7.2.

Let us assume that any order is quite useful for a manufacturer in a certain prospect. Note that the greater the order scope and order fulfilment time is, the greater utility such order will have. In such a way, the possibility of order fulfilment in future certainly has a positive utility and should yield a respective gain.

On the other hand, the closer the due date for an order is, a manufacturer starts facing certain difficulties. The utility value starts decreasing. However, if the manufacturer manages to perform the order when due, the order utility remains positive till the very time of order execution, and utility becomes zero at the time of completion. If the manufacturer fails to meet the due date, he usually gets into troubles with this. The order becomes the source of losses, thus having a negative utility.

As we can see, if slack time is available for an order, a manufacturer generally counts on gain while in case of tardiness it incurs losses. The behavior of utility as a potential gain/loss function is described in numerous sources. Generally, all research results can be reduced to two options shown in Fig. 2.7.

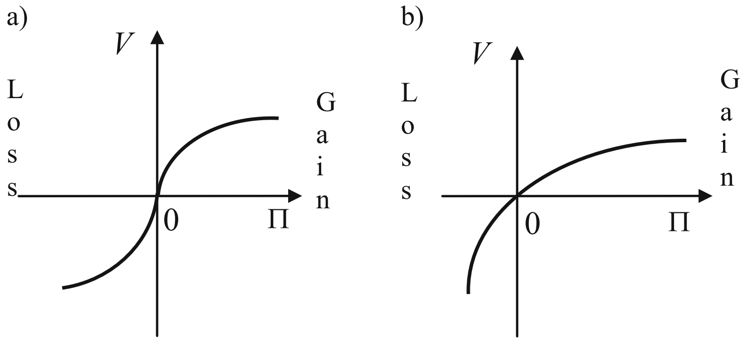


Fig. 2.7 Charts of utility: potential gain/loss

The gain value is plotted on the X-line (potential gain); the gain utility is plotted on the Y-line in the positive part of the X-line, and the loss utility is plotted on the Y-line in the negative part of the X-line. The curve in Fig. 2.7a is quite similar to Campbell curve shown in Fig. 1.12; the only difference is that its breakpoint coincides with or is close to the point of origin. The curve in Fig. 2.7b has no breakpoint.

The curve in Fig. 2.7a has been known as S-curve after well-known research of Kahneman and Tversky (1984) who were awarded the Nobel Prize in the field of economics in 2002. Their research proved that people usually tend to take risks (refer to Sect. 1.7.2 above) if there is a probability of potential loss (left-hand part of the chart in Fig. 2.7a). In this case, the concave left-hand part of the curve represents negative risk aversion—the second derivative has a positive sign at this interval of 2.7a curve.

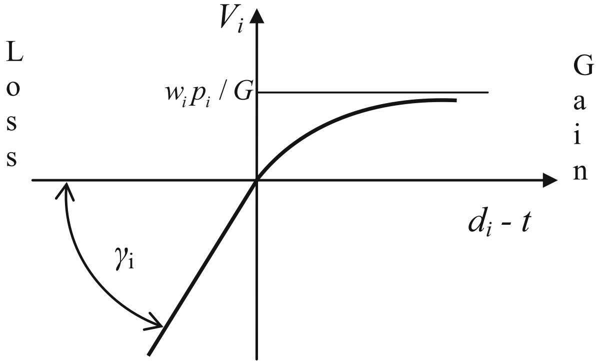
Unlike 2.7a curve, 2.7b curve always has a positive risk aversion no matter whether there is a probability of gain or loss. This curve resembles curves shown in Fig. 1.14. Please note that the chart of 2.7b type or Grayson–Bird utility function (Keeney and Raiffa 1976) was developed in 1957 that is much earlier than 2.7a chart. Differences between 2.7a and 2.7b curves probably arise from the pools of respondents and areas of money allocations. Studies conducted by Kahneman and Tversky in 1984 featured polls with low-income respondents whose income were insignificant and mainly used for personal consumption, while Grayson's studies in 1957 dealt with large corporate investments.

Let us use the above results of economic and psychological studies in order to build a function of dynamic utility of orders. Let us assume that the current utility of i-order is as follows:

$$V_i = \frac{w_i p_i}{G} - H_i, \tag{2.29}$$

where (as before) p_j represents the processing time in days (hours) that remains before job completion, w_i is the weight coefficient of priority, G is slack time in

Fig. 2.8 Dynamic order utility curve



days (hours) during the planning period, and H_i is the production intensity associated with the order (job).

Let us see how the dynamic utility function depends on slack time available till the established order due date (Fig. 2.8) assuming that the orders have not yet been started, i.e. p_j does not depend on time.

The depicted curve in the positive part ($d_i - t \geq 0$) approaches the asymptote.

$$V_i = w_i p_i / G. \tag{2.30}$$

In the negative part ($d_i - t \leq 0$), the curve gets transformed into a straight line for which:

$$\begin{gathered} tg\gamma_i = \frac{d(w_i p_i / G - H_i)}{d(d_i - t)} = -\frac{dH_i}{d(d_i - t)} \\ \text{and} \\ tg\gamma_i = -\frac{d\left(\frac{w_i p_i}{G}((t - d_i)/\alpha G + 1)\right)}{d(d_i - t)} = \frac{w_i p_i}{\alpha G^2}. \end{gathered} \tag{2.31}$$

If we compare the curve in Fig. 2.8 with the curves in Fig. 2.7, we can see that the slack time available for a job determines gain or loss in Fig. 2.8. The presence of an order is a substantial gain in a far prospect, but the rate of gain growth decreases with the farness. In this part, the behavior of the order utility function curve is absolutely similar to both curves shown in Fig. 2.7.

The negative part of Fig. 2.8 is similar to the loss part in Fig. 2.7. In this part, the curves in Fig. 2.7 have different shapes. In order to check if the curve in Fig. 2.8 is correct in the negative part, let us assume human behavior as described above. As dynamic utility functions are used by corporate managers, their behavior will be closer to the curve in Fig. 2.7b rather than 2.7a curve as they are unlikely to take risks even in the loss part.

However, this is formally impossible to use Fig. 2.7b curve as the order utility function as positive risk aversion typical of this curve results in a sharp increase of

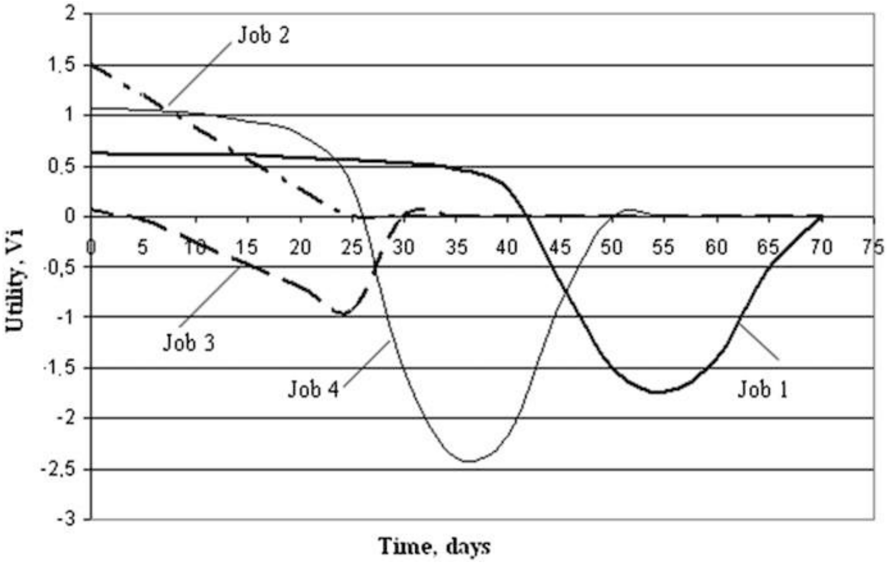


Fig. 2.9 Time-based utility dynamics for planned jobs

negative utility and, consequently, limits the potential loss value. The order utility function shall not have such a limit as, actually, any tardiness is possible during order performance. Therefore, the correct behavior will be a linear drop of the dynamic order utility function where risk aversion (the second derivative of the function) equals to zero. Figure 2.8 shows exactly this dependency.

As an example, let us build utility curves for orders as per Table 2.7.

The diagram shown in Fig. 2.9 assumes that the job sequence is {2, 3, 4, 1} (refer to the example described in the previous section). As we can see, utility of job 2 that is done first decreases almost linearly during the runtime as the processing time decreases with time, too. The utility of job 3 that is delayed from the very start decreases rapidly and rapidly goes to zero right after the start of this job; the reason for this is a small processing time of this job. Initially, utility values of jobs 4 and 1 decrease slowly as these jobs have quite a large slack time (especially for job 1). However, then the utility of job 4 rapidly gets negative; its absolute value rises and this can cause serious troubles. The same situation applies to job 1 (thought to a lesser extent).

Now let us consider the cumulative utility of multiple parallel orders. If the number of order is N at the planning horizon, their cumulative utility equals to the sum of utilities of each order as orders are generally independent. Thus, we get the cumulative order dynamic utility function as follows:

$$V = \sum_{i=1}^{N} V_i = \frac{1}{G}\sum_{i=1}^{N} w_i p_i - \sum_{i=1}^{N} H_i. \tag{2.32}$$

Application of the cumulative order utility function will be described further.

2.5 More Complex Models of Linear Optimization

The problem described above in Sect. 2.1.2 deals with the linear optimization model that allows continuous changes of variables. However, this is not true in many situations where variables can take certain values only, for instance, integers.

2.5.1 Integer Linear Optimization Model

Let us take the following example to consider such model. We assume that a production shop has 100 machines of type 1 and 200 machines of type 2; machines of each type can produce items A_1 and A_2. Table 2.8 shows machine capacities, the cost of item for each type, and minimum daily output plan.

The planning task is to calculate the number x_{ij} of machines of j-type, $j = 1,\ 2$, which should be allocated to manufacture items A_i, $i = 1,\ 2$ provided that the maximum price $f(x)$ of output products manufactured per day is achieved.

In this situation, the objective function (the daily output price) shall tend to the maximum value, i.e.

$$\begin{aligned} f(x) = {} & 20 \times 6 \times x_{11} + 15 \times 6 \times x_{12} + 35 \times 4 \times x_{21} + 30 \times 4 \times x_{22} \\ & \to \max. \end{aligned} \tag{2.33}$$

The limits on machine count are as follows:

$$\begin{gathered} x_{11} + x_{21} = 100 \\ \text{and} \\ x_{12} + x_{22} = 200. \end{gathered} \tag{2.34}$$

Minimum output limits are as follows:

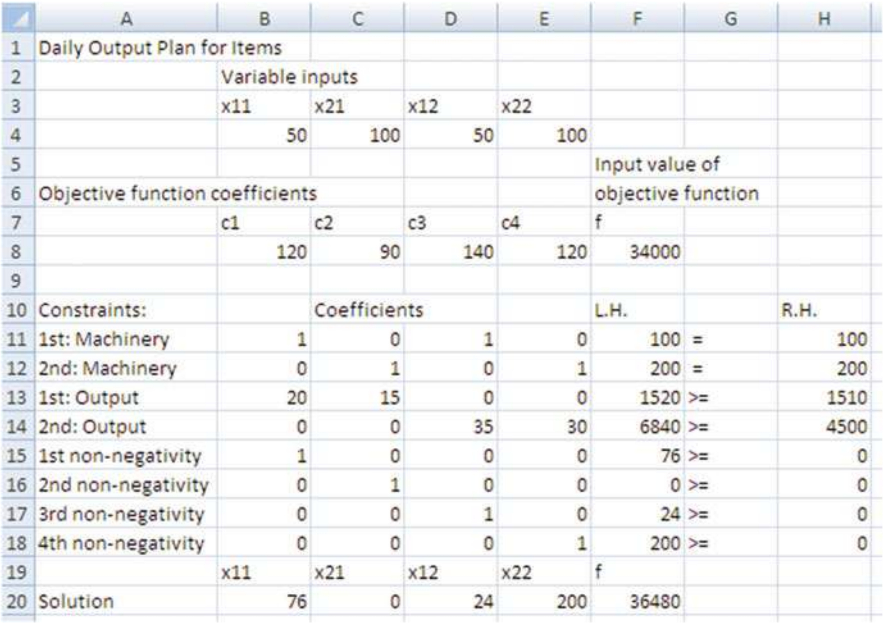
$$\begin{gathered} 20 \times x_{11} + 15 \times x_{12} \geq 1510 \\ \text{and} \\ 35 \times x_{21} + 30 \times x_{22} \geq 4500. \end{gathered} \tag{2.35}$$

Besides, we need to specify that x_{ij} can take positive values only

$$x_{ij} > 0 \tag{2.36}$$

and must be integers.

Figure 2.10 shows how this task is solved using MS Excel. The solution is very much alike to the solution obtained in Sect. 2.1.2 (Fig. 2.3). The difference is that we need to apply the constraint that each variable shall be non-negative. The constraints that variables must be integers are entered directly into MS Excel in



	A	B	C	D	E	F	G	H
1	Daily Output Plan for Items							
2		Variable inputs						
3		x11	x21	x12	x22			
4		50	100	50	100			
5						Input value of		
6	Objective function coefficients					objective function		
7		c1	c2	c3	c4	f		
8		120	90	140	120	34000		
9								
10	Constraints:		Coefficients			L.H.		R.H.
11	1st: Machinery	1	0	1	0	100	=	100
12	2nd: Machinery	0	1	0	1	200	=	200
13	1st: Output	20	15	0	0	1520	>=	1510
14	2nd: Output	0	0	35	30	6840	>=	4500
15	1st non-negativity	1	0	0	0	76	>=	0
16	2nd non-negativity	0	1	0	0	0	>=	0
17	3rd non-negativity	0	0	1	0	24	>=	0
18	4th non-negativity	0	0	0	1	200	>=	0
19		x11	x21	x12	x22	f		
20	Solution	76	0	24	200	36480		

Fig. 2.10 Solution to the integer optimization task

the field containing the list of all constraints in addition to the inputs shown in Fig. 2.10.

The review of the obtained solution shows that in this situation we should keep the output of A_1 to the minimum required level and increase the output of A_2 items as much as possible. Despite lower price of the latter items, greater production capacities can yield better economic performance.

2.5.2 Integer Linear Optimization Models with Binary Variables

This model is widely used to solve the so-called assignment tasks. As an example, let us consider the task to distribute multiple jobs between three machines that have

Table 2.9 Balanced assignment task

	Jobs		
Machine	1	2	3
1	32	45	58
2	39	50	65
3	27	41	54

similar process functions but different capacities. If the number of jobs to be assigned equals to the number of machines, the task is called balanced; otherwise, the task is non-balanced.

First, let us consider the case where there are three jobs, i.e. the task is balanced. Table 2.9 shows runtime values for each job (in hours).

We assume that all machines have the same cost of 1 h of operation. Then, it seems practical to distribute jobs between machines in such a way that ensures the minimum total operating time of all machines. In order to solve assignment tasks, we need to introduce the so-called binary variables that may take one of the following possible values only, i.e. 0 or 1. Let us denote such variables as δ_{ij} to describe if it is possible to assign i-job to j-machine. If such assignment is possible, then $\delta_{ij} = 1$; otherwise, $\delta_{ij} = 0$.

The presence of binary variables in a model automatically limits possible values of such variables. If a job is assigned to a certain machine, such job cannot be assigned to another machine and, accordingly, a busy machine cannot be used for another job.

The objective function will be as follows:

$$\begin{array}{l} f(\delta) = 32 \times \delta_{11} + 45 \times \delta_{21} + 58 \times \delta_{31} + 39 \times \delta_{12} + 50 \times \delta_{22} \\ +65 \times \delta_{32} + 27 \times \delta_{13} + 41 \times \delta_{23} + 54 \times \delta_{33} \end{array} \rightarrow \min. \tag{2.37}$$

Machine load limits will be:

$$\begin{array}{l} \delta_{11} + \delta_{21} + \delta_{11} = 1; \\ \delta_{12} + \delta_{22} + \delta_{32} = 1; \\ \delta_{13} + \delta_{23} + \delta_{33} = 1. \end{array} \tag{2.38}$$

Job assignment limits will be:

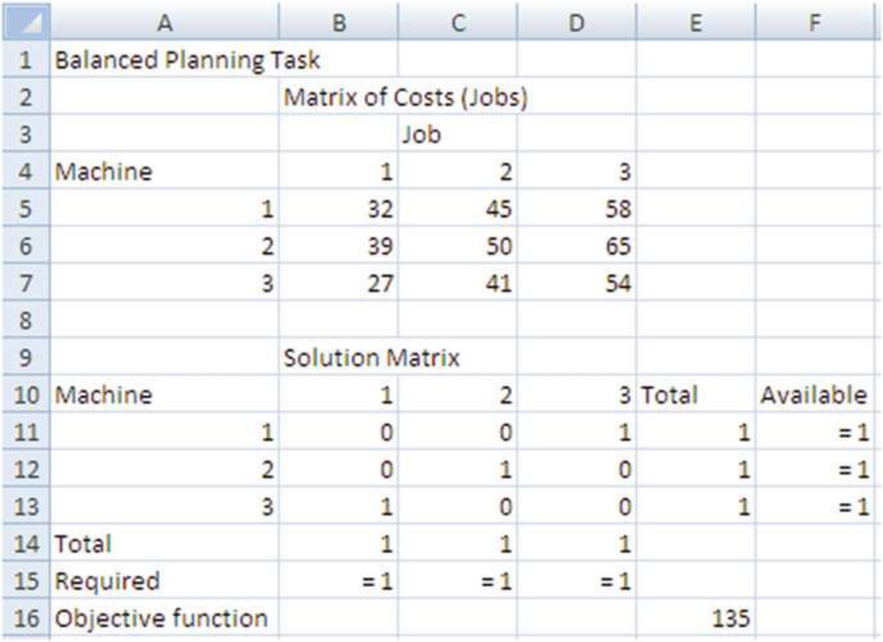
$$\begin{array}{l} \delta_{11} + \delta_{12} + \delta_{13} = 1; \\ \delta_{21} + \delta_{22} + \delta_{23} = 1; \\ \delta_{31} + \delta_{32} + \delta_{33} = 1. \end{array} \tag{2.39}$$

And we also need to add constraints of variable’s binary.

Figure 2.11 shows the task solution using MS Excel. As we can see, job 3 is assigned to machine 1, job 2 is assigned to machine 2, and job 1 is assigned to machine 3. Actually, this plan seems quite surprising. On the face of it, it appears that the most productive machine 3 should have assigned the most prolonged job 3 and machine 2 with low capacity should do the least prolonged job 1. However, detailed calculations disprove this intuitive assumption.

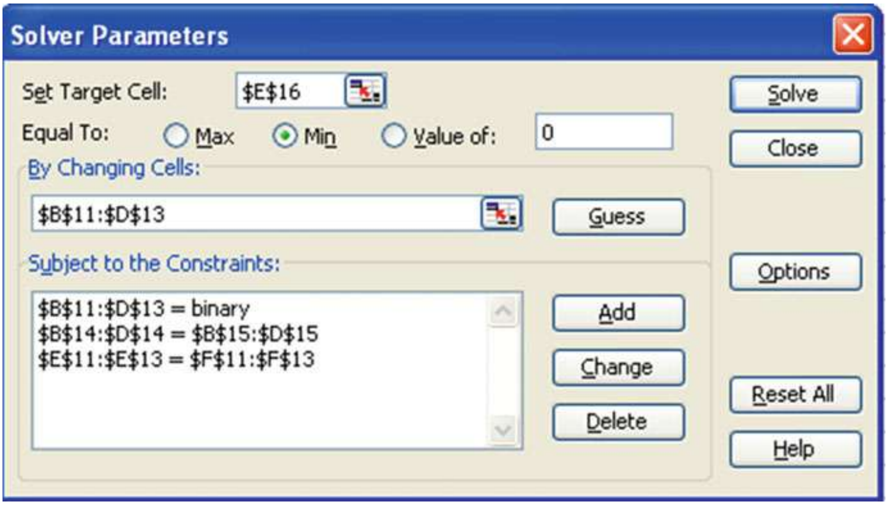
For your reference, Fig. 2.12 shows the objective function settings and task constraints in MS Excel.

Now, let us add another job to this task to make more complex (refer to Fig. 2.13). The task is non-balanced now and we have to assign two jobs to one machine. In this situation, the objective function (2.37) will have 12 δ_{ij} variables



	A	B	C	D	E	F
1	Balanced Planning Task					
2		Matrix of Costs (Jobs)				
3			Job			
4	Machine	1	2	3		
5	1	32	45	58		
6	2	39	50	65		
7	3	27	41	54		
8						
9		Solution Matrix				
10	Machine	1	2	3	Total	Available
11	1	0	0	1	1	=1
12	2	0	1	0	1	=1
13	3	1	0	0	1	=1
14	Total	1	1	1		
15	Required	=1	=1	=1		
16	Objective function				135	

Fig. 2.11 Job distribution between three parallel machines



Solver Parameters

Set Target Cell: $E$16

Equal To: ○ Max ● Min ○ Value of: 0

By Changing Cells:

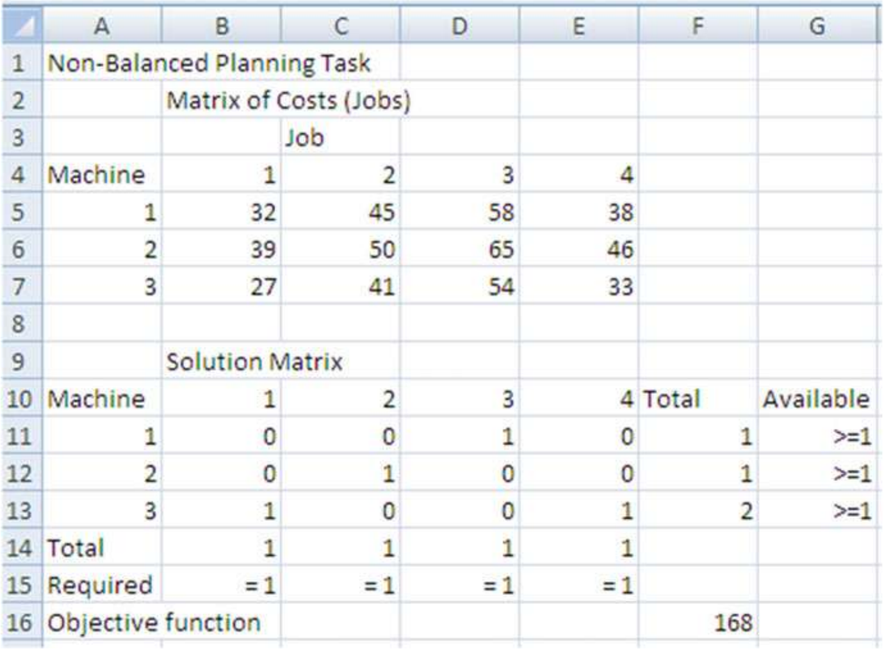
$B$11:$D$13

Subject to the Constraints:

$B$11:$D$13 = binary
$B$14:$D$14 = $B$15:$D$15
$E$11:$E$13 = $F$11:$F$13

Fig. 2.12 Constraints screen

instead of 9. The number of constraints (2.38) will remain the same (3) according to the number of jobs; however, the left-hand part will have 4 δ_{ij} variables and it can be equal or more than 1. Constraints (2.39) will remain the same, but their number will increase to 4 (one constraint per each job).



	A	B	C	D	E	F	G
1	Non-Balanced Planning Task						
2		Matrix of Costs (Jobs)					
3			Job				
4	Machine	1	2	3	4		
5	1	32	45	58	38		
6	2	39	50	65	46		
7	3	27	41	54	33		
8							
9		Solution Matrix					
10	Machine	1	2	3	4	Total	Available
11	1	0	0	1	0	1	>=1
12	2	0	1	0	0	1	>=1
13	3	1	0	0	1	2	>=1
14	Total	1	1	1	1		
15	Required	=1	=1	=1	=1		
16	Objective function					168	

Fig. 2.13 Distribution of four jobs between three machines

The task solution displayed in the lower part of Fig. 2.13 shows that two jobs are assigned to the most productive machine 3 (as expected) and remaining machines have one job each.

2.6 Fixed Job Sequence Models

At the beginning of this chapter we named three main planning challenges, i.e. What products? What quantity? When? Task and solving methods described above do not deal with the practical and reasonable choice of product types. The above tasks mainly deal with the optimal quantity of products to be manufactured or delivered based on a certain criterion. The job sequence can be determined under different rules. This causes a wide spread of values of the objective function. Besides, any of the applied rules does not warranty an accurate and explicit solution to the optimization task (except for a number of special cases described in Sect. 2.3.2). In this section, we will describe the so-called branch-and-bound method that allows obtaining accurate and explicit solutions.

As a general matter, there is no universal algorithm that yields an optimal solution. Therefore, various planning situations require different versions of the branch-and-bound method that are the most appropriate under given circumstances.

2.6.1 Branch-and-Bound Method with Minimum Cumulative Tardiness T^w

The branch-and-bound method includes two procedures: branching and bounding (finding value limits). *Branching* is to split the feasible region (set of candidate solutions) into smaller subsets. Its recursive application results in a *search tree* or the *branch-and-bound tree*. The *nodes of this tree* represent subsets of candidate solutions. *Bounding* is to compute upper and/or lower bounds to find an optimal solution within feasible subsets.

Branching and bounding procedures can be different for different tasks. First of all, branching and bounding can start either at the final or initial stage of job performance. In the first case, it is better to use the algorithm described in (Sule 2007).

Let us see how to apply the branch-and-bound method using the example as per Table 2.7. As the objective function of planning, the same as in Sect. 2.3.1 we take minimized T^w that is the cumulative weighted tardiness (2.18). As the required sequence includes 4 jobs, the search tree will also have 4 levels.

In order to build the search tree let us remember that any of the four jobs as per Table 2.7 can, in fact, go to the last in the job sequence. Now let us determine which of the jobs will have the least effect on T^w if performed last, i.e. we will find the lower bound of T^w. The best way to graphically display step operations to find a solution is to build a search tree (refer to Fig. 2.14).

Level 4 contains 4 nodes that reflect all possible jobs as per Table 2.7. In order to determine T^w for each node at this level, let us calculate the total processing time of all jobs:

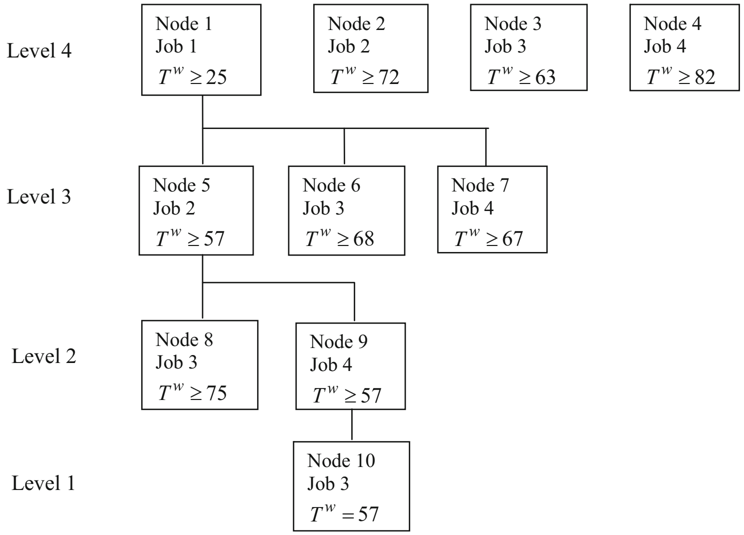

Level 4
Node 1 Job 1 $T^w \geq 25$
Node 2 Job 2 $T^w \geq 72$
Node 3 Job 3 $T^w \geq 63$
Node 4 Job 4 $T^w \geq 82$
Level 3
Node 5 Job 2 $T^w \geq 57$
Node 6 Job 3 $T^w \geq 68$
Node 7 Job 4 $T^w \geq 67$
Level 2
Node 8 Job 3 $T^w \geq 75$
Node 9 Job 4 $T^w \geq 57$
Level 1
Node 10 Job 3 $T^w = 57$

Fig. 2.14 Search tree for cumulative tardiness T^w

$$\sum_{i=1}^{4} p_i = 20 + 25 + 4 + 18 = 67\,\text{days}.$$

If job 1 goes last, it will be completed only 67 days after the day of starting the whole job package (we assume that the start date is the same as the planning date). In this case, tardiness on this job from the due date (42 days after the planning date) will be $67 - 42 = 25$ days. Given that the weight coefficient of job 1 $w_1 = 1$:

$$T_1^w = \ 1 \times 25 = 25.$$

Applying the similar procedure to the remaining nodes of level 4, we get:

$$T_2^w = 2 \times (67 \ - \ 31) = 72;$$

$$T_3^w = 1 \times (67 \ - \ 4) = 63;$$

$$T_4^w = 2 \times (67 - 26) = 82.$$

As we can see, we will get the least weighted tardiness if job 1 goes the last. Besides, in any other case, no matter which job sequence is selected, we cannot get tardiness less than T_1^w. So, in our situation, T_1^w is the lowest bound of T^w for the whole search tree. Therefore, it is viable to continue branching to the next level from node 1 (refer to Fig. 2.14).

This node will have three outgoing branches corresponding to three remaining jobs. Then, we need to calculate T_i^w at each resulting node. First, let us calculate the total processing time of jobs p_i at created nodes: $\sum_{i=1}^{3} p_i = 25 + 4 + 18 = 47$. At each of the nodes 5, 6, and 7, T_i^w will exceed the sum of T_1^w and the weighted tardiness value for a job at a respective node. For instance,

$$T_5^w \geq 25 + 2(47 - 31) = 57$$

and similarly, $T_6^w \geq 25 + 1(47{-}4) = 68$, $T_7^w \geq 25 + 2(47{-}26) = 67$.

After we have arranged level 3 nodes, we get 5 nodes that have no outgoing search branches. These nodes include "dead-end" nodes at level 3 and "dead-end" nodes 2, 3, and 4 at level 4. Node 5 at level 3 (job 2) has the least T_i^w. That is why we continue branching from this node 5. As a result, we get nodes 8 and 9. The remaining processing time is $\sum_{i=1}^{2} p_i = 4 + 18 = 22$, $T_8^w \geq \ 57 + 1(22{-}4) = 75$, and $T_9^w \geq 57 + \max\left(2(22{-}26), 0\right) = 57$. So, the last branch should go from node 9 that represents job 4.

Finally, we get only job 3 at node 10 (level 1); T_{10}^{w} is no longer the lowest bound of potential weighted tardiness but its precise value. This solution matches the solution resulted in Sect. 2.3.1 from the EDD rule, thus confirming Theorem 5 described in Sect. 2.3.2.

2.6.2 Branch-and-Bound Method with Maximum Average Utility $\overline{V}$

Now, let us consider the same job example as per Table 2.7 and build the search tree at the initial stage of job performance. The search tree is shown in Fig. 2.15.

At the start point of job performance (node 0) utility of outstanding jobs is at its maximum and is calculated using formula (2.32). Similar to as in Sect. 2.4.1, we

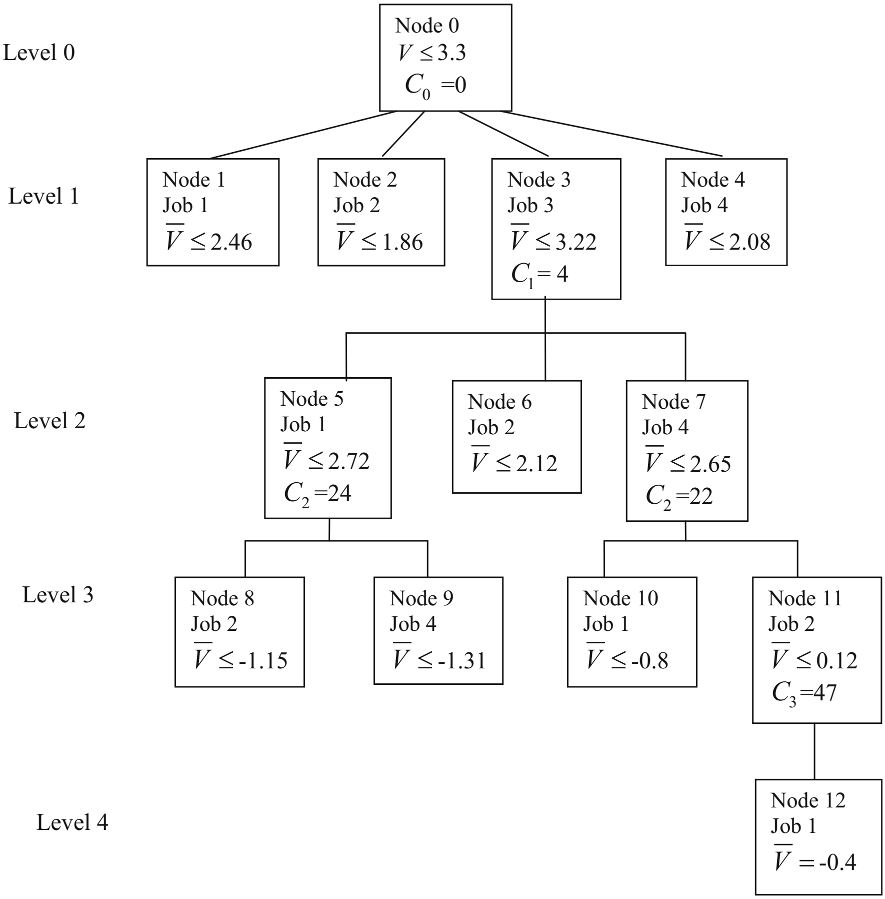


Fig. 2.15 Search tree for maximum average utility $\overline{V}$

assume that planning period $G = 30$ days and psychological coefficient $\alpha = 0.1$; then, using H_i resulting from iteration 1, we get:

$$V_0 = \sum_{i=1}^{n} V_i = \frac{1}{G}\sum_{i=1}^{n} w_i p_i - \sum_{i=1}^{n} H_i \quad \text{at} \quad t = 0$$

or

$$\begin{aligned} V_0 &= \frac{1 \times 20 + 2 \times 25 + 1 \times 4 + 2 \times 18}{30} - (0.044 + 0.14 + 0.06 + 0.12) \\ &= 3.66 - 0.36 = 3.3. \end{aligned}$$

In fact, any job from the job package can go first. That is why level 1 includes 4 nodes corresponding to these jobs. During the runtime of each job, intensity is rising and, consequently, utility is dropping.

Let us assume that at a certain time point $C_l l$-jobs are completed and k-job starts. Now, let us find the average utility of the whole job package for the whole time interval C_l+p_k from the start date to k-job:

$$\overline{V}_{l+1,k} = \frac{1}{C_l + p_k}\int_0^{C_l+p_k} V dt = \frac{1}{C_l + p_k}\left(\overline{V}_l C_l + \int_{C_l}^{C_l+p_k} V dt\right). \tag{2.40}$$

In expression (2.40), $\overline{V_l}$ equals to the average utility of the whole planned job package from the start time $t = 0$ till time point C_l representing the completion of l jobs. For instance, at the start time $t = 0$ the number of completed jobs $l = 0$; so, $C_0 = 0$. As the first stage (level) allows any job to be performed, let us assume that job 3 will go first. In this case, after the completion of this job, the average utility will be determined as:

$$\overline{V}_{1,3} = \frac{1}{p_3}\int_0^{p_3} V dt.$$

If the average utility for job 3 exceeds that of any other job, i.e. $\overline{V}_{1,3} = \max\left(\overline{V}_{1,i}\right)$ at $i = 1,\ 2,\ 3,\ 4$, this job will go first, and the maximum average utility $\overline{V}_1$ at level 1 will be equal to $\overline{V}_{1,3}$.

At the next level (level 2) $l = 1$, $C_1 = p_3$ and, for instance, for job 2:

$$\overline{V}_{2,2} = \frac{1}{p_3 + p_2}(\overline{V}_1 p_3 + \int\limits_{p_3}^{p_3+p_2} Vdt) \text{ etc.}$$

As we can see, in order to calculate the average utility values, we need to calculate integrals in expression (2.40). For this purpose, let's introduce the complete set J of all jobs to be planned, and the set J_l for the jobs that have already been sequenced. In our example, $J = \{1, 2, 3, 4\}$. If, for instance, job 3 is sequenced first, then at level 2 $J_l = J_1 = \{3\}$.

Now, let us use formula (2.32):

$$\int\limits_{C_l}^{C_l+p_k} Vdt = \frac{1}{G}\int\limits_{C_l}^{C_l+p_k} \sum_{i\in J-J_l} w_i p_i dt - \int\limits_{C_l}^{C_l+p_k} \sum_{i\in J-J_l} H_i dt, \tag{2.41}$$

Note that in expression (2.41) summation applies to those jobs from J-set that are not included into J_l set of jobs planned before.

Each sum in expression (2.41) has two components, i.e. jobs for which $i \neq k$ and job with $i = k$. For the first integral in (2.41) we get:

$$\begin{gathered}\frac{1}{G}\int\limits_{C_l}^{C_l+p_k} \sum_{i\in J-J_l} w_i p_i dt = \frac{p_k}{G}\sum_{i\in J-J_l} w_i p_i \quad \text{at} \quad i \neq k \\ \text{and} \\ \frac{w_k}{G}\int\limits_{C_l}^{C_l+p_k} (p_k - (t - C_l))dt = \frac{w_k p_k^2}{2G} \quad \text{at} \quad i = k.\end{gathered} \tag{2.42}$$

Using the time-based dependence of H_i at $i \neq k$ and $d_i - C_l - p_k \geq 0$ under expressions (2.28), we get:

$$\int\limits_{C_l}^{C_l+p_k} H_i dt = \alpha w_i p_i \ln\left(\frac{(d_i - C_l)/\alpha G + 1}{(d_i - C_l - p_k)/\alpha G + 1}\right). \tag{2.43}$$

For the job performed, i.e. at $i = k$ and $d_k - C_l - p_k \geq 0$, assuming that this job becomes less linear with time, we get:

$$\int\limits_{C_l}^{C_l+p_k} H_k dt = \alpha w_k \left[p_k - (d_k - C_l - p_k + \alpha G)\ln\left(\frac{(d_k - C_l)/\alpha G + 1}{(d_k - C_l - p_k)/\alpha G + 1}\right)\right]. \tag{2.44}$$

If C_l+$p_k - d_i > 0$ and $d_i > C_l$, integral splits into two parts:

$$\int_{C_l}^{C_l+p_k} H_i dt = \int_{C_l}^{d_i} H_i dt + \int_{d_i}^{C_l+p_k} H_i dt. \tag{2.45}$$

At $i \neq k$ the first part will be:

$$\int_{C_l}^{d_i} H_i dt = \alpha w_i p_i \ln\left(\frac{d_i - C_l}{\alpha G} + 1\right)$$

And the second part will be : (2.46)

$$\int_{d_i}^{C_l+p_k} H_i dt = \frac{\alpha w_i p_i}{2}\left[\left(\frac{C_l + p_k - d_i}{\alpha G} + 1\right)^2 - 1\right].$$

For the job performed $i = k$ at $C_l + p_k - d_k > 0$ and $d_k > C_l$ we get:

$$\int_{C_l}^{d_k} H_k dt = \alpha w_k \left[d_k - C_l + (C_l + p_k - d_k - \alpha G)\ln\left(\frac{d_k - C_l}{\alpha G} + 1\right)\right]$$

and (2.47)

$$\int_{d_k}^{C_l+p_k} H_k dt = \frac{\alpha w_k (C_l + p_k)}{2}\left[\left(\frac{C_l + p_k - d_k}{\alpha G} + 1\right)^2 - 1\right]$$
$$-\frac{w_k}{2G}\left(1 - \frac{d_k}{\alpha G}\right)[(C_l + p_k)^2 - d_k^2] - \frac{w_k}{3\alpha G^2}[(C_l + p_k)^3 - d_k^3].$$

If C_l+$p_k - d_i > 0$ and $d_i \leq C_l$, first two components in (2.46) and (2.47) will become zero.

As an example, let us determine the average utility $\overline{V}_{1,3}$ for job 3 at level 1. Here, $l = 0$, the set of jobs to be planned is empty $J_l = J_0$ and the start time of a new job $C_l = C_0 = 0$. So:

$$\overline{V}_{1,3} = \frac{1}{p_3}\int_0^{p_3} V dt = \frac{1}{G}\sum_{i \in J, i \neq 3} w_i p_i + \frac{w_3 p_3}{2G} - \frac{1}{p_3}\sum_{i \in J, i \neq 3}\int_0^{p_3} H_i dt - \frac{1}{p_3}\int_0^{p_3} H_3 dt. \tag{2.48}$$

The first and the second components in (2.48)

$$\frac{1}{G}\sum_{i\in J,\, i\neq 3} w_i p_i = \frac{1}{30}\times(1\times 20 + 2\times 25 + 2\times 18) = 3.53;$$

$$\frac{w_3 p_3}{2G} = \frac{1\times 4}{2\times 30} = 0.07.$$

The third component:

$$\frac{1}{p_3}\sum_{i\in J,\, i\neq 3}\int\limits_0^{p_3} H_i dt = \frac{1}{p_3}\left(\int\limits_0^{p_3} H_1 dt + \int\limits_0^{p_3} H_2 dt + \int\limits_0^{p_3} H_4 dt\right). \tag{2.49}$$

As due dates for jobs 1, 2, and 4 exceed $p_3 = 4$, we need to use expressions (2.43) in order to calculate integrals in (2.49). For example:

$$\frac{1}{p_3}\int\limits_0^{p_3} H_1 dt = \frac{\alpha w_1 p_1}{p_3}\ln\left(\frac{(d_i - C_l)/\alpha G + 1}{(d_i - C_l - p_k)/\alpha G + 1}\right)$$

or

$$\frac{1}{p_3}\int\limits_0^{p_3} H_1 dt = \frac{0.1\times 1\times 20}{4}\ln\frac{(42-0)/(0.1\times 30)+1}{(42-0-4)/(0.1\times 30)+1} = \ 0.05.$$

And similarly, $\frac{1}{p_3}\int\limits_0^{p_3} H_2 dt = 0.15,\ \frac{1}{p_3}\int\limits_0^{p_3} H_4 dt = 0.13.$

For the last component in (2.48) we should use formula (2.44) that results in $\frac{1}{p_3}\int\limits_0^{p_3} H_3 dt = 0.04.$

Now, let us gather all the components of (2.48) and get:

$$\overline{V}_{1,3} = 3.53 + 0.07 - 0.05 - 0.16 - 0.13 - 0.04 = 3.22.$$

As we can see from Fig. 2.15, other nodes at level 1 have smaller utility values than for job 3. That is why we should continue branching from job 3, i.e. $\overline{V_1} = \overline{V}_{1,3} = 3.22$. After job 3 is completed, the number of completed jobs $l = 1$ and, consequently, $C_1 = 4$, and the set of already planned jobs $J_1 = \{3\}$.

Now, let us determine the average utility at level 2, for example, for job 1, i.e. $\overline{V}_{2,1}$. Using expression (2.40), we get:

$$\overline{V}_{2,1} = \frac{1}{C_1 + p_1}\left(\overline{V}_1 C_1 + \int\limits_{C_1}^{C_1+p_1} V dt\right) = \frac{\overline{V}_1 C_1}{C_1 + p_1}$$
$$+\frac{p_1}{(C_1 + p_1)G}\sum_{i\in J-J_1,\, i\neq 1} w_i p_i + \frac{w_1 p_1^2}{2(C_1 + p_1)G} - \frac{1}{C_1 + p_1}\int\limits_{C_1}^{C_1+p_1}\sum_{i\in J-J_1} H_i dt. \tag{2.50}$$

The integral in (2.50) has three components corresponding to jobs 1, 2, and 4 with job 1 being performed; so, C_1+p_1 = 4+20 = 24. As the due dates for jobs 2 and 4 (Table 2.8) exceed C_1+p_1 = 24, we should use expressions (2.43) to calculate integrals in (2.50). The due date of job 1 also exceeds C_1+p_1; so, we should calculate the respective integral in (2.50) using formula (2.44). Our calculations will result in:

$$\overline{V}_{2,1} = 0.55 + 2.38 + 0.28 - 0.02 - 0.23 - 0.24 = 2.72.$$

After calculating average utility bounds for jobs 1, 2, and 4 at level 2, we can see that $\overline{V}_{2,1}$ is the greatest; so, we take it as $\overline{V}_2$. Consequently, the next branch should go from node 5 (job 2). As we can see from Fig. 2.15, we get nodes 8 and 9 with quite low (negative) values of average utility. As such values are lower utility values at nodes 6 and 7 that have not been branched yet, we need to branch them too. Note that node 7 will have priority for branching as it has greater upper bound $\overline{V}$.

Branching from node 7 gives us nodes 10 and 11 with the latter having greater $\overline{V}$. Technically, as node 11 has greater $\overline{V}$ than node 6 (level 2) that has not been branched yet, we would need to branch this node. However, some reasonable assumptions may reduce the number of branches.

For example, we can assume that if $\overline{V}_{li}$ for a certain job is lower than for another job $\overline{V}_{lk}$ at the same level, such relation will be consecutively true for some levels down and if the difference between these values at subsequent levels does not decrease, we can skip branching from the nodes corresponding to job $\overline{V}_{li}$.

Now, let us consider such relations at levels 1 and 2 (Fig. 2.15). For jobs 1 and 4, $\overline{V}_{1,1} > \overline{V}_{1,4}$ at level 1; at level 2, $\overline{V}_{2,1} > \overline{V}_{2,4}$, however, the difference is $\overline{V}_{1,1} - \overline{V}_{1,4} = 2.46 - 2.08 = 0.38$ and $\overline{V}_{2,1} - \overline{V}_{2,4} = 0.07$, i.e. the difference substantially decreased. That is why we should do branching from node 7 representing job 4. However, there is no need of branching from node 6 (job 2) as $\overline{V}_{1,4} - \overline{V}_{1,2} = 0.22$ and $\overline{V}_{2,4} - \overline{V}_{2,2} = 0.53$, i.e. the difference increased rather than decreased.

So, we get only job 1 at the last level (level 4) and the average utility for the whole job package will be $\overline{V} = -0.4$. Generally speaking, as we can see from Fig. 2.15, the average utility of the whole job package gradually decreases as far as jobs progress. First of all, this happens due to the decrease in the total processing time of jobs to be done. Another reason is intensity that rises as far as the job due date gets closer. However, the average utility may rise in certain situations. For example, average utility values increase if jobs 1, 2, and 4 are performed at level

2 instead of level 1. The reason for this is a sharp decrease of intensity if job 3 is performed first.

Please note that the resulting job sequence $J = \{3,\ 4,\ 2,\ 1\}$ matches the job sequence resulting from the EDD and ATC rules described in Sect. 2.3.1. However, it does not match the sequence $J = \{2,\ 3,\ 4,\ 1\}$ resulting from the intensity-based rule described in Sect. 2.4.1. This demonstrates that despite high intensity associated with a job because of a high priority assigned to such job we must carefully check all assigned priorities. This is an often case that an urgent performance of one job can lead to sharp increase in intensity associated with other jobs. In our example, if high-priority job 2 is completed before job 3, this causes a sharp increase of job 3 intensity and decrease in utility functions as shown in Fig. 2.9.

As you can see, this optimization method is much difficult than other methods described in this chapter. However, it yields a precise solution and can be applied to highly complicated planning task as will be described below.

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Production Bottlenecks Models 3

3.1 Theory of Constraints

Modelling of production bottlenecks refers to the earliest historical methods of production planning. Despite the introduction of a number of different ways of planning within recent decades, the planning for bottleneck remains in the spotlight and being developed effectively. Currently, the best known such method is the so-called Theory of Constraints. This theory appeared first as means of solving purely industrial problems, but then its use was extended to a variety of general management issues.

3.1.1 Fundamentals of Theory of Constraints

Theory of Constraints (TOC) considers organizations as systems consisting of resources interconnected by ongoing processes in the system. The goal of the organization is the main indicator of success. Within this system, the constraint is defined as something that limits the system in achieving higher performance with regard to its target. Distribution of interdependencies within the organization creates an analogue of chain or chain network, which consists of the processes of the system. Just as the strength of the chain is determined by its weakest link, from the point of view of the theory of constraints, the ability of any organization to achieve its goals depends on one or more constraints.

Originally, the theory of constraints was developed as Optimized Production Technology (OPT), controlling the flow of material at the enterprise. Already in this technology system, which has not been widespread though due to a number of reasons, the accounting rules for the capacity constraints and identifying bottlenecks were used. Theory of Constraints has extended these ideas to any activity of a commercial company and showed that the results of this activity depend greatly on the weakest (unproductive) link. Moreover, improvements in the work of other units lead to not only non-improvement of company's

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performance but also degradation of its performance, as in this case material and other supplies can accumulate superfluously.

In the theory of constraints, it is proved that in any system there are only a few real constraints (bottlenecks). Very quickly after the initial feedstock, the material starts to be processed in the production system; the material begins to accumulate in one form or another in one of the bottlenecks. After passing the first bottleneck the situation can be repeated elsewhere in the processing. These bottlenecks of the system determine its important (from the point of view of the theory of constraints) property—throughput. In the theory of constraints "throughput" refers to the number of products that can be produced and sold in the market.

The throughput is always limited in one of the three directions:

- Market that limits the production volume
- Suppliers that cannot provide the enterprise with raw materials
- Internal resources of the enterprise—the resource undercapacity or lack of competence.

If the direction with the constraint is determined then the correct planning can weaken the impact of this constraint and increase throughput. For example, if the constraint is the sales, it is obviously necessary to enhance the level of customer service, i.e. to reduce the time for order fulfilment, if possible. With constraints in the amount of raw materials the production should be planned so that stocks of unfinished products were as small as possible. Of course, the possibilities of planning with the first two kinds of constraints are not so great, which is vice versa impossible to say about the possibilities of planning of internal resources use. The main scope of advanced planning is exactly in the area of optimal use of equipment, personnel, tools and other internal resources.

Methodology of Theory of Constraints includes five steps of throughput improvement by reducing the influence of constraints (Goldrat 1987).

Step 1: identify of the specific constraint in the system.
Step 2: determine methods to weaken this constraint.
Step 3: establish the subordination for all processes, which are not constraints to the process-constraint.
Step 4: take measures to weaken impact of the constraint according to the methods defined in step 2—purchase equipment, hire personnel, etc.
Step 5: since the weakening of one constraint always causes new constraint, it is necessary to go back to step 1 for the next improvement of the system.

The feature of TOC is the method used in Step 3—the so-called Drum–Buffer–Rope process (DBR) to synchronize the flow of products. This method is a combination of some "push" and "pull" methods of production management, referred to in Sect. 1.2.2. Like conventional "push" method the production process is started from one point where raw material is supplied. This point is called the point of planning. However, the moment, when the start must be made, is not set by the original plan but determined by the time of next scheduled operation of the equipment, which is the bottleneck of production. Such equipment is a resource limiting system capacity or Capacity Constraint Resource (CCR), and according to the theory of constraints, it should be considered as a point of control of material flow.

When the control point gets information about the beginning of certain job, the starting point of planning gets the command to launch a new batch of raw material. This process is similar to the "pull" mode of production and called "Rope". Since the CCR dictates the rhythm of the entire production system, the schedule of its work is called "Drum". The throughput of the system is entirely determined by the capabilities of the CCR, so the stocks in the production system should be distributed in the way that the CCR always could start a new job without delay. For this purpose, the system is provided with "Buffer" immediately before the CCR.

The buffer, created before CCR, is of timed nature in the sense that its size, i.e. the number of objects in the buffer, is determined based on the reserve time required to protect the CCR from downtime. This downtime can be caused by a delay in the previous work. Figure 3.1 shows a diagram of a production system where "drum–buffer–rope" process is exercised. The circles in the diagram designate machines and the triangles designate buffers. The bottleneck (CCR) of the system is machine 4, before which the CCR buffer is provided. At the end of the next scheduled job, the "rope" is triggered that starts processing. This control mechanism reduces the in-process stocks significantly, as it prevents "logjam" in front of the bottleneck.

One buffer is not sufficient for the system in Fig. 3.1. Besides the CCR there are always other items in the system that are actually also bottlenecks and should be protected by buffers (Schragenheim and Ronen 1990). These items (filled with black) are called critical and include:

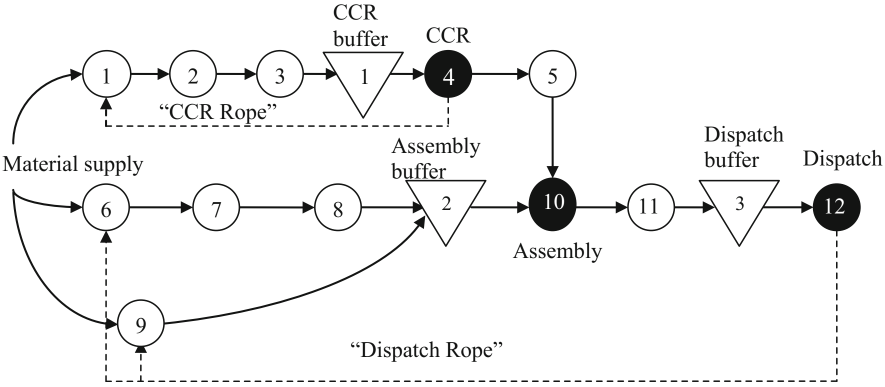


Fig. 3.1 Diagram of "Drum-Buffer-Rope" process

- Resource itself with limited capacity—item 4 in Fig. 3.1
- Any subsequent stage where the part, which is processed by the limiting resource, is assembled with other parts—item 10
- Dispatch of finished products containing the parts processed by the limiting resource—item 12.

Assembly buffer ensures the timely supply of the assemble objects, the manufacture of which is not limited by constraints, so that the rate of assembly is not less than the rate of CCR operation. The buffer eliminates idling of finished goods at the dispatch, which also allows having the dispatch rate not less than the rate of CCR. Note that in the diagram in Fig. 3.1, all production chains, in which there are no constraints, are started by a signal of the start of the finished products dispatch. This signal is made by special “rope”, which is shown in the diagram.

3.1.2 Bottleneck Operation Planning

Theory of Constraints provides that the planning of the entire production system starts with the planning of the bottleneck. To do this, first of all, the master plan of finished-product output must be prepared and lot size of the manufactured objects must be determined. Often when using Theory of Constraints they believe that lot sizes of each object must accurately meet the demand for this object for output (ordered) lot of finished products—the so-called lot-for-lot planning.

However, the increase in lot size improves operation with CCR significantly, because this reduces the time spent on changeovers. Therefore, in determining the number of objects to be produced within a certain timeframe, it is reasonable to consider these objects in all current orders. For this, we consider the explosion tree of finished products.

In his paper Goldrat (1987) states that during planning it makes sense to combine data on bill of materials and technological processes in one tree specification. The diagram in Fig. 3.2 illustrates this idea. The diagram example assumes that there are two parallel orders in production and for order 1 products of type A are produced and for order 2 type B. The nodes of the tree specification represent assembly units and parts, and some of these objects are included simultaneously in the structure of both products. Product A elements are described as A1, A2, etc., and product B as B1, B2, etc. As can be seen from Fig. 3.2, product B includes elements A3, A10, A11, and A12, taken from product A.

In the nodes that display assembly units, there is branching to other units and parts, for example, A1, B1, etc. For every part a node chain is connected to the tree and displays relevant process steps. In Fig. 3.2, an entry in this node consists of a part designation and code (number) of the machine on which the current operation is performed; at that, the chain of process nodes is connected to a tree in the reverse sequence of operations. For example, for part A4 four operations should be performed, the last of which is performed on machine type 4.

Since the machine codes are ranged in order matching the process operation sequence for this particular part, the designations in the nodes are presented as

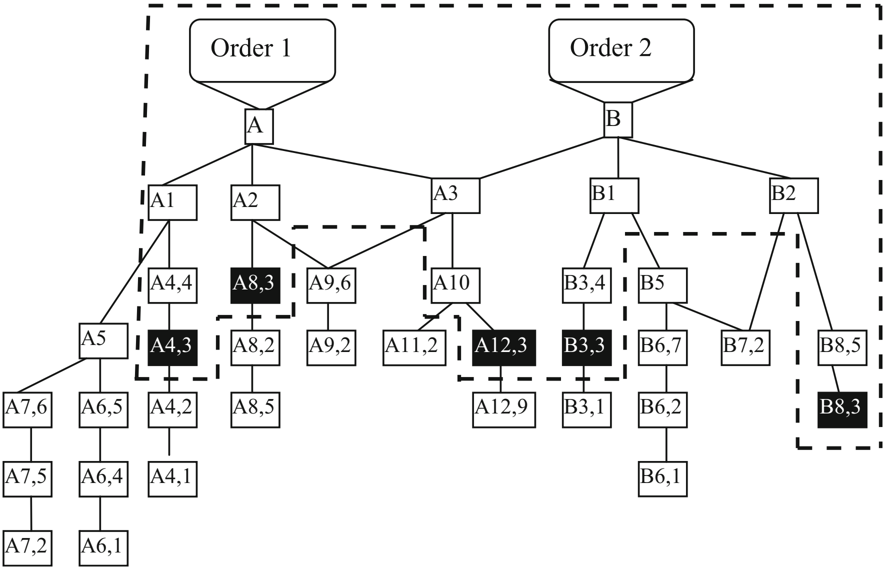


Fig. 3.2 Tree specification for application of Theory of Constraints

follows: A4,4; A4,3; A4,2; A4,1. For any other part the sequence of machines can be in various orders. For example, part B3 should be processed sequentially on machines 1, 3, and 4, which is reflected in Fig. 3.2 as a set of nodes arranged in the reverse order.

Assume that the bottleneck in the production is a machine with code 3, the corresponding nodes of which are marked by black fill. According to Theory of Constraints, the entire tree network in Fig. 3.2 should be divided into two sections, which are marked by dashed line in the figure. The section inside the dashed line includes the CCR (machine 3) and all nodes to operations, performed after passing the bottleneck. For these operations according to Theory of Constraints, the scheduling is "forward" in time. For other operations outside the dashed line, the scheduling is "backwards" in time. Both of these calculations are based on the initial calculation of the CCR operation, i.e. machine 3 in this case.

When scheduling it is assumed that all the parts of the same type, required according to the built tree specifications, are processed by the Capacity Constraint Resource (CCR) in the same lot. At the same time, the transfer to the subsequent operations (not being limitations) to reduce the cycle time can be made in smaller lots. Table 3.1 shows the input data for this calculation, including machine code and (with slant) processing time for each operation.

In addition to the processing time, Table 3.1 presents data on the dispatch of finished products, which includes parts passing through the CCR. If the parts of one type are intended for different finished products such as the part A12, then the minimum time of shipment is indicated for it. The products dispatch, as it was

Table 3.1 Duration of operations for the parts passing the CCR

Part code	Machine code and processing time of the lot in hours for operation no.				Duration of subsequent operations before dispatch	Dispatch according to master plan	
	1	2	3	4		Date	Hour
A4	1/3	2/4	3/4	4/1	8	Day 5	32
A8	5/2	2/1	3/3	–	3	Day 5	32
A12	9/1	3/2	–	–	7	Day 5	32
B3	1/2	3/4	4/2	–	10	Day 6	40
B8	3/4	5/4	–	–	7	Day 6	40

shown above, requires the time buffer, which in this case is assumed to be 8 h. The duration of subsequent operations before dispatch is considered to be the longest duration by all the branches from the end of the lot processing to dispatch.

It can also be assumed that the working time fund of machine 3 is 8 h and the machine is loaded with pre-scheduled work to hour 4 of the second day. In determining the order of works performance on machine 3, we will use the rule of job with earliest due date (EDD) (Sect. 2.3.1). To do this it is necessary to set the time of termination for each job required to perform all subsequent jobs.

For example, to determine the required due moment for part A4 processing on machine 3 it is necessary to subtract duration of dispatch buffer, duration of subsequent assembly operations with part A4, as well as lot processing time after the passage of the bottleneck.

$$d_{A4} = 32 - 8 - 8 - 1 = 15.$$

As the deadlines for processing parts A8, A12, B3, B8 are 21, 17, 20, 21, respectively, we accept the sequence {A4, A12, B3, A8, B8}. In this sequence for jobs A8 and B8, having the same required due moment, the job with shortest processing time A8 goes first, i.e. the priority rule SPT (Sect. 2.3.1), is applied. Figure 3.3 shows a diagram of the CCR work in the form suggested in Vollmann et al. (2005).

In Fig. 3.3, light grey filling designates the time range of pre-scheduled jobs. New jobs are scheduled for the end of the second day, the third and fourth day before noon. Due to the fact that the time remaining by the end of the third day after completion of jobs A12, B3, and A8 is small, it makes no sense to schedule new job for this day. Therefore, the rest of the third day of 1 h is shown by dark grey filling, as a reserve.

After scheduling the CCR jobs it is necessary to schedule forward for the tree nodes inside the area limited by the dashed line in Fig. 3.2. The results of this scheduling based on data of Table 3.1 are listed in Table 3.2. For example, for part A4 according to Fig. 3.3 the processing on machine 3 finishes at moment 16. Considering the duration of the further processing equal to 1 and the duration of

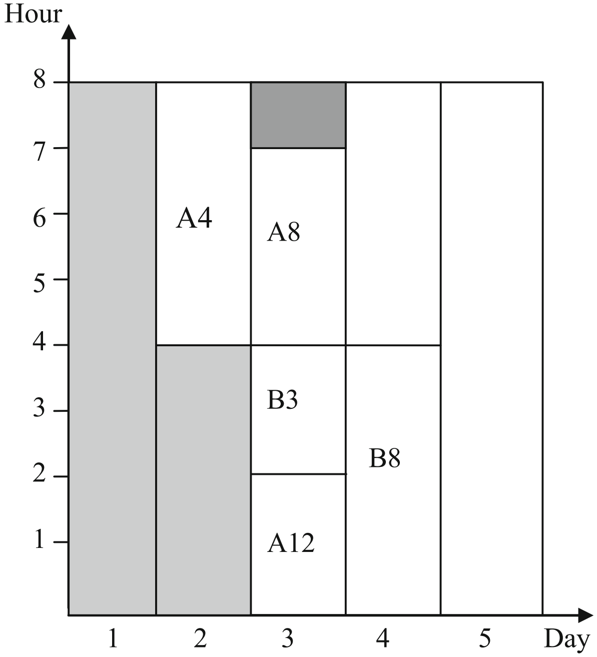


Fig. 3.3 Diagram of bottleneck loading—machine 3

follow-up operations equal to 8 we find that part A4 gets into the dispatch buffer at moment 16 + 1 + 8 = 25.

According to Table 3.2, arrival of finished products A to the dispatch buffer is determined by the duration of the jobs involving parts A8 and must occur within hour 26. This lead time for dispatch is longer than the set standard buffer reserve, which was accepted in amount of 8 h. Therefore, in column "Adjusted plan of dispatch" there is value 34. In practice, most probably, the plan for dispatch of products A need no adjustment. The dispatch buffer for ordered products B, as can be seen from Table 3.2, is not provided by the CCR job plan, and the dispatch plan requires adjustment.

3.1.3 Planning for Buffers, Ropes, and Non-bottleneck Machines

The parts of each type in Table 3.2 before processing on the CCR (machine 3), as it follows from Fig. 3.2, pass through different workflows. In some of them, for example, for parts A4 and A8, the machine code immediately preceding the CCR is the same and in this case equals 2. For part A12 preceding the CCR it is machine 9, for part B3 preceding the CCR it is machine 1, and part B8 starts to be processed on machine 3 at all.

Table 3.2 Time of dispatch in working hours

Part code	Readiness for dispatch (arrival to buffer)	Moment of dispatch according to master plan	Adjusted plan of dispatch (including buffer)
A4	25	32	34
A8	26	32	34
A12	25	32	34
B3	32	40	47
B8	39	40	47

Table 3.3 Estimated moments of "rope" activation for two production cycles

Part code	Activation moment, h
A4	7
A12	11
B3	13
A8	15
B8	19
A4	23
A12	27
B3	29
A8	31
B8	35

The value of the buffer according to Schragenheim and Ronen (1990) should be about three times the value of the average processing time for a lot on all machines immediately preceding the CCR. In this case, we obtain

$$b = 3 \times \frac{4 + 1 + 2 + 1 + 0}{5} \approx 5 \text{ hour.}$$

The CCR "rope" determines the start of the first machine in the process flow, where the CCR is. "Rope" is activated at the moment, when the lot of parts in the time buffer on one of the machines prior to the CCR is transferred to the CCR for processing. Obviously, if the production is quite stable and is repeated cyclically, then at the time of "rope" activation the production should be supplied with a new lot of those parts that come in the buffer and wait for their turn to be processed at the CCR. Therefore, the estimated launch of a new lot of these parts occurs earlier than the scheduled start of processing of the current lot on the CCR by the value of the buffer.

Assuming that machine 3 regularly processes only the parts of the above example, then the cycle of processing is 16 h according to Fig. 3.3. Table 3.3 shows the activation moments for two cycles. For example, for part A4 according to Fig. 3.3 the processing should be initiated at moment 12. Accordingly, start of a lot (rope activation) must occur earlier considering the duration of stay in the buffer equalling to 5, i.e. at moment 7.

As far as the author knows, there are no practical recommendations on the size of the required buffer of assembly and dispatch in the literature. According to various examples cited, these values are generally within half to one working shift.

For the machines that are not the bottleneck, Schragenheim and Ronen (1990) suggested to use the rule of greatest complexity (LPT—longest processing time) and, if possible, not to interrupt the initiated work prior to its completion. It is believed that in this case the possibility of unforeseen delays in the operation of the machines that are not the bottleneck reduces, which in turn reduces the size of the CCR buffer.

If a number of different objects, which are waiting for processing at the CCR, can be simultaneously in the CCR buffer, then it is (Vollmann et al. 2005) often useful to control these objects manually. To do this, the entire set of these objects is usually divided into three zones: red, yellow, and green. The red zone should include the objects, the processing of which according to the plan should begin immediately or very soon. If at least one of these objects is not in the buffer, it should immediately be an alarm with relevant organizational measures.

In the yellow zone of the buffer, the absence of some objects may be acceptable, and necessary decisions should to be taken concerning the others. The absence of objects in the green zone of the buffer is usually not a reason for any arrangements.

Consider a situation that may occur in the CCR buffer of our example. The number of objects in the buffer at different times will be different. For example, at the moment equal to 15 working hours (Fig. 3.3), as the time length of the buffer is equal to 5 h, lots of parts A12 and B3 must be there. In this case, apparently, parts A12 must be in the red area, i.e. mandatory, and B3 are possibly in the yellow zone. Therefore, in the absence of parts A12 in the buffer immediate actions are required, and for parts B3 the actions are likely advisable too.

Consider the situation with 16 h right before putting parts A12 to work on the CCR. At this point three lots of parts must be in the buffer—A12, B3 and A8. Obviously, not only A12, but also B3 must be in the red area, parts A8 may be either in the yellow or green area. After putting parts A12 into operation, parts A8 should most probably go to the yellow zone.

3.1.4 Simple Example of Theory of Constraints in Application

As a simple example of TOC application in the scheduling of discrete production we consider the problem described in Pegels and Watrous (2005). In this chapter, the scheduling of car lighting equipment production is considered. The bottleneck in this production is automatic moulding machines, which produce the bodies for lighting devices. The increase of the capacity in this case can only be achieved through better use of working time fund of these injection moulding machines.

To produce parts of a particular nomenclature provided by the master plan the relevant mould must be installed on the automatic moulding machine. The machines are serviced by the maintenance team, whose correct sequence of work eventually provides the machines operation without downtime. In fact, achieving

Table 3.4 Plan of site operation for January 20

Machine code	Mould type	Processing time of next order, h	Shift number of the next order start
112	M15	35	3
109	M20	86	1
211	M20	54	3
214	M20	125	2
210	N23	45	2
165	N23	78	3
200	P27	30	2
306	P30	44	1
513	Q50	70	2
706	Q70	71	2

the highest possible throughput of the production is defined by the rules of priority when selecting a specific machine queuing for the installation of the mould. To let the maintenance team be able to make the right choice, Pegels and Watrous (2005) suggest to make shift-day target like it is shown in Table 3.4.

For example, for the second shift five changeovers are required—for machines 214, 210, 200, 513, and 706. If on some of these machines the previous work has been completed and changeover is required at the same time, the question of its sequence arises. For example, if it is required simultaneously for machines 214, 210, and 200, the priority is given to machine 214, where the complexity of work is the highest, i.e. rule is used longest processing time or LPT (Sect. 2.3.1). The choice of this rule is based on the intuitive assumption that since the possibility of unforeseen delays of high complexity jobs is significantly higher, then for delays elimination the jobs must be fulfilled as soon as possible. This example shows that even very simple actions can have a significant effect provided the position of the limiting resource is well identified.

3.1.5 Theory of Constraints in Process Manufacturing

Despite the fact that the theory of constraints appeared as a response to the problems arising in the discrete production, it is quite possible to use this theory in the process production. According to Sect. 1.3.1, the main difference between the process production and discrete one is the ability to produce the core and by-products simultaneously. There is one more significant difference. In the discrete production, the size of shipment lot, transferred from one operation to another, is different from the size of the process lot only in rare cases. The situation is just the opposite in process production—changeover of the machines is usually performed after a certain, often considerable, quantity of shipment lots. The dimensions of the latter are usually determined by the size of machines or transport devices.

In the paper of Schragenheim et al. (1994), two variants of this TOC application in the process production are described. In the first variant, the capacity constraint resource (CCR) is the sales of finished product. In this case, the use of "drum–buffer–rope" approach means creation of a direct link between the demand and the first (earliest) stage of the production process. Here the demand (shipment) acts as "drum" that sets the process rhythm. At the start of shipment the link with the machine at the beginning of the process is activated, i.e. "rope" is on. In case of the second variant the "drum" constraint is one of the machines of the current process and, obviously, its job has to manage the "rope" of production start.

Figure 3.4 shows the diagram of "drum–buffer–rope" method of the first variant.

The main feature of method "drum–buffer–rope" is in the buffer composition and its management. If the constraint is sales, the buffer is finished goods inventory. This reserve is required if the production cycle time is greater than the time the customer agrees to wait for the order fulfilment. The amount of the required reserve, in general, depends upon its rate of depletion, fluctuations in market demand, and fluctuations of the production cycle duration. It is often assumed that the minimum value of this inventory must meet the demand for the time of production of the next lot of finished product.

If the process production produces several kinds of products, the inventory in the buffer must meet the demand of each kind of product for production cycle of each product lot. This cycle includes not only manufacture of the relevant product but also manufacture of lots of other products, as well as the changeover time from one product to another.

The smaller the process lots of the product, the smaller inventory can be stored in the buffer. However, with decrease in the size of lots the changeover time increases and no longer sales but the production of products on one of the machines can become constraint on—we get the second variant of the theory of constraints. It should be noted that the changeover time often depends on the consistency of manufactured products, which also affects the throughput.

In general, if the system has the CCR, the scheme of "drum–buffer–rope" method for the process production is actually similar to Fig. 3.1. In this case, the "assembly" might be needed, which should prevent the processing of the product by the machine subsequent to the CCR when putting other products (being not constraints) to this machine.

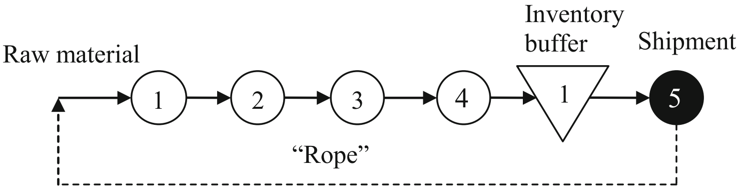


Fig. 3.4 Diagram with limitation in sales

3.1.6 Review of TOC Applications

A number of literature materials on the application of the theory of constraints were processed and analysed in Balderstone and Mabin (1998). This chapter studies the process performance indicators such as duration of the production cycle, timely fulfilment of orders, inventory levels, and production profits. As a result, the authors make the following conclusions.

- After considering more than a hundred cases of application, completely unsuccessful applications have not been identified.
- On the average, inventory level decreased by 50 % and duration of the production cycle by 60 %. It is significant that the inventory reduction was followed by simultaneous decrease in time of a production cycle that does not usually occur with other methods.
- In most cases, the theory of constraints was applied in every company for individual processes; there is almost no data on extensive application throughout the company.
- Technical solutions needed for the results application are usually quite simple and do not require big investments.
- The main difficulty in applying the theory of constraints is as always "Business as Usual" thinking and as a result the antagonism of performers. In this regard, it should be noted that the TOC methods have been widely used in support of the armies of the United States and Israel.

Table 3.5 shows some data on the results of TOC application in the 1990s.

Table 3.5 Improvement of performance after application of TOC [based on Balderstone and Mabin (1998)]

Company	Year	Duration of production cycle, %	Stock level, %	Throughput, %
Modine Manufacturing	1992	75	70	
Renton Coil	1996			48
Tiger Brend	1993		50	50
Ketema A&E	1997	30	40	
BHP	1991	20	20	
Toyo Tanso	1998	50		
Dresser Industries	1997	77		
Morton Automotive	1996	50	50	
EMC Technology	1998	75		
Harris Semiconductor	1995	50	40	

3.2 Theory of Logistic Operating Curves

Attempts to use the criteria described in Sect. 2.2.1 for optimal planning often face their inconsistency. Indeed, the best performance criterion K4 requires to increase the size of production lots; on the contrary, to reduce criterion K3 (cycle time) the lot sizes should be reduced. This contradiction has been called "dilemma of planning" (Nyhuis and Wiendahl 2009) and led to generation of a new approach to logistics called Logistic Operating Curves Theory.

3.2.1 Production (Logistics) Variables

Logistic Operating Curves Theory considers the dependencies between the variables describing the production process in order to establish their optimal ratio for each particular case. For this purpose, it is necessary to describe these variables first.

Production logistics processes are characterized by several factors: cost, the value of work-in-progress, capacity, cycle time, and deviation from the production plan. When constructing logistic curves some parameters can be considered as independent variables, while others are dependent on the former. In this theory, work-in-process (utilization rate) abbreviated as WIP is selected as an independent variable.

This term refers to the total operation hours required to fulfil all orders in production. It should be noted that in modern literature relating to the load management system (see Sect. 3.5.1 below), term "workload" as a synonym for "work-in-process" is often used. Production loading is measured in operation hours. It can refer to a separate unit of equipment or to a workshop or an enterprise in general and be used for all processes of production logistics.

In the general case, the job processing time (work content) for lot Q_i of i-th products processed on i-th machine is defined as follows:

$$p_{ij} = \frac{Q_i \tau_{ij} + s_{ijk}}{60} \text{ hours,} \tag{3.1}$$

where τ_{ij} is run time per piece in min;

s_{ijk} is changeover time to i-th product form k-type product on j-type machine in minutes.

If the changeover time does not depend on the type of product preceding i-th product, then index k in designation of changeover time can be omitted. Logistic Operating Curves Theory considers mainly production processes relating to one machine. Therefore, we will omit index j in presentation of the theory and its applications.

Processing time for the lots with different products on the same machine is a variable random value, which can be always defined by mean value and variance:

$$\overline{p} = \frac{1}{n}\sum_{i=1}^{n} p_i \tag{3.2}$$

and

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n} (\overline{p} - p_i)^2. \tag{3.3}$$

Ratio of root-mean-square deviation σ to mean value

$$\nu = \frac{\sigma}{\overline{p}} \tag{3.4}$$

is called variation coefficient of processing time.

The processing time is usually measured in hours. At the same time for the purpose of problem solution, we will need to define the time of loading the machine in days of operation of that machine, which can be found as follows:

$$\hat{p}_i = \frac{p_i}{P_{\max}} \text{ working days,}$$

and accordingly the mean value of processing time in working days

$$\hat{p} = \frac{\overline{p}}{P_{\max}}, \tag{3.5}$$

where $P_{\max}$ is the most possible for use operation time of one machine, hours per day. Upper limit $P_{\max}$ is full working time for all shifts per day.

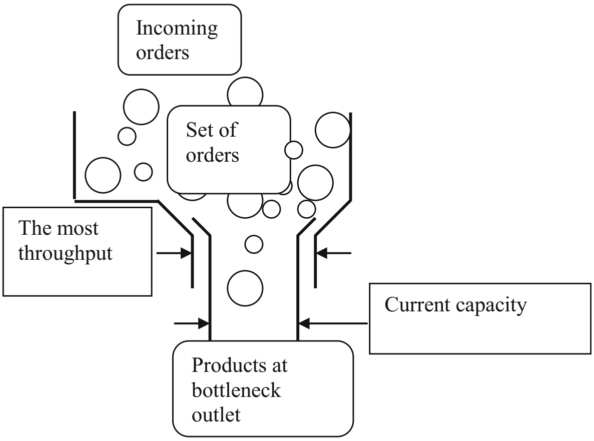
The third of the considered variables of the production process producing i-th product is duration of its cycle F_i in working days. The cycle duration is also a random value with mean value

$$\overline{F} = \frac{1}{n}\sum_{i=1}^{n} F_i. \tag{3.6}$$

One of the basic concepts of the logistic curves theory, as well as in the TOC, is throughput. The key to this concept is a widespread bathtub or funnel model, as shown in Fig. 3.5.

In the model of Fig. 3.5 in the set of incoming orders going to the production system, a queue of their consistent fulfilment forms. Then orders pass through a hole of bathtub (funnel), which is a bottleneck of processing. In this case, the actual capacity of equipment is always less than the maximum possible throughput.

Fig. 3.5 Bathtub (funnel) model



This model resembles "drum–buffer–rope" model of the theory of constraints, except that the latter has no "rope". Indeed, the funnel acts as the "drum" setting the rhythm, and the queue of accumulated orders is a buffer. The total processing time of these orders in the buffer and directly during processing, i.e. work-in-process (WIP), is the main independent variable in the logistic curves theory.

"Funnel" model allows describing the production capacity of any type as a set of order input, work-in-process, and production output. The funnel neck determines the actual output rate of the production system, which is measured in the amount of processing time for the product output per working day. Upon entering the order, the production is supplied with the main and auxiliary materials, components, tools, documentation, etc., required to fulfil this order.

Logistically, the order input into production is characterized first of all by processing time of its fulfilment. With this input, the production loading increases abruptly by the amount of the processing time. On the other hand, when the lot of finished products is released, the production load is reduced by the processing time of this lot. The diagrams in Fig. 3.6 can illustrate the process of changes in the production load associated with the orders input and production output.

Figure 3.6, which was built in the coordinates processing time p in hours—the time t in working days, reflected changes in production load W for time z, occurring as a result of order input I and product output O. Both at order input and product output the production load changes abruptly by an amount corresponding to processing time of the production lots start and release. At the end of the accounting period z the production load W_2 may differ from the load at the beginning of period W_1. This is possible if the angle of the line, tangent of which corresponds to average rate $\overline{S}$ of processing time change of the orders that are in production, is not equal to the slope angle of a line of average rate of output $\overline{P}$ processing hour/working day.

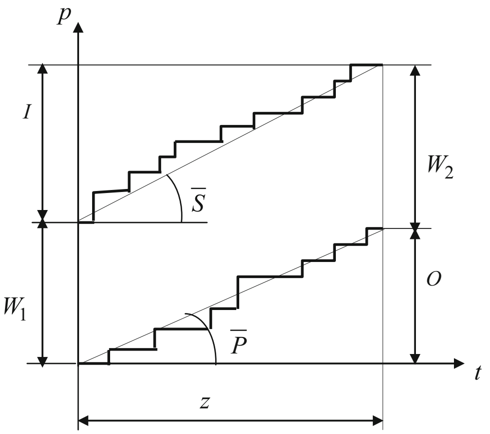


Fig. 3.6 Diagram of processing time flow in production

Let us define the average production loading $\overline{W}$ for period z as ratio of area A in-between the diagrams of order input and product output to duration Z

$$\overline{W} = A/z. \tag{3.7}$$

Let us introduce ratio $\overline{R}$ of area A to processing time O of all orders fulfilled within period z

$$\overline{R} = A/O. \tag{3.8}$$

As mean output rate $\overline{P}$, according to Fig. 3.6, is equal to

$$\overline{P} = O/z,$$

it is obvious that

$$\overline{R} = \frac{\overline{W}}{\overline{P}}. \tag{3.9}$$

Expression (3.9) is called “funnel formula”. Value $\overline{R}$ (Mean WIP Range) is time in working days which is necessary to perform mean production loading $\overline{W}$ in processing hours at mean output rate $\overline{P}$ in hours per working day. Funnel model presented in Fig. 3.5 allows illustrating the concept of mean coefficient of machine work load as

$$\overline{K} = \overline{P}/P_{\max}. \tag{3.10}$$

In the logistic curves theory, the concept of weighted mean duration of production cycle is also used, which is defined as

$$\overline{F}^{w} = \frac{\sum_{i=1}^{n} (F_i p_i)}{\sum_{i=1}^{n} p_i} \text{ working days.} \tag{3.11}$$

Nyhuis and Wiendahl (2009) show that

$$\overline{F}^{w} \approx \overline{R}. \tag{3.12}$$

3.2.2 Some Notions Used in Queuing Theory

Theory of queue is widely used for solving the problems of orders maintaining going through the bottleneck; at that it is supposed that order input occurs randomly. This theory considers the relationship among waiting time of orders, queue length, and machine work load coefficient.

To simplify the description the numerous existing queuing models, the classification of these models is widely used in the form of records of type A/B/X/Y/Z. The first two characters A and B designate kind of the probability distribution of the time of order arrival and method for processing orders in the system. Symbol X is the number of parallel machines processing the orders, and symbol Y describes the capacity of these machines. Last letter Z determines the priority rules for orders processing.

Since, for the most part, FIFO priority rule is used (Sect. 2.3.1), i.e. “First in, first out”, and the issues of the machine capacity arise quite rarely, the first three components in the queuing record are usually enough. When considering the operation of one machine one of two models M/M/1 or M/G/1 is used. The first symbol M in both of these models means that for the time of arrival the statistical distribution of Poisson is applied. The same distribution is applicable for the first model and order fulfilment. For the second model, the distribution of the time of order fulfilment can vary greatly including the Poisson distribution as well.

It is to be recalled that the Poisson distribution for the orders going to the system at mean rate λ describes probability P of receipt of n orders with time interval t as follows

$$\mathrm{P}(n) = \frac{(\lambda t)^n}{n!} e^{-\lambda t}, \quad n = 0,\ 1,\ 2,\ \ldots \tag{3.13}$$

For the commonly used model M/G/1 formula of Pollaczek and Khinchine is valid. It defines mean duration of queuing. Using this formula in terms of the above designations of the variables gives the following expression for waiting time $\overline{w}$ equalling the difference between the duration of production cycle $\overline{F}$ and processing time $\hat{p}$:

$$\overline{w} = \overline{F} - \hat{p} = \frac{\hat{p}\,\overline{K}(1 + \nu^2)}{2(1 - \overline{K})}. \tag{3.14}$$

From Eq. (3.14), according to definition (3.6), it follows that the mean duration production cycles $\overline{F}$ is more than the value of mean processing time $\hat{p}$ in working days by the value dependent on the mean coefficient of machine loading $\overline{K}$ and variation coefficient ν, defined by formula (3.3).

Little's formula is very important in the theory of queues. According to this formula the mean duration production cycle is

$$\overline{F} = \frac{\overline{N}}{\lambda}, \tag{3.15}$$

where $\overline{N}$ is mean quantity of orders present in system; and λ is mean rate of order receipt, i.e. quantity of orders received per day.

Nyhuis and Wiendahl (2009) clarify similarity and differences of expression (3.15) in comparison with funnel formula (3.9). The main difference of these dependencies is that the funnel formula uses not just the quantity of orders, like in Little's formula, but mean production loading $\overline{W}$, the value of which depends not only on the quantity of orders but on their work content. If we evaluate the mean production loading just as average quantity of orders in work $\hat{W}$, which are either in processing or waiting for it, then using expressions (3.14) and (3.15) we can obtain

$$\hat{W} = \overline{K}\left[1 + \frac{\overline{K}(1 + \nu^2)}{2\left(1 - \overline{K}\right)}\right]. \tag{3.16}$$

With increase of mean machine loading $\overline{K}$ from 0 to 1 the quantity of orders waiting for fulfilling will grow fast and become large. The length of queue for processing according to Eq. (3.16) depends very heavily on variation coefficient ν, defined by expression (3.3). The calculations show that, e.g. increase of ν from 0.5 to 1.5 leads to increase of queue length by 1.5–2 times, and bigger increase relates to higher values of load coefficient.

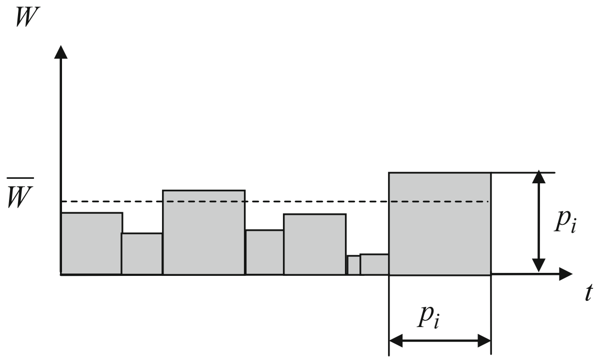


Fig. 3.7 Diagram of ideal production load [based on Nyhuis and Wiendahl (2009)]

3.2.3 Plotting Logistic Operating Curves

Logistic operating curves can be constructed both for a particular unit of equipment and the equipment group, workshop, or enterprise. In most simple cases, such curves are constructed, for example, for one machine working independently. For such a machine or workstation the so-called ideal logistic operating curve is determined first.

When constructing the ideal curve the following two main assumptions are made: (a) one order is set for processing on every machine at any given moment; and (b) each order is fulfilled immediately after its receipt, which occurs immediately after fulfilment of the previous order. In such ideal conditions, the diagram of changes in production load (work-in-process) with time will look like in Fig. 3.7.

Since under the assumptions made that in the production is always only one order, the value of the production load W at any time is exactly equal to the processing time of the current order. At the same time, the duration of manufacturing under the same conditions is also equal to the processing time. Therefore, in Fig. 3.8, each i-th order is represented in the form of a square with a sides equal to p_i.

Let us introduce the concept of ideal minimum of production load $W_{\min}$, which is equal to average value W in the diagram. this value is equal to the area of the diagram divided into the time of monitoring, meaning

$$W_{\min} = \sum_{i=1}^{n} (p_i \times p_i) / \sum_{i=1}^{n} p_i. \tag{3.17}$$

Value $W_{\min}$ can be calculated using the above parameters of order processing time as follows:

$$W_{\min} = \overline{p}\left(1 + \nu^2\right). \tag{3.18}$$

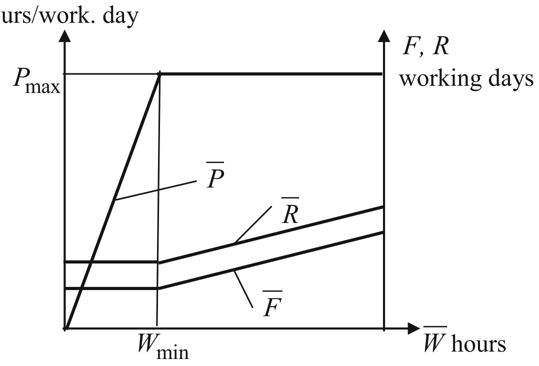


Fig. 3.8 Ideal logistic curves [based on Nyhuis and Wiendahl (2009)]

Using concept $W_{\min}$ we construct the so-called ideal logistic curves of mean output rate $\overline{P}$, mean duration of production cycle $\overline{F}$, and mean WIP range $\overline{R}$ (Fig. 3.8).

According to Fig. 3.8 it follows that in the ideal case the mean output rate $\overline{P}$ grows linearly with increase of mean work-in-process $\overline{W}$ to a value of $W_{\min}$, until the processing time of the set of ideal orders is equal to the machine operation time fund. Then $\overline{P}$ remains constant and equal to throughput $P_{\max}$. Durations $\overline{F}$ and $\overline{R}$ do not change, while $\overline{W}$ is less than $W_{\min}$, and then increase because the orders are put into queue for processing. Herewith as it can be illustrated, $\overline{R}$ is always bigger than $\overline{F}$.

Real logistic curves differ from the ideal ones by the fact that instead of the break at point $W_{\min}$ the initial sections of the real curves at $\overline{W} < W_{\min}$ transfer smoothly into the sections that are valid at $\overline{W} > W_{\min}$. To construct a real logistic curve $\overline{P}$ its setting in parametric form is usually used:

$$\overline{W} = W_{\min}\left(1 - \left(1 - \sqrt[4]{t}\right)^4\right) + W_{\min}\alpha_1 t;$$

$$\overline{P} = P_{\max}\left(1 - \left(1 - \sqrt[4]{t}\right)^4\right). \tag{3.19}$$

Additional variable t changes from 0 to 1. It is obvious that point $t = 0$ corresponds to the origin of coordinate and point $t = 1$ corresponds to the point at which the mean output rate $\overline{P}$ reaches values $P_{\max}$. Coefficient α_1 determines how much the real curve is stretched along axis $\overline{W}$. According to a number of studies, this coefficient is usually close to 10.

Logistic curve $\overline{R}$ is constructed according to curve $\overline{P}$ using the funnel formula (3.9). To construct logistic curve $\overline{F}$ the following relation is used:

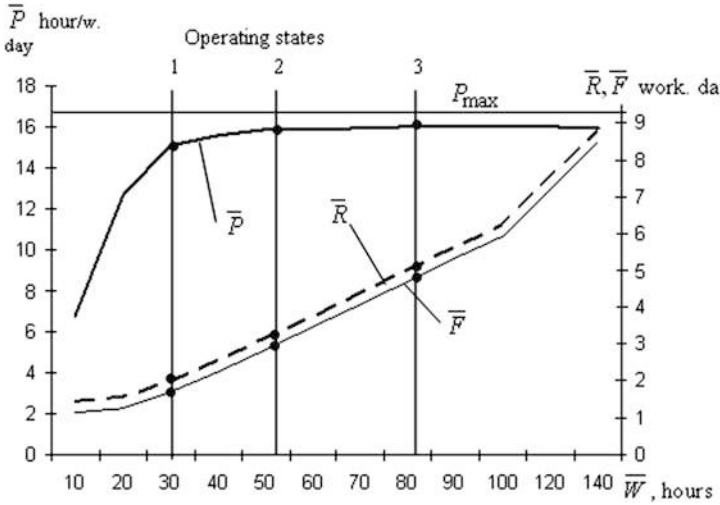


Fig. 3.9 Example of construction of logistic curves for one machine [based on Nyhuis and Wiendahl (2009)]

$$\overline{F} = \overline{R} - \frac{\overline{p}\nu^2}{P_{\max}}. \tag{3.20}$$

According to Eq. (3.9), it is obvious that $\overline{F} < \overline{R}$, where curve $\overline{F}$ is similar to $\overline{R}$, because the second component in this formula does not depend on $\overline{W}$.

Figure 3.9 shows an example of constructing curves $\overline{P}$, $\overline{R}$, and $\overline{F}$ for a machine (multispindle machine) as diagram in MS Excel. Horizontal line $P_{\max}$ corresponds to throughput (capacity) of the machine.

The machine under study had two working stations running in parallel. The constructed histogram of production lots load showed that the mean processing time of a lot appeared to be 6.68 h, and variation coefficient ν was equal to 0.86. Then using formula (3.17) value $W_{\min}$ for one work station was defined (11.72 h). Because of two workstations available it was accepted that $W_{\min} = 23.44$ h. To calculate values $\overline{P}\left(\overline{W}\right)$ relationships (3.19) were used, where t took values from 0 to 1, and coefficient α_1 was set at 5 due to two workstations available. Values $\overline{R}$ and $\overline{F}$ were determined using relationships (3.9) and (3.20).

The diagram's bold points on each of the curves show three operating states which correspond sequentially to underload 1, medium load 2, and overload 3. The diagram shows that mean output rate $\overline{P}$ grows first with increase of $\overline{W}$ and then approaches the value of throughput. Values $\overline{F}$ and $\overline{R}$ at low values of $\overline{W}$ do not change and then begin to increase. This is due to the fact that at sufficiently big load the orders queue up, where the waiting time increases with increase of $\overline{W}$.

3.2.4 Main Properties of Logistic Curves

The following are details of some properties of logistic operating curves. Understanding of the properties described increases significantly the opportunities of application of these curves to control bottleneck operation.

The most important and equally rather obvious feature is the balancing of order input and product output processes in due course. These processes are described by stepped curves in Fig. 3.6. Accordingly, input rate $\overline{S}$ on a sufficiently long time interval usually corresponds to mean output rate $\overline{P}$.

As was shown above, such characteristics of logistic curves as time $\overline{R}$ required to perform mean work-in-process $\overline{W}$ at mean output rate $\overline{P}$, as well as duration of the production cycle $\overline{F}$, are directly dependent on ratio $\overline{W}/\overline{P}$.

Changing of machine load coefficient $\overline{K}$ influences greatly both production load $\overline{W}$ and duration of the cycle $\overline{F}$. With heavy loaded equipment, even relatively small reduction of $\overline{K}$ can seriously reduce load $\overline{W}$. Since in the real conditions, big fluctuations in production loads are often observed, then a buffer is necessary to stabilize high load equipment. As you can see, this property actually repeats the suggestion of using the buffer in the Theory of Constraints (Sect. 3.1.1).

Mean duration of the production cycle $\overline{F}$ can seriously be influenced by the established priority rule in fulfilment of orders (Sect. 2.3.1). Generally speaking, the minimum duration $\overline{F}$ occurs when the rule of “first in, first out” (FIFO) is applied. It should be borne in mind that increase in the cycle time difference, when instead of FIFO, for example, rules SPT or LPT are used, depends on the processing time dispersion. The more is the variation in the processing time of the individual jobs, the more significant increase in the mean duration of the cycle.

3.3 Application of Logistic Operating Curves

Theory of logistic curves enables to carry out various improvements of the production process. Mostly, these improvements are carried out to reduce current production load $\overline{W}$. It is possible to decrease $\overline{W}$ due to the rational planning in particular, where the processing time of the individual jobs is equalled as much as possible. In general, the greatest effect of the logistics operating curves can be seen in their use for the so-called logistic positioning and bottleneck analysis.

3.3.1 Logistic Positioning

The purpose of logistic positioning is to determine the most appropriate ratio in a given situation between such contradictory process indicators as its duration $\overline{F}$ and machine load coefficient $\overline{K}$. Let us demonstrate the meaning of the logistic positioning by the example of logistic curves $\overline{P}$, $\overline{R}$, and $\overline{F}$ in Fig. 3.9.

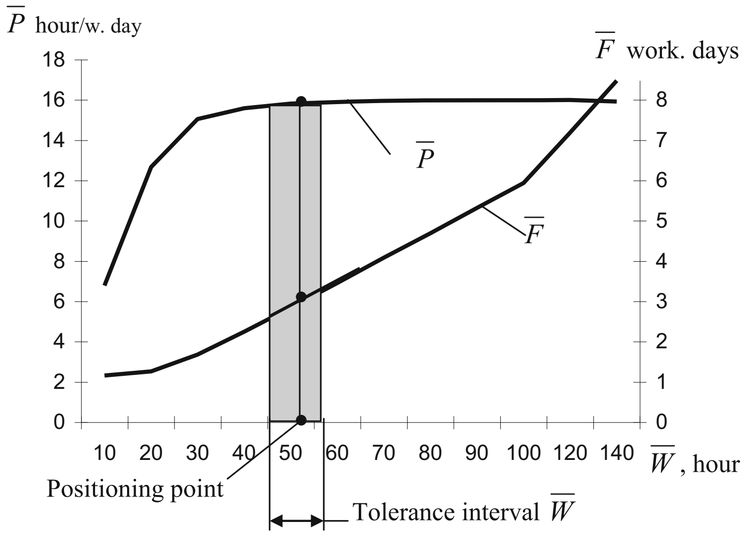


Fig. 3.10 Example of logistic positioning

For this purpose (Fig. 3.10) in the theory of logistic curves, it is assumed that each value of production load $\overline{W}$ corresponds to a certain value of production costs, besides the costs increase with increase of $\overline{W}$. In logistic positioning, an acceptable value of costs and its corresponding value $\overline{W}$—positioning point can be set by expertise.

By varying the production load within the tolerance interval, mean output rate $\overline{P}$ changes slightly in this example. At the same time, duration of the production cycle $\overline{F}$ can vary significantly. This situation is typical if the positioning point is selected in the area of high values of machine load coefficient $\overline{K}$, i.e. in operating state 3 in Fig. 3.9. When selecting the positioning point in operating state 1, i.e. at low load, the situation is reversed. In this case, mean output rate $\overline{P}$ increases rapidly even with small increase of $\overline{W}$, and duration of the cycle $\overline{F}$ varies slightly.

3.3.2 Bottleneck Analysis and Improvements

The logistics analysis of bottlenecks is based on the manufacturing process data, and the mode of operation of individual units of equipment can affect each other. When collecting the necessary information it is necessary to define a number of parameters that determine the mode of the production:

(a) for the work assignment—the value of the mean processing time of production lot processing, the variation coefficient, and the minimum inter-operation time

(b) for the production cycle duration—the mean value, the variation coefficient and the mean value of the time for production load performance
(c) for production rate—the average value in hours per day and the number of fulfilled orders
(d) for the work-in-process—the mean value in hours and number of orders
(e) to analyse the schedule adherence—relative delay and its variation coefficient

During the analysis, various logistic targets have to be defined, which can sometimes be inconsistent. The starting point for establishing the target priority is the logistic positioning described above. Actually, this means the determination of what is most important in the current situation—mean output rate $\overline{P}$ or cycle duration $\overline{F}$.

Bottlenecks in the production flow cause an increase in the production cycle duration $\overline{F}$, so it is natural to tend to reduce this value somehow. Methods of bottleneck "pointing" are well known, but here it is reasonable to consider them in terms of logistics analysis. Naturally, the increase of the maximum throughput $P_{\max}$ (at least temporary) is the most effective. If it is not possible, you should delay the order transferring for processing, if practical. In this case, the mean value of production load $\overline{W}$ reduces, and, as it follows from Fig. 3.10, duration of the cycle $\overline{F}$ can reduce.

More complex methods of influencing the equipment performance, which is the "bottleneck", are based on the so-called harmonization of orders. First of all, it is an attempt to reduce the spread of the processing time of production lots and, therefore, to reduce the variation coefficient ν. Value ν, pursuant to formula (3.18), influences greatly the ideal minimum of production load $W_{\min}$, which in turn, according to expression (3.19), leads to a significant decrease of the mean production load $\overline{W}$.

Besides, $W_{\min}$ essentially depends on the mean processing time of the production lot $\overline{p}$ (Eq. 3.18) and the sizes of the lots can be reduced in extreme cases. Of course, such method for accelerating the order fulfilment leads to an increase in setup time and, in general, an increase in the labour input and it should be used with caution. From this perspective, it is advisable not only to reduce the size of a production lot supplied for processing but also to distinguish between the sizes and quantity number of input lots and output lots. This can reduce the bottleneck's effect on the business greatly in many cases.

3.3.3 Evaluation of Overall Production Performance

In the framework of the logistic curves theory, we can construct a logistic curve describing the efficiency of a production process both for a single machine and for the whole enterprise. To do this, we use mean production load $\overline{W}$ as an independent variable of the process, which is usual for this theory. Efficiency of the process is characterized by the following basic parameters. Firstly, it is cost characteristics—

income received as a result of the production load performance and production costs for time of the production load performance. Secondly, the efficiency of operation is essentially determined by the load level of the equipment available at the enterprise. Finally, the evaluation of the operation quality depends on the ability of the enterprise to fulfil the received orders in time.

Let us introduce the complex criterion of production process efficiency

$$E = \frac{f\overline{K}}{c\overline{H}}, \tag{3.21}$$

where f is profit for time $\overline{R}$ of production load performance,

c is production costs for that period,

$\overline{K}$ is mean coefficient of the equipment utilization, and

$\overline{H}$ is mean intensity of obligations performance under the contracts with consumers.

Criterion E allows consideration of several aspects of production activity simultaneously: profitability, capacities utilization, and real possibilities to perform contractual obligations.

Taking into account that the profit is equal to the difference of sales G and costs, then formula (3.21) will look as follows:

$$\frac{E = (G/C - 1)\overline{K}}{\overline{H}.} \tag{3.22}$$

All the components in expression (3.22) depend on value of production load $\overline{W}$. When $\overline{W} = 0$, meaning there are no orders for products, the value of profit is 0, and the value of costs is not equal to 0, as the costs are required to support the enterprise activity. Machine load coefficient is also equal to 0, and to calculate the value of criterion E it is required to define value $\overline{H}$.

It has to be recalled that the concept of intensity H_i for i-th order, introduced in Sect. 2.4.1 by formula (2.23), for the case when the delivery time is not over yet, is defined as

$$H_i = (T_{1i} + T_{2i})/z_{1i},$$

where T_{1i} is a component defined by the processing time not performed at the moment of scheduling;

T_{2i} is a component appeared due to necessity of transferring of job to the remaining operations;

z_{1i} is the estimated production time reserve against to the target.

This intensity is variable in time and can be determined for any moment of the actual fulfilment of an order. However, to assess the overall effectiveness of production there is no need to rely on the intensity of each order. For this purpose, it makes sense to determine the mean intensity as follows

$$\overline{H} = \frac{\overline{F}}{\overline{z} + \overline{w}}, \tag{3.23}$$

where $\overline{F}$ is the average duration of the production cycle, $\overline{z}$ is the average stock of finished products in production days, and $\overline{w}$ is the average waiting time which is acceptable for the customer from the moment of the order placement to its delivery.

Suppose that all the produced products can sold without delays and the value of sales is

$$G = a\overline{P} \ \ \$/\text{work. day}, \tag{3.24}$$

and the costs are

$$c = b_1 + b_2\overline{P} + b_3\overline{P}\overline{z} \ \ \$/\text{work. day}, \tag{3.25}$$

where $\overline{P}$ is the mean output rate hours per day; a, b_1, b_2, b_3 are constant coefficients.

The costs in expression (3.25) are made of three parts: constant component b_1, direct expenditures $b_2\overline{P}$ for products manufacturing, and the costs related to the storage of the finished products $b_3\overline{P}\overline{z}$. Pursuant to formula (3.10), we obtain the following from expressions (3.22–3.25):

$$E = \frac{(\overline{z} + \overline{w})[a\overline{P}/(b_1 + b_2\overline{P} + b_3\overline{P}\overline{z}) - 1]\overline{P}}{P_{\max}\overline{F}}, \tag{3.26}$$

where $P_{\max}$ is the biggest possible output rate in hours per day and where $\overline{P}$ and $\overline{F}$ are connected with production load $\overline{W}$ by relations presented in Fig. 3.7.

Let us make relationship of criterion $E(\overline{W})$ (Fig. 3.11), by putting the following values of coefficients into Eq. (3.26): $\overline{z} = 5$ days; $\overline{W} = 20$ days; $a = 20$ \$/h; $b_1 = 3500$ \$/day; $b_2 = 5$\$/h; $b_3 = 0.5$ \$/h/day of storage. Let the value of throughput $P_{\max}$ for the site for 49 workplaces equal to 440 h/work. day, mean processing time of one job $\overline{p}$ approx. 16 h, variation coefficient ν 0.87, and mean time for production load performance $\overline{R}$ 9 h. To define the mean duration of the production cycle $\overline{F}$ we use formula (3.20). Considering that in formula (3.20) the throughput of one workplace is involved, we get N workplaces in this case

$$\overline{F} = \overline{R} - \frac{\overline{p}N\nu^2}{P_{\max}} = 9 - \frac{16 \times 49 \times 0.87^2}{440} = \ 7.5 \text{ days.}$$

According to Fig. 3.9 when $\overline{W} = 0$, $\overline{P} = 0$, and $\overline{F} > 0$, and so from Eq. (3.26) it follows that $E = 0$. With increase of production load $\overline{W}$ value E is negative unless the sales G becomes bigger than costs c—i.e. to the breakeven point, and then it will increase gradually. The highest value of criterion E is at point $\overline{W} = \overline{W}^*$, equalling to 2650 h in this case.

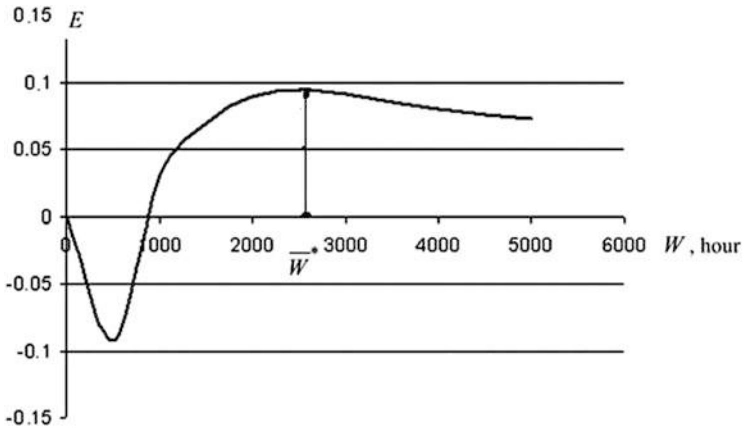


Fig. 3.11 Curve of efficiency complex criterion

Let us consider the change in the intensity value for various values of $\overline{z}$ and $\overline{w}$. For the values set for the construction of the curve in Fig. 3.11

$$\overline{H} = \frac{\overline{F}}{\overline{z} + \overline{w}} = \frac{7.5}{5 + 20} = 0.3.$$

Average waiting time $\overline{w}$, which is acceptable for the customer, essentially depends on the type of market conditions, which were described in Sect. 1.6.1. With the growing market, the customer is usually willing to wait long enough and value $\overline{w}$ can be comparable to or even exceed duration of the production cycle $\overline{F}$. In this case, intensity $\overline{H}$ is small and can reduce the number of working days $\overline{z}$ of warehouse stock to 0. On the stable market, the acceptable waiting time is determined by the competition and usually requires some or sometimes a substantial margin.

When operating on the mature market, there are large fluctuations in the flow of orders, and not to lose an order its fulfilment should be provided fast enough. At the same time, under these conditions, it is not economically justified to store large amounts, since it is difficult to predict when they are demanded. In addition, large fluctuations in demand lead to an increase in the duration of the production cycle $\overline{F}$, which also leads to an increase in intensity and reduction of effectiveness E.

Under these conditions, transition from the strategy “make-to-order” to strategy “assembly-to-order” should be considered as the main way to enhance efficiency. Using this strategy can dramatically reduce fluctuations in production load $\overline{W}$, reduce cycle time and intensity, and ultimately, raise the value of efficiency criterion E.

3.4 Optimal Lot Sizing for Production Bottlenecks

In the literature on the production scheduling, a simple formula is often recommended, which determines the lot size Q_i for the i-type product as the minimum allowable for the high load equipment:

$$Q_i = \frac{s_i(1-\delta)}{\tau_i \delta}, \tag{3.27}$$

where s_i, τ_i, like in expression (3.1), are setup time and processing time per piece, accordingly, and δ is a norm coefficient of acceptable loss of time for changeover.

Since formula (3.27) does not take into account the value of demand D_i for the manufactured product, Karmarkar et al. (1992) suggested to perform the so-called approximate calculation of a lot

$$Q_i = \varphi D_i, \tag{3.28}$$

using other norm coefficient φ, defining the ratio of a lot size to some, e.g. annual, demand.

Formulas (3.27) and (3.28) are vague because of the uncertainty of norm coefficients. There are other difficulties with the application. For example, it is difficult to set the demand horizon; even when these formulas can be used the difference in two values obtained may be so big that the actual lot sizing becomes difficult, etc.

At the same time, wide application of these relationships suggests two things. Firstly, there is a serious need for a method of calculating the lot size, taking into account the equipment load level. Secondly, this method should use production process parameters that are used in expressions (3.27) and (3.28). In the 1990s, a series of studies were made aimed at the development of this method, and the results in the studies of Karmarkar et al. (1992) set forth in the following paragraph received the greatest recognition.

3.4.1 Lot Sizing Heuristic

The methods discussed here are based on the fact that the waiting for processing is usually a large part of the production cycle duration. This leads to an increase of lead time inventories, to lengthening of time for order fulfilment, etc., and therefore it is quite useful to reduce the waiting time for processing. The reduced waiting time is usually associated with the reduced size of production lots, but with too small lots the average waiting time in the queue may increase due to the increasing time loss of setups. Therefore, it makes sense to try to calculate such lot sizes, which provide mathematical minimum of the mean waiting time.

Mean waiting time $\overline{w}$, in terms of Karmarkar et al. (1992) approach, which considers processing queues in the framework of M/G/1 model described in

Sect. 3.2.2, is a function of vector Q (set of values) of lots Q_i of various i-th products that must undergo processing on the same machine. Value $\overline{w}(Q)$ is inherently equal to the difference of mean cycle time $\overline{F}$ and mean value of processing time $\hat{p}$ in working days.

The basis for the described approach is formula (3.14) of Pollaczek and Khinchine, given above in Sect. 3.2.2, for mean waiting time $\overline{w}(Q)$. After some transformations, the formula is given by

$$\overline{w}(Q) = \frac{\lambda E(p^2)}{2\left(1 - \overline{K}\right)}. \tag{3.29}$$

In formula (3.29), $\overline{K}$ is still coefficient of load, $E(p^2)$ is mean value of the square of processing time p^2, and value λ, the same as above in Sect. 3.2.2, represents the rate of order arrival for processing. Here this rate is define as follows

$$\lambda = \sum_{i=1}^{n} \lambda_i = \sum_{i=1}^{n} \frac{D_i}{Q_i}, \tag{3.30}$$

meaning λ is the total number of all lots n of different products coming within the time, for which the demand is established. To be specific, we assume that the production intensity is equal to demand D_i and is defined in the pcs/h.

The time for performance of one job (lot) p_i in hours is defined as a function of lot size Q_i

$$p_i = s_i + \frac{Q_i}{P_i}, \tag{3.31}$$

where, as above, s_i is the setup time in hours and P_i is the machine capacity in pcs/h.

Whereas the probability of arrival of i-th product lot is equal to

$$\mathrm{P}_i = \lambda_i / \lambda, \tag{3.32}$$

time of processing of random value p_i has discrete distribution with mean value

$$\overline{p} = E(p) = \sum_{i=1}^{n} \mathrm{P}_i p_i. \tag{3.33}$$

From expressions (3.30–3.33) it follows that the mean coefficient of load is equal to

$$\overline{K} = \lambda \overline{p}. \tag{3.34}$$

The so-called second-order moment of the random discrete value p_i, which is used in formula (3.29), is defined as

$$E(p^2) = \sum_{i=1}^{n} \mathrm{P}_i p_i^2. \tag{3.35}$$

Since the purpose of this problem is to determine the optimal lot sizes, all the values appearing in expression (3.29) are inserted into (3.29) as functions from Q_i. The problem under consideration is formulated mathematically as finding the minimum waiting time

$$\overline{w}^* = \min_{Q_i > 0} \overline{w}(Q) \tag{3.36}$$

at multiple values of Q_i with constraints

$$\sum_{i=1}^{n} (D_i/P_i + D_i s_i/Q_i) < 1, \tag{3.37}$$

meaning that load coefficient $\overline{K}$ must be less than one.

From expressions (3.31–3.35), we can obtain

$$\overline{w}(Q) = \frac{\sum_{i=1}^{n} \frac{D_i}{Q_i}\left(s_i + \frac{Q_i}{P_i}\right)^2}{2\left[1 - \sum_{i=1}^{n} \frac{D_i}{Q_i}\left(s_i + \frac{Q_i}{P_i}\right)\right].} \tag{3.38}$$

To find the minimum of function $\overline{w}(Q)$ for each variable Q_i, it is necessary to define the partial derivative with respect to this variable and set it equal to zero. Since all the equations for each variable have the same form, it is sufficient to solve one of them. The most compact form of this equation is represented by Kuik and Tielemans (1998)

$$\sum_{i=1}^{n} u_i \sqrt{s_i^2 + 2 s_i \overline{w}^*} = (1 - u)\overline{w}^* - \sum_{i=1}^{n} u_i s_i, \tag{3.39}$$

where the operation load (without time for setup) of machine u is a combination of lot processing load of each i-type product

$$u = \sum_{i=1}^{n} u_i = \sum_{i=1}^{n} \frac{D_i}{P_i}. \tag{3.40}$$

Equation (3.39) includes the optimal value of mean waiting time $\overline{w}^*$, which is associated with the relevant optimal values Q_i^* by expression (3.38). Hence, it follows that the value for one i-th product Q_i^* cannot be found separately from the values of optimal lots for other products, and it does not allow obtaining exact analytical solution of the problem.

For the approximate solution, in Karmarkar et al. (1992) heuristic methods are suggested for estimating the boundaries, within which the values of optimal lots should be. Among the three proposed methods, the so-called The second heuristic became the most widespread, the results of which are shown below. According to Karmarkar et al. (1992) and Missbauer (2002), this method is well supported under heavy load of a machine.

In the described heuristic method, from the analysis of Eq. (3.39), it is supposed that if $\overline{w}_i$ is much higher than s_i the approximate equation can be applied

$$\overline{w}^* \approx \frac{Q_i^2}{2P_i^2 s_i} \tag{3.41}$$

for each i-th product. This assumption leads to the expression for optimal lot

$$Q_i^* = \frac{2P_i}{1 - \sum_{j=1}^{n} u_j} \sqrt{s_i \sum_{j=1}^{n} s_j u_j}, \tag{3.42}$$

where

$$u_j = \frac{D_j}{P_j}. \tag{3.43}$$

3.4.2 Analysis of Heuristic Solutions

We begin our analysis of the described method with calculating the optimal lot sizes of two different products manufactured on the same machine. Initial data for the calculation are given in Table 3.6.

Taking into account the dimensions in Table 3.6, expressions (3.41) and (3.42) will have forms

$$\overline{w}^* \approx \frac{Q_i^2 G^2}{2P_i^2 s_i}, \tag{3.44}$$

$$Q_i^* = \frac{2P_i}{G\left(1 - \sum_{j=1}^{n} u_j\right)} \sqrt{s_i \sum_{j=1}^{n} s_j u_j}, \tag{3.45}$$

where G is the duration of a working day.

Table 3.6 Products manufactured on the machine

Product no.	Setup time s_i, h	Demand D_i, pcs/day	Machine capacity P_i, pcs/day
1	2	40	100
2	1	10	35

Table 3.7 Optimal lots with different capacities

	Capacity P_i, pcs/day		
Parameter	$P_1 = 80, P_2 = 27$	$P_1 = 100, P_2 = 35$	$P_1 = 120, P_2 = 45$
Optimal lot of product 1 Q_1^* pcs	196	117	90
Optimal lot of product 2 Q_2^* pcs	52	29	24
Machine load coefficient $\overline{K}$	0.91	0.81	0.71
Mean optimal waiting time $\overline{w}^*$, h	96	22	9

Assume that in this case the duration of the working day $G = 8$ h. Then, for example, Q_1^* according to Eq. (3.42) will be

$$Q_1^* = \frac{100}{8} \times \frac{2}{1 - (40/100 + 10/35)} \sqrt{2 \times \left(\frac{2 \times 40}{100} + \frac{1 \times 10}{35}\right)} = 117 \text{ pcs.}$$

The calculation results by formulas (3.44) and (3.45) for the machines with different capacity are provided in Table 3.7.

The load coefficient is defined pursuant to Eq. (3.34). For example, in this case, capacity $P_1 = 100$, $P_2 = 35$ taking into account the dimension we have

$$\overline{K} = \sum_{i=1}^{2} \lambda_i p_i = \frac{D_1}{Q_1^*}\left(\frac{s_1}{G} + \frac{Q_1^*}{P_1}\right) + \frac{D_2}{Q_2^*}\left(\frac{s_2}{G} + \frac{Q_2^*}{P_2}\right) = 0.81.$$

The mean optimal waiting time is defined pursuant to Eq. (3.44) by parameters of any of the products, for example, the first one. In this case we have

$$\overline{w}^* = \frac{Q_1^2 G^2}{2P_1^2 s_1} = \frac{117^2 \times 8^2}{2 \times 100^2 \times 2} = 22 \text{ hours.}$$

As you can see from Table 3.7, with high load coefficient $\overline{K} = 0.91$ the optimum lot sizes become large, and the waiting time increases dramatically. Reduced load coefficient $\overline{K} = 0.71$ makes it possible to obtain small waiting time, but this will require increasing the frequency of setups. For example, if the load coefficient is 0.81, the processing of product 1 must be performed without setup for about 9.4 h, and if the load coefficient is 0.71, the operation with this product without setups will be only 7.2 h to obtain optimal waiting time.

Now, we consider the nature of the ratio between the optimal lot sizes of various products. Using expression (3.42), we obtain

$$\frac{Q_i^*}{Q_k^*} = \frac{P_i}{P_k}\sqrt{\frac{s_i}{s_k}}. \tag{3.46}$$

From Eq. (3.46) it follows that the optimal ratio between the optimal lots does not depend on the machine load, i.e. the size of demand D_i. In this case, for example, the ratio between capacities P_1 and P_2 varies in a small range—from 2.7 to 3.0. Therefore, the ratio of the optimal lots, as can be seen from Table 3.7, changes slightly as well—from 3.7 to 4.0.

The results presented in the previous paragraph also determine the direction, in which it is advisable to work on reduction of the setup time. For example, if it is decided to develop relevant devices that accelerate the setup, it makes sense to determine for which type of the products the setup time reduction is more effective. This question has been studied in detail in the paper of Kuik and Tielemans (1998).

Obviously, in the case where, for example, the setup time for the i-th product affects the changes in the mean waiting time $\overline{w}$ more than the time of setup for k-type product, the following condition shall be satisfied:

$$\frac{\partial \overline{w}}{\partial s_i} > \frac{\partial \overline{w}}{\partial s_k}, \tag{3.47}$$

i.e. the partial derivative from the waiting time for the i-th product shall be bigger than for the k-type product. Since with the equality of these derivatives there is the same effect of both products, so for the solution of this problem it is necessary to define a set of parameters of the problem corresponding to this equation—the so-called indifference curve.

In Kuik and Tielemans (1998), the indifference curve is defined as function

$$u_i = \phi(u_k, s_i, s_k), \tag{3.48}$$

where u_i, u_k are variables defined by expression (3.43).

Analysis of the results of Kuik and Tielemans (1998) paper shows that functions (3.46) depend little on the setup time, in most cases, i.e. the indifference curve may be replaced by a straight line without prejudice for practical use

$$u_i = u_k. \tag{3.49}$$

Figure 3.12 shows the indifference line for the example with data in Table 3.6. Since point A with coordinates $u_1 = 0.4$; $u_2 = 0.28$, corresponding to the process state in Table 3.6, is below the indifference line, then the reduction in setup time for product 1 affects the mean wait time more than a similar decrease in setup time for product 2. Table 3.8 presents three possible combinations of setup time for products from Table 3.6. Data from Table 3.8 show that setup time reduction for the first

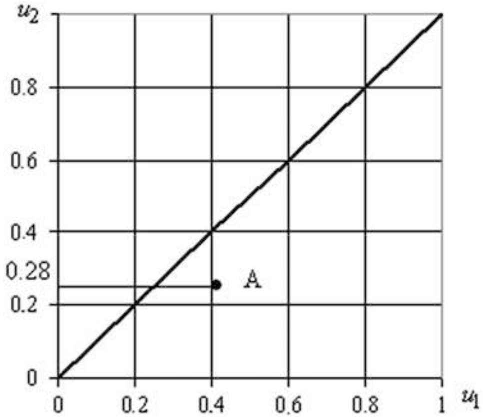


Fig. 3.12 Indifference line of influence of two products

Table 3.8 Mean waiting time for different values of setup time

Variant no.	Time s_1, h	Time s_2, h	Waiting $\overline{w}^*$, h
1	2	1	22
2	1.6	1	18.7
3	2	0.6	19.7

product indeed influences the waiting time reduction more than a similar reduction in setup time for product 2.

From a practical point of view, it is essential to know the degree of sensitivity of the mean waiting to the deviation lot size from the optimal value. The paper of Karmarkar et al. (1992) gives the example of the curve describing the change in the mean waiting time, depending on the ratio of the actual size of the lot to the optimal one *Q*/*Q**. This curve appears to be similar to the curve in Fig. 2.2 showing the change in relative costs. As in Sect. 2.1.1, the upward variance of the lot size has significantly less deterioration of optimization criterion than the downward variation.

3.5 Hierarchical Approach to Machinery Load Management

The logical evolution of the theories and methods of bottleneck management in production was the development of so-called Workload Control Concept (WLC concept). The basic idea of this concept is to provide systematic prediction, monitoring and maintenance of the management environment in a production unit, which allows responding quickly and adequately to current production requirements. It is assumed that planning at the operational level is not very itemized but has a certain freedom of control.

3.5.1 Principles of Workload Control Concept

The WLC concept provides two basic methods of such control (Henrich et al. 2003):

- Creation of buffers protecting from fluctuations in production load
- Aggregation of jobs and equipment into groups

Sets of jobs grouped by any ground together are called "workload" in terms of this concept. The WLC concept is expected to provide quite small cycle time by adjusting workloads within certain established norms. At the same time, such regulation should ensure the timely execution of jobs in accordance with the existing contracts. For this purpose, the software supporting the WLC concept should enable the acceptance of orders and determine their due dates and start dates.

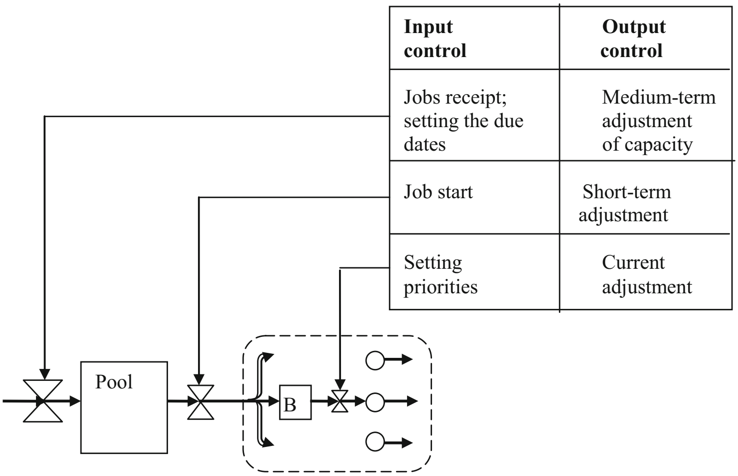
In theory, this concept is based on the logistic relationship between production load W and duration of the production cycle F, described in Sect. 3.2.1. According to this relationship, when the production load is within the established norms, the duration of the cycle will also be within acceptable limits.

To make the load control possible, it is detailed. For this purpose, first of all, it is assumed that in the unit (shop) the operation is performed in two levels. The first is the level of accounting, planning, and delivery of incoming jobs (pool), and then it is the level of the performers (shop floor). Accordingly, the total production load, which must be performed on the j-type work center, is divided into two components, W_j^p and W_j^f. Each of these components has its own duration F_j^p and F_j^f. Moreover, when more precise control is needed, the load of performers W_j^f can be divided into the load being performed in the current time period, waiting for performance and already performed. These loads can be associated with the corresponding durations of the process and checked for their compliance with standards in place.

Figure 3.13 shows the hierarchical control scheme at the shop floor level in the described concept by Land (2004). The circles in the diagram designate machines and the rectangles designate buffers. The material flows are indicated by bold arrows and the control lines are indicated by regular arrows. Material feed into the pool, from the pool to the work center B pool and the latter to processing are controlled by the hierarchical system of load shown in the upper part of Fig. 3.13. This system allows controlling both the material flow input and product output from processing and adjust the outgoing flow by changing the capacity of the equipment.

The decisions made at the level of the job input are used to control the entire volume of the gained load. The start level controls the load of work centers, and the level of scheduling allows setting the priority of each job.

The main purpose of the controlled load concept is to maintain minimal loaded condition at the level of lean shop floor. When controlling by the scheme showed in Fig. 3.13, the job does not accumulate in queues (logjam) before work centers and does not compete for the right of processing, but wait for the start in the workshop



Input control	Output control
Jobs receipt; setting the due dates	Medium-term adjustment of capacity
Job start	Short-term adjustment
Setting priorities	Current adjustment

Fig. 3.13 Hierarchal scheme of machine loading control [based on Land (2004)]

buffer until the relevant control command comes. Thus, the workshop pool is intended not only to smooth the manufacturing process but also has an important role to reduce the size of the queue of jobs directly on the equipment.

The controlled start maintains the small and constant size of the queues for processing. The main task of scheduling is to maintain the stability of material flow and respond to some abnormalities associated with the peculiarities of each job.

3.5.2 Example of Application of Controlled Load Approach

For the methodology used and described in this section (Great Britain) the special software complex Workload Control is developed in Lancaster University. Stevenson's paper (2006) describes the application of this complex for controlling the loading in the relatively small workshop containing 23 machines.

First of all, the machines in the shop were grouped into several work centers, the data of which were transferred to the system and served for decision making concerning jobs start. The grouping of machines would improve the system control significantly since it would allow reducing the number of standard indicators, for example, constraints in delaying the orders which must be traced.

Grouping of the interchangeable machines also allows (Henrich et al. 2004) making decisions concerning each machine load with some postpone, which provides more flexibility in the system. Despite the fact that due to some differences of the machines of the same work center in manufacturing date, capacity etc., as a rule, they are not fully interchangeable; however, the system's flexibility increases sufficiently. In particular, this is true for the case under consideration where the shop personnel have qualification allowing each of them to work on every machine belonging to the same work center.

In the illustrated case, all the machines were divided into 12 work centers with 1–4 machines in each center. Interchangeability of the machines in one work center averaged about 80 %. Then for each work center, its capacity was determined. The capacity was determined based on the number of employees, their classification, and the possibility to work in different shifts and overtime. At the same time, the possible transfer of employees from one work center to another in accordance with their qualifications is analysed.

After that, the average performance of the work center per 1 h of work by an employee is calculated. For example, for one of the work centers this indicator amounted to 1.5 processing hour per 1 h of an employee's working time. This indicator, as well as time real resources of the employees, was monitored by the system on a regular basis.

After the above preparation, the system allowed users to control loads in the following modes:

- Customers' requests control
- Introduction of new jobs
- Launching of jobs in production
- Capacities control
- Loads scheduling

To evaluate the effectiveness of the system activity, the performance indicators with a set of quite standing orders of one customer were studied. The orders accounted for about 10 % of the total work content of production. This evaluation was performed in three directions:

- Changes in the data structure on consumers' requests
- Changes in the scheduling
- Changes in the nature of delays of orders

Usage of the system showed that there is a significant difference in the perception of the quality of receiving orders from the company management and the customer. While the management believes that only 15 % of the customer's requirements is not met, the customer believes the percentage is equal to 85. Consequently, measures had been taken to consider the customer's requirements to the full.

During scheduling, it was found that when allocating jobs from the date of completion required under the contract by reverse calculation up to the present moment, it often appears that the lead time or capacity is not enough. Sometimes, there are situations when the required date appears to pass already at the moment of data input.

For correct calculation in scheduling as early as the stage of due date negotiation, the standards were used for the jobs queuing to the work center and the duration of the presence of the jobs in the workshop buffer. In such cases, the system helped to hold additional negotiations to adjust the due dates. The study of order delays nature revealed the need to monitor performance daily, when the duration of the production cycle is quite small. Otherwise, it is possible to do weekly check of the order status.

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Multi-criteria Models and Decision-Making 4

4.1 Basic Concepts in Multi-criteria Optimization Theory

This theory aims to develop an optimal solution of the complicated problem under the conditions where there is no one clear criterion for the quality of such solution. In the theory of multi-criteria optimization, the decision methods are being developed to adequately reconcile various criteria that are important from the point of view of the decision-maker (DM). The full range of these methods can be conditionally divided into two groups:

- Methods of finding a decision
- Methods of decision support

While in the first case the decision is completely determined in the framework of the theory, in the second case, the theory only suggests possible decisions to the DM, leaving the right of choice to DM. In reality, multi-criteria optimization methods always contain the elements of both these approaches to any extent.

4.1.1 Definition of Multi-criteria Optimization Problems

We begin with discussing the problem of the linear optimization similar to the problem described in Sect. 2.1.2 for the case of a single objective function. Recall that in this problem it was necessary to find the values of a set of variables x_1, x_2, and x_3, which determine the monthly production of paint of the first, second, and third types in order to get the highest value of the total profit $f(x)$.

In multi-criteria problem, there are several objective functions $f_1(x), f_2(x), \ldots, f_m(x)$, which reach their maximal (or minimal) values in various points of admissible set of values of variables $x_1, x_2, \ldots, x_n$. To make a decision on selecting the best combination of these variables the DM must have information that describes:

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Table 4.1 Rate of resource use and prices

Parameter	Product 1	Product 2	Resource price	Resource reserve
Resource 1	1	2	1	20
Resource 2	1	1	2	15
Resource 3	3	1	3	39
Product price	14	10	–	–
Profit per product unit	2	3	–	–

- Acceptable range of values of the variables
- Type of each objective function
- Principles of selection of optimal decision

Consider the example suggested in the book of Minyuk et al. (2002). Suppose that there are two types of products in the manufacturing and three types of resources are used for them. Table 4.1 shows the rate of resource use and prices for resources and finished products.

Assume that two goals are set in the production: to achieve the maximum output and maximum profit.

Determining from Table 4.1 that the profit per unit of product 1 is 2 units, and per unit of product 2 is 3 units, we formulate the following problem:

find these quantities x_1, x_2 of products 1 and 2 accordingly, which allow obtaining maximum of functions

$$f_1(x) = 14x_1 + 10x_2$$

and

$$f_2(x) = 2x_1 + 3x_2 \tag{4.1}$$

with resource constraints

$$\begin{aligned} x_1 + 2x_2 &\leq 20; \\ x_1 + x_2 &\leq 15; \\ 3x_1 + x_2 &\leq 39 \end{aligned} \tag{4.2}$$

and constraints on variables' values

$$x_1 \geq 0; x_2 \geq 0. \tag{4.3}$$

We shall construct the range of acceptable values of variables x_1, x_2, limited by constraints (4.2), (4.3). To do this, we draw three straight lines in the rage of positive values:

line AA′ with equation $x_2 = 10 - x_1/2$;
line B′C′ with equation $x_2 = 15 - x_1$;
line DD′ with equation $x_2 = 39 - 3x_1$.

The range limited by line segments AB, BC, and CD (Fig. 4.1) is a range of acceptable values of variables x_1, x_2 because constraints are realized inside of it (4.2).

In Sect. 2.1.2, it was shown that every optimal decision corresponds to a number of "coupled" constraints. Recall that the coupled constraints are the constraints, the values at the left side of which are equal to the values on the right side in case of optimal decision. This indicates that the optimal decisions occur at the points of the range, where simultaneously at least two of the inequations (4.2) become equal, i.e. at the intersection points of the straight lines that limit the acceptable range of variables.

Indeed, in the theory of linear programming it is proved that the peak values of the objective functions take place at the points called vertices which in this case are points A, B, C, and D. For these points, we calculate the values of functions $f_1(x)$ and $f_2(x)$ (Table 4.2).

Analysing the results in Table 4.2 we can see that the greatest value of production output $f_1(x)$ is at point C with coordinates $x_1 = 12, x_2 = 3$. At the same time, the highest profit is achieved at point B with output of product 1 equalling to $x_1 = 10$

Fig. 4.1 The range of acceptable values of variables

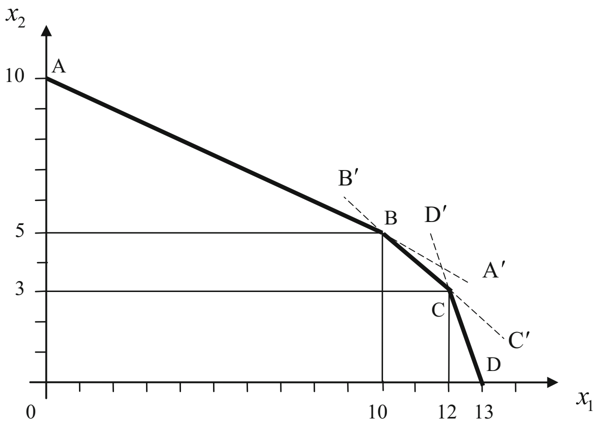


Table 4.2 Values of the objective functions at vertices of acceptable range

Function	Vertex A	Vertex B	Vertex C	Vertex D
$f_1(x)$	100	190	198	182
$f_2(x)$	30	35	33	26

and product 2 equalling to $x_2 = 5$. It is obvious that the decision-maker has to make his choice between these two possibilities. The rest of this chapter is devoted to various approaches to this decision-making.

4.1.2 Pareto Optimality

When choosing optimal output volumes of two different products in the previous example it is not necessary to be confined to those found peak points of functions $f_1(x)$ and $f_2(x)$. Moreover, as a rule, the decision to be made is between these values. At the same time, it is obvious that the made decision should be optimal in the sense that it cannot be improved simultaneously by both criteria under consideration.

We analyse various possible decisions of the previous example with the help of the so-called level lines. Each line of the level represents the set of points in the space of variables for which the value of objective function is constant. For example, in Fig. 4.2 four level lines of function $f_1(x)$ (dashed lines) are drawn with values ranging from 186 to 198 and four level lines of function $f_2(x)$ (solid lines) with values ranging from 32 to 35. As you can see, for analysis the level lines are used with the values of the objective functions, a bit smaller than the previously identified peaks since the high values do not satisfy the problem constraints. The bold line in Fig. 4.2 shows a section of the boundary of the rage of possible values for the variables shown in Fig. 4.1.

Let us consider a few of the intersection points of the level lines and the area boundaries. At points B and C, as stated above, there are peak values of functions $f_2(x)$ and $f_1(x)$, respectively. At point B, maximum line $f_2(x) = 35$ intersects also with the level line $f_1(x) = 190$. Similarly, at point C, maximum line $f_1(x) = 198$ intersects the boundary and level line $f_2(x) = 33$.

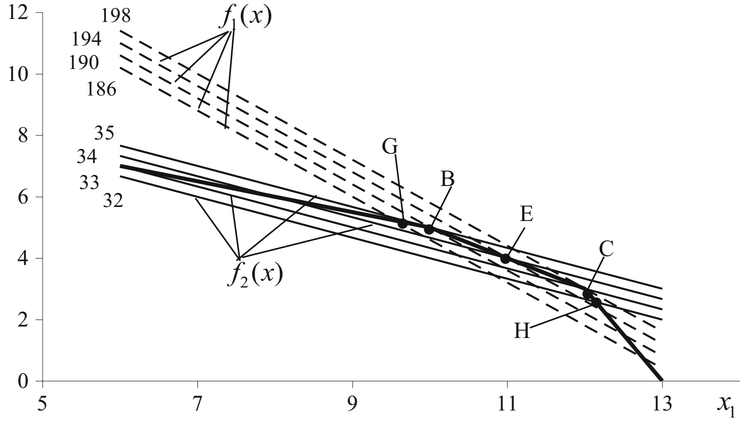


Fig. 4.2 Lines of objective functions level

At point E, which is on the boundary segment BC, the level lines $f_1(x) = 194$ and $f_2(x) = 34$ intersect as well. Obviously, at this point, $f_1(x)$, though not reaching the maximum equal to 198, is larger than the value at point C, equal to 190. Similarly, the position of point E is observed for function $f_2(x)$ as well—here its value is 34, which is intermediate between the maximum value 35 and value 33 at point C, which is the maximum point of function $f_1(x)$.

A completely different situation occurs at points G and H. At point G, in which the level line of function $f_1(x) = 186$ crosses the area boundary, the function value is in the range from 34 to 35. Thus, at this point, the values of both functions $f_1(x)$ and $f_2(x)$ are simultaneously less than their values at point B, equal to 190 and 35, accordingly. The situation is similar at point H. Here $f_2(x) = 32$, $194 < f_1(x) < 198$, while at point C the value of objective functions $f_2(x) = 33$ and $f_1(x) = 198$.

They say that decision f^1 dominates decision f^2, if the values of all objective functions of the first decision is greater or equal to the values of the second one, and at least one of the values of the objective functions of the first decision is greater than that of the corresponding function of the second decision. In the above case, decision f^B dominates decision f^G and decision f^C dominates f^H. At the same time, decision f^E is not dominated by any other decision.

A number of non-dominated decision is called the set of Pareto optimality or simply Pareto set. Each non-dominated decision $f(x)$ in space m of variables corresponds to point x with coordinates, $x_1, x_2, \ldots, x_m$. These points are called effective (Sobol and Statnikov 1981). In the above example, $m = 2$, and the set of efficient points is segment BC of the border of the range, in which values of variables are acceptable.

Accomplishing the discussion of the example in Fig. 4.1, we observe that the level lines of the objective functions in this example are straight lines, because the problem equations (4.1) and (4.2) are linear. For this problem, as it was mentioned above, the largest (or smallest) values are always achieved at the boundary of the acceptable range.

A significant part of planning problems, however, is non-linear, and for these problems extremum points are within the range of variables. In addition, unlike the example considered, for multi-criteria planning problems the "planning dilemma" (mentioned in Sect. 3.2) is typical, which consists in the inconsistency of criteria. For example, to achieve the best productivity it is required to increase the size of production lots, and in order to reduce the production cycle time the lot sizes must be reduced. For the optimal solution of such problems, it is extremely important to identify effective points in the space of variables and construct the corresponding Pareto set of non-dominated decisions.

The case of the two objective functions is the most common and at the same time easily illustrated in graphics. Let the number of independent variables be equal to n. Curve E in Fig. 4.3 is the projection of the curve of effective points in space with dimension n to the plane of certain two variables out of the total of n. The corresponding curve $\widetilde{E}$ in Fig. 4.3b contains Pareto set of non-dominated decisions in the space of criteria (Sobol and Statnikov 1981). The shaded area D of possible

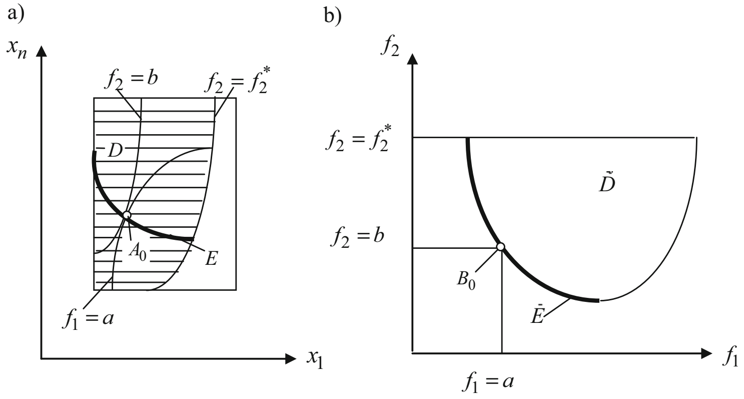


Fig. 4.3 Effective points in the space of variables and Pareto points in the space of criteria

values on the plane of variables x_1x_n (Fig. 4.2a) is limited by certain values x_1 and x_n, as well as the curve with a given criterion value$f_2 = f_2^*$. We choose any point A_0 on the set (curve) of efficient points. Each such point is at the same time the point of contact of the level lines of criteria f_1 and f_2 passing through this point.

On the plane of criteria (Fig. 4.3b), area D goes into area$\widetilde{D}$, the curve E goes into curve $\widetilde{E}$, and A_0 point goes to point B_0 with coordinates a and b. Area $\widetilde{D}$ itself is an area of acceptable but not enough effective decisions, and the boundary of this area $\widetilde{E}$ is a set of effective decisions.

Line $\widetilde{E}$ is called a trade-off curve. All points of the trade-off curve represent non-dominated decisions. At transition from one point to another point of this curve the value of one of the criteria increases, and the value of another certainly decreases. This kind of trade-off curve is typical for the case where it is desirable to simultaneously increase or decrease both optimization criteria. If one of the criteria must be increased, and another must be decreased, the trade-off curve also exists but has a slightly different form, as will be shown below.

Unlike the trade-off curve, curve E can be of any nature and even become a straight line. As the final decision, one of the points of trade-off curve should be taken, the combination of the criteria values of which is the most acceptable from the point of view of the decision-maker.

4.1.3 Main Methods of Solving Multi-criteria Planning Problems

Most of the methods for solving multi-objective problems belong to one of three groups:

- Methods of theory of utility
- Methods based on criteria space metric
- Methods of hierarchal analysis

The methods of the first group were mentioned above in Sect. 1.7.3. Methods of the first and second groups are used when all the relevant criteria have comparable value. These will be considered further in the various chapters of this book. In cases when some of the criteria are much more important than the others, it makes sense to use the methods of hierarchical analysis. In this book, these methods are not considered.

The process of setting a multi-objective problem consists mainly of three steps (T'Kindt and Billaut 2005). The first of them determines a number of possible solutions; the second step describes the criteria and variables (attributes) of the problem; and the third stage defines the utility function of each criterion. In the process of solving the problem the attribute values are searched, which maximize the utility of criteria. In the theory of multi-criteria optimization, it is proved that the optimal solution can be found almost always.

In solving the problems of planning the set of possible solutions usually consists of a set of lots of different products that can be released for manufacturing in specific time periods. Optimal solution should obviously consist in determination of the lot sizes in each period, and for the development of such a decision, the quality criteria must be set.

Above Tables 2.5–2.7 show exemplary sets of criteria for each of the basic types of production. Section 2.2.3 gives the analysis of these tables, from which it follows that the quality of a particular plan is mainly determined by the values of direct or indirect costs and performance of various obligations. Such obligations may be not only the need to fulfil orders, but also to maintain the size of reserves.

In the simplest cases, the criterion of order fulfilment is defined as makespan $C_{\max}$ or a minimum value of the maximum delay of orders fulfilment $T_{\max}$; at that the first criterion is preferable for "to stock" strategy and the second "to order" strategy. The criteria are calculated at the time of planning for a specific duration of the plan implementation—for some planning horizon.

In order to achieve the calculated values of the criteria, there must be strict adherence to the plan, which is difficult to perform, at least for "to order" strategy. This situation is due, firstly, to the difficulty in determining the initial conditions of the plan—time of readiness of processed products and equipment. Secondly, during the time of the plan implementation new jobs can appear, the conditions of equipment operation and personnel work can change, etc. Therefore, in practice, at least at the level of operational plans, we have to use not only the optimization planning using $T_{\max}$ or $C_{\max}$, but rather the planning by the priority of jobs. In multi-criteria planning, it is useful to use utility functions as criteria.

Instead of the criterion of direct costs (K1 in Tables 2.5–2.7) the function of negative utility of costs (function of loss) can be introduced to perform n jobs on the planning horizon

$$U = \frac{1}{c}\sum_{i=1}^{n} c_i, \tag{4.4}$$

where c_i is value of costs, e.g. the changeovers from one job to another,, and
c is the cost of one shift.

If the quantity of the orders on the planning horizon h is equal to n, their total current utility is equal to the sum of utilities of each since the orders are usually independent. Then the total value of the current utility function of the orders according to Eq. (2.32):

$$V = \sum_{i=1}^{n} V_i = \frac{1}{G}\sum_{i=1}^{n} w_i p_i - \sum_{i=1}^{n} H_i, \tag{4.5}$$

where, as above, w_i is the priority coefficient of i-type job, p_i is the processing time of the i-th job, G is the average quantity of days in planned period, and H_i is the production intensity of i-th order.

In contrast to the examples in Sect. 4.1.2, the utility of criteria U and V has a different focus. In fact, when planning we should try to have costs value U as low as possible and current utility V, on the contrary, as high as possible. In this case, the dilemma of planning leads to another kind of trade-off curve: when moving along the points of the curve, both criteria simultaneously either increase or decrease depending on the direction of motion.

Using Table 1.2 of production scale corresponding to the production types, and Tables 2.5–2.7, which describe the hierarchy of costs criteria K1–K6 and timely performance of obligation C1 and C2, you can set a definite connection between functions U and V (Mauergauz 2012). Figure 4.4 shows Pareto diagram of functions U and V for master plan and Fig. 4.5 for the operational plan.

The curves in Figs. 4.4 and 4.5 are a set of Pareto points, showing the quality of the specific formed plan as a combination of the values of the loss function due to the costs and current utility function. On these curves, there are the points corresponding to different production scales and strategies.

Figures 4.4 and 4.5 make it obvious that the smaller the scale of production and lower the level of the plan hierarchy, the greater the effect of the current utility function of obligations performance. In this case, its optimal value becomes higher; while the acceptable value of the loss function of the cost also increases. From this result, we can draw several conclusions which directly define the setting of multi-criteria optimization problems of planning.

- In mass and large-serial production, where it is primarily important to reduce the costs, decrease of order utility can be assumed and thus the point of decision on the Pareto diagram is in the area of small values of function V and U. In these cases, for the purpose of optimization it is advisable to use one objective function that minimizes the amount of costs, and the need to fulfil obligations

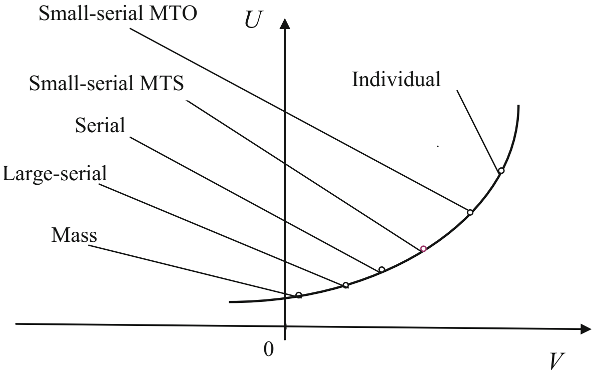


Fig. 4.4 *UV* diagram for master plans

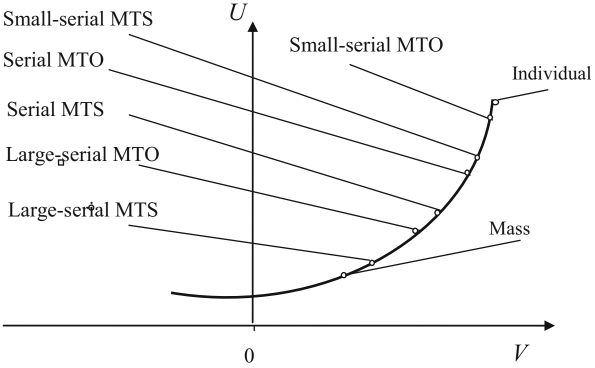


Fig. 4.5 *UV* diagram for operational plans

can be taken into account using some limitations. This applies to both the master and operational plans.

- For operational plans in small-serial production it also makes sense to use one objective function, but minimizing the losses because of non-fulfilment of obligations. In this variant, the decision point is in the area of big values of V and U.
- When finding the decision point in the middle section of the curve it is not possible any more to be limited to one optimization criterion, in general case. In some cases, for example, during developing master plan for serial, small-serial, and project productions, one objective function of costs can remain by including fines derived from non-fulfilment of obligations into it. However, in developing the plans of the lower level—material requirement and, in particular, operational plans, the cost issues fade into insignificance. For example, economic criteria can be considered only when determining the size of the production lots and the lead time is determined on the basis of the minimum delay in the performance of obligations.

Yet, in some cases, for example, when planning operation of the machine for cutting sheets (serial or small-serial production), it is impossible to be limited by one criterion. The main criterion of the plan when fulfilling the orders of machine shops, as seen in Table 2.5, is criterion C1 of timely provision of these shops with blanks. In this case, however, the need to consider other criteria is clearly seen: perhaps the less labour input of cutting and delivery of the sheets K1 and material saving when cutting sheet K2.

4.1.4 Analytical Method of Constructing a Trade-Off Curve

As mentioned above it follows that to solve the multi-criteria problem, you must first construct a trade-off curve and then select the point of the curve that fits the real requirements of the problem in the best way. Let us consider the problem with two criteria that need to be let to go to a minimum by the optimal choice of set of independent variables x_i. Suppose for definiteness, for example, the increase of values of any of the parameters leads to decrease of value $f_1(x)$ and increase of value $f_2(x)$.

In formulating the multi-criteria problem we can always assume that one of the criteria serves as an objective function of the one-criterion problem, and the rest of the criteria are constraints. Suppose that one of the criteria, for example, $f_2(x)$, is the objective function and the second $f_1(x)$ is the constraint with the set value $f_1(x) = a$ (Fig. 4.6).

Along the coordinate axes, similar to Fig. 4.3b, on Fig. 4.6 the criteria values are plotted. At the point of X-axis with $f_1 = a$ we draw a vertical straight line, which is the locus of all the states of the system with $f_1 = a$. Let us consider a state of the system (point B_1), characterized by a set of values (a, b_1).

Now we will increase by one of the independent variables. For example, suppose that if x_1 change the system will transfer from state B_1 to state L_1, if x_2 change—to

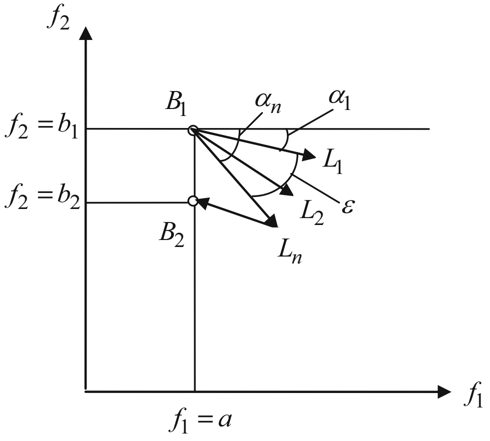


Fig. 4.6 Algorithm to construct points of the trade-off curve

state L_2, if x_n change—to state L_n. Now by linking point B_1 to new points L_i we get a sheaf of vectors characterizing the transitions from the initial state to some other one by varying the independent variables of the system. The larger the vector's angle of inclination towards the X-axis, the greater the decrease of $f_2(x)$ at the same increases of $f_1(x)$. Therefore, out of the entire set of vectors, the "best" effect on the system is produced by the vector with the greatest inclination to the X-axis and the "worst" with the smallest inclination. In this case, this "best" vector is B_1L_n and the "worst" vector is B_1L_1.

When moving from point B_1 to L_n the value of $f_2(x)$ decreases, but the value of criterion $f_1(x)$ increases, and thereby the accepted condition of constancy of limit $f_1 = a$ is violated. To maintain the constant limit value it is obviously necessary to go back to the state illustrated by the point on line $f_1 = a$.

This yields the idea that in order to "improve" the state of the system it is advisable first to go from point B_1 to point L_n, i.e. pass along the vector with the largest angle of inclination α_n, and then come back from point L_n to line $f_1 = a$ along the vector with the smallest inclination α_1. In this case, the state of the system will be clearly better than the initial, because limit $f_1 = a$ will be valid, and the values of criterion $f_2(x)$ at obtained point B_2 will be clearly smaller than the initial one. This process can be continued until angle ε of the opening of vector sheaf equals 0.

Given that the tangent of the inclination of each vector is represented by a derivative from $f_2(x)$ by $f_1(x)$ at changing only one of the independent variables, the equal-zero condition of angle opening ε will be as follows:

$$\frac{\partial f_2(x_1)}{\partial f_1(x_1)} = \frac{\partial f_2(x_2)}{\partial f_1(x_2)} = \ldots = \frac{\partial f_2(x_n)}{\partial f_1(x_n)}. \tag{4.6}$$

The derivatives in expression (4.6) depend on the parameters. Using the differentiation formula of the implicit function

$$\frac{df_2(x)}{df_1(x)} = \frac{df_2(x)}{dx} \bigg/ \frac{df_1(x)}{dx}$$

we obtain the equation system

$$\frac{\partial f_2}{\partial x_i} \times \frac{\partial f_1}{\partial x_k} = \frac{\partial f_2}{\partial x_k} \times \frac{\partial f_1}{\partial x_i}, \quad i = 1 \ldots n, \quad k = 1 \ \ldots n, \quad i \neq k. \tag{4.7}$$

The number of linearly independent equations (4.7) with any number of independent variables is always one less than number n of independent variables x_i. So their solution gives not one specific optimal set of variables, but the set of these values form a trade-off curve. Selection of the most appropriate point on the trade-off remains with the decision-maker.

From the geometric point of view, equation (4.7) describes the curves lying in the planes which at the same time touch surfaces $f_2(x_1, x_2, \ldots x_n)$ and $f_1(x_1, x_2, \ldots x_n)$.

Therefore, the method is called the method of common tangents (Mauergauz 1999). The example of use of equations (4.7) for the problems of planning is presented below in Sect. 4.2.2.

4.2 Optimized Multi-criteria Lot Sizing

The scientific literature describes two methods of multi-criteria optimization of lot sizes. In the first case, the solution of the optimization problem with one objective function, which is a combination of several criteria, is searched. The second approach is based directly on construction of the trade-off curve and the solution is selected on this curve. We begin consideration of the first method.

4.2.1 Lot Sizing Based on Costs and Equipment

In the paper of Missbauer (2002), the optimization of lot sizing is based simultaneously on minimization of the cost of setup and storing similar to EOQ model, described in Sect. 2.1.1, and minimization of the waiting time similar to the model described in Sect. 3.4.1. At the same time, as usual, the constant demand for each product D_i is assumed. Every possible solution is represented by a set of values of lots of each i-th product, characterized by the costs for setup and storage, as well as the average waiting time of the lot for processing.

Workload of machine u consists of processing load of each i-th product lot

$$u = \sum_{i=1}^{n} u_i = \sum_{i=1}^{n} \frac{D_i}{P_i},$$

where, the same as above, P_i is the machine capacity pcs/h.

To construct the trade-off curves "cost–waiting time" the following problem is to be solved:

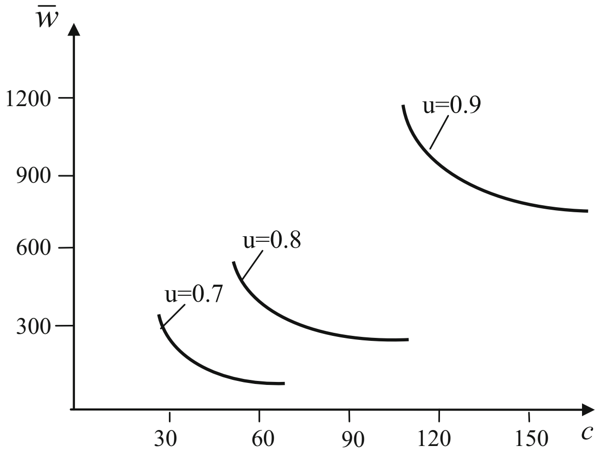
$$(1-\gamma)\sum_{i=1}^{N}\left(\frac{c_{hi}Q_i}{2} + \frac{c_{oi}D_i}{Q_i}\right) + \gamma\overline{w}(Q), \tag{4.8}$$

with constraint

$$\sum_{i=1}^{n}(D_i/P_i + D_i s_i/Q_i) < 1 \tag{4.9}$$

and at any (or at least several) values of weighing factor γ, which can be changed from 0 to 1. Constraint (4.9) means that the average load coefficient $\overline{K}$ should be less than one. In expressions (4.8) and (4.9) in analogy with Sects. 2.1.1 and 3.4.1 c_{oi} is costs for i-th order fulfilment, c_{hi} is costs for storage of i-th product, Q_i is the size of

Fig. 4.7 Trade-off curves of criteria $c - \overline{w}$ (costs – mean waiting time) at various values of workload



i-th product, $\overline{w}(Q)$ is the mean waiting time, and s_i is the time of setup for i-th product production.

Figure 4.7 shows the exemplary trade-off curves approximately reflecting some results of numerical solution of problem (Eqs. 4.8 and 4.9) performed in Missbauer (2002). The qualitative analysis of the trade-off curves shows that with increasing load both the cost of production and the waiting time increase and this happens when $u > 0.8$ is very fast. At the same time, the lot sizes also increase substantially.

When changing weighting factor γ within 0 to 1 the optimal value of a lot changes from the value determined by formula EOQ (2.4) to the value set by the minimum waiting formula (3.42). In this transition in Missbauer (2002) in order to determine the optimal lot size it is suggested to use the value determined by formula EOQ (2.4), multiplied by the so-called correction factor χ_i, i.e.

$$Q_i^* = \chi_i \sqrt{\frac{2c_{oi}D_i}{c_{hi}}}. \tag{4.10}$$

In Missbauer (2002), it is also proved that for the case $\gamma = 1$ the correction factor can be presented in the form of the product of two components

$$\chi_i = \overline{\chi} \times \overset{\frown}{\chi}, \tag{4.11}$$

where $\overline{\chi}$ depends only on the value of workload u and does not change depending on the product type. If the output proportion of the various products at the change of their output volume does not change, then $\overset{\frown}{\chi}_i$ does not depend on this volume, but depends on the type of product.

In the case when the cost of storage per product unit c_{hi} is a portion of the total cost of this product c_i, and the costs value c_{oi} is proportional to setup time s_i, expression (4.10) can be as follows:

$$Q_i^* = \hat{\chi}_i \sqrt{\frac{s_i D_i}{c_i}}, \tag{4.12}$$

where factor $\hat{\chi}_i$ is analogous to correction factor χ_i. Below, we will show how expressions (4.10) and (4.12) can be used in different ways of lot sizing.

4.2.2 Analytical Lot Sizing with Two Criteria: Setup Time and Cost

Let us consider the problem of determining the size of production lot of the parts of several kinds to make both the setup time and the value of work-in-progress as low as possible. Since the time of parts processing does not depend on the size of the lots, it is possible to reset the problem as follows (Mauergauz 1999):

Find the set of lots $Q_1, Q_2, \ldots Q_n$, giving both the minimum for the function of setup time and costs per the time unit

$$s = \sum_{i=1}^{N} s_i \frac{D_i}{Q_i} \tag{4.13}$$

and average cost of work-in-progress for schedule period G

$$\overline{c} = \sum_{i=1}^{N} \overline{c}_i \frac{D_i G}{Q_i}. \tag{4.14}$$

In expression (4.14), value $\overline{c}_i$ is the average value of one lot of parts in progress, referred to the schedule period. Assuming that the manufacturing cost of the lot can be considered as linearly increasing with time, and the value of costs linearly decreasing, we obtain a graph of cost variance, as shown in Fig. 4.8.

From Fig. 4.8, it follows that the cost of the lot increases during the lead time t_{mi} of the lot from 0 to overall cost $c_i Q_i$, and then drops to 0 for the time t_{si} of the lot utilization. To determine average cost $\overline{c}_i$ of work-in-progress belonging to the schedule period, the area under the triangle of the cost coming out of the manufacture and utilization of the lot should be divided by the duration of schedule period G.

$$\overline{c}_i = \frac{c_i Q_i (t_{mi} + t_{si})}{2G} = \frac{c_i Q_i (p_i Q_i + Q_i / D_i)}{2G}. \tag{4.15}$$

Hence, the average cost of the work-in-progress according to Eq. (4.14):

$$\overline{c} = \sum_{i=1}^{N} c_i Q_i \frac{(p_i D_i + 1)}{2}. \tag{4.16}$$

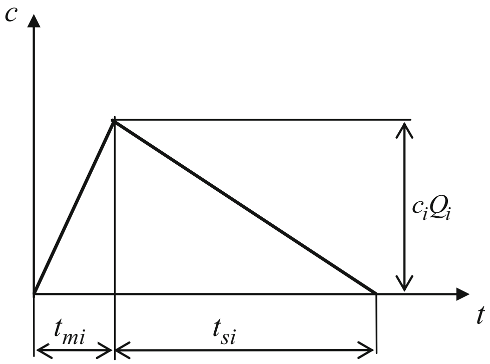


Fig. 4.8 Graph of cost variance of the lot in progress

Let us find the derivatives for the lot sizes Q_i from setup time s (Eq. 4.13) and cost $\overline{c}$ (Eq. 4.16):

$$\begin{aligned}\frac{\partial s}{\partial Q_i} &= -\frac{s_i D_i}{Q_i^2},\\ \frac{\partial \overline{c}}{\partial Q_i} &= \frac{c_i(1+p_i D_i)}{2}\end{aligned} \tag{4.17}$$

and insert them into Eq. (4.7). We obtain the equation system as follows:

$$\frac{s_i D_i c_k(1+p_k D_k)}{Q_i^2} = \frac{s_k D_k c_i(1+p_i D_i)}{Q_k^2}, \tag{4.18}$$

the number of which equals $N-1$.

If we divide both parts of the equations by $c_i(1+p_i D_i)$ and $c_k(1+p_k D_k)$, we shall obtain:

$$\frac{s_i D_i}{Q_i^2 c_i(1+p_i D_i)} = \frac{s_k D_k}{Q_k^2 c_k(1+p_k D_k)}. \tag{4.19}$$

In this equation, the left side depends only on the variable with number i and the right side only on the variable with the number k. This is possible only if both the left and right sides are equal to some arbitrary constant A. In this case, with any i we have the expression for the optimal lot size:

$$Q_i^* = A\sqrt{\frac{s_i D_i}{c_i(1+p_i D_i)}}. \tag{4.20}$$

In those cases when the amount of the consumed product within the manufacture time of the product unit is much less than one, expression (4.20) is simplified:

Table 4.3 Products manufactured on the machine

Product no.	Setup time s_i, h	Demand D_i, pcs/day	Product cost c_i, \$/pcs
1	2	40	80
2	1	10	20
3	1	20	50

$$Q_i^* = A\sqrt{\frac{s_i D_i}{c_i}}. \tag{4.21}$$

Accordingly, expression (4.16) is simplified:

$$\overline{c} = \frac{1}{2}\sum_{i=1}^{N} c_i Q_i. \tag{4.22}$$

By comparing Eqs. (4.12) and (4.21) we can see that constant A in formula (4.21) coincides with the value of correction factor $\hat{\chi}$, and in this case this factor does not depend on the type of product i. Let us consider the example of formula (4.21). Suppose that a machine manufactures three products (Table 4.3).

By setting various values of correction factor A we build the diagram of effective points and level lines of criteria in the space of variables Q_i (Fig. 4.9a) and the trade-off curve for criteria s and $\overline{c}$ (Fig. 4.9b).

Figure 4.9a shows projection E of the effective points line in the space of variables to plane Q_1Q_2. The diagrams in Fig. 4.9 are similar to the relevant diagrams in Fig. 4.3 with the difference that, in this case line E is straight, as in accordance with Eq. (4.21)

$$Q_2^* = Q_1^*\sqrt{\frac{s_2 D_2 c_1}{s_1 D_1 c_2}}.$$

At each point of straight line E, for example, at point A_0 with coordinates $Q_1 = 100$ and $Q_2 = 70$, level lines s and $\overline{c}$ are tangents to each other, as shown in Fig. 4.9a. Level lines $\overline{c} = \text{Const}$ are the straight lines because according to Eq. (4.16) $\overline{c}$ depends on the lot sizes $Q_1, Q_2, \ldots Q_n$ linearly. Level lines $s = \text{Const}$ have the form of hyperboles defined by relationship (4.13). Point A_0 of line E corresponds to point B_0 on the trade-off curve $\widetilde{E}$ with coordinates $\overline{c} = 6300$ and $s = 1.25\,\text{h}$.

From the above Sects. (4.2.1) and (4.2.2), it follows that the mathematical solution of the problem of multi-criteria optimization is to build a trade-off curve, but this is not enough to obtain specific values of the optimal lots. In fact, the mathematical solution allows the user choosing from a limited set of options that do not dominate each other. The result is that the decision-maker must make a final choice on some additional grounds. The methods of this choice are discussed below in this chapter.

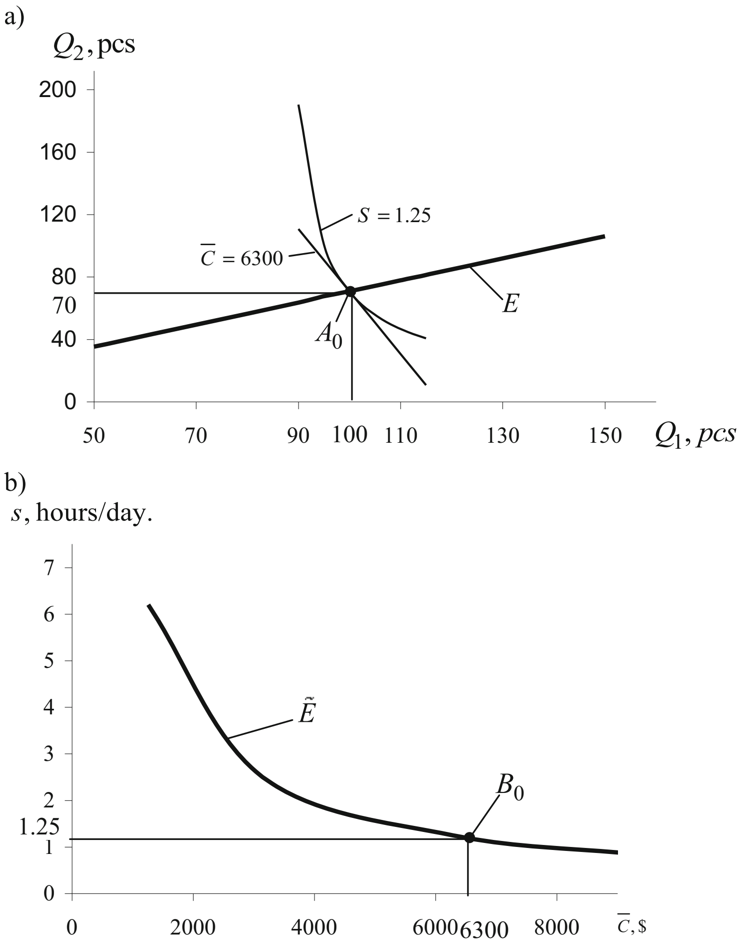


Fig. 4.9 (**a**) Diagram of effective points; (**b**) trade-off curve

4.3 Example of Multi-scheduling Problem

Let us consider one of simple scheduling problems with two criteria. Suppose that several jobs need to be performed on a single machine. For each of the jobs the required due date d_i is set. We will seek optimization of the problem simultaneously by two criteria: the minimum of mean duration of each job $\overline{F}$ and the minimum value of the maximum delay $T_{\max}$. According to the classification of scheduling problems described in Sect. 2.2.2, the classification formula of this problem has the following form

$$1\left|d_i\right|\overline{F}, T_{\max}. \tag{4.23}$$

To solve this multi-criteria problem it is necessary, as above, to construct a trade-off curve of non-dominated solutions. The algorithm of the solution was developed in the paper of VanWassenhoven and Gelders (1980) and is essentially based on the concept of the so-called special ε-neighbourhood of effective points.

4.3.1 Special ε-Neighbourhood of Efficiency Points

If every i-th criterion f_i must tend to increasing then in neighbourhood of every effective point A_0 it is always possible to build a certain area, at the set of points A of which the values of all criteria are less than their values at the effective point, increased by the value ε, corresponding to this area, meaning

$$f_i(A) < f_i(A_0) + \varepsilon. \tag{4.24}$$

If the criteria must tend to descending, then set A defined by relation (4.24) is an area, in which the values of all the criteria are more than their values at the effective point, increased by the corresponding value ε of this area. Here is an example of ε-neighbourhood of the effective point for the problem discussed above in Sect. 4.2.2.

In this problem both criteria s and $\overline{c}$ should tend to minimum. That is why ε-neighbourhood is formed by level line $s = s(A_0) + \varepsilon$ and level line $\overline{c} = \overline{c}(A_0) + \varepsilon$ (Fig. 4.10a).

The shaded area in Fig. 4.10 is formed by the level lines of the problem criteria. The criteria values at the level of these lines according to Eq. (4.24) exceed the corresponding values of the criteria at point A_0 by value of ε. The points of ε-neighbourhood can be used to find the actual position of point A_0. To do this it

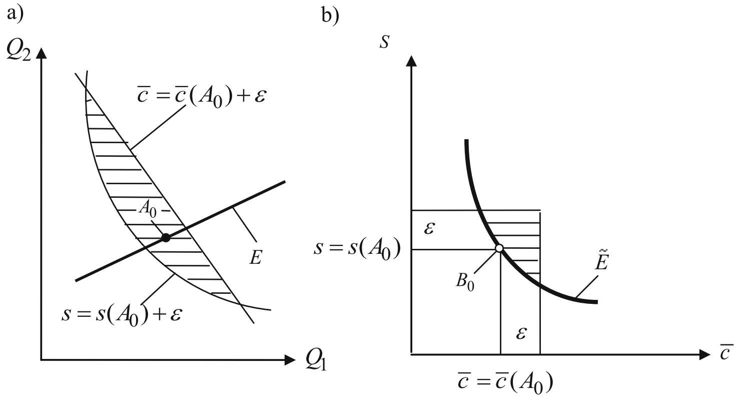

a)
Q_2
$\overline{c} = \overline{c}(A_0) + \varepsilon$
A_0
E
$s = s(A_0) + \varepsilon$
Q_1
b)
S
ε
$s = s(A_0)$
B_0
$\tilde{E}$
ε
$\overline{c}$
$\overline{c} = \overline{c}(A_0)$

Fig. 4.10 (**a**) Special ε-neighbourhood in the space of variables; (**b**) illustration of ε-neighbourhood in the space of criteria

Table 4.4 Data on the planned jobs

Job no.	Processing time p_i, work. days	Due date d_i
1	2	24
2	4	22
3	5	18
4	7	23
5	10	21

is obvious that one has to set the initial point of ε-neighbourhood and then using the relevant algorithm to decrease value ε gradually, pulling the shaded area to the point.

Figure 4.10b shows that special ε-neighbourhood of effective point A_0 in the space of criteria meets the one-sided pyramid (shaded) neighbourhood of Pareto point B_0.

4.3.2 Solving Algorithm

We continue to solve the problem set above in this section by the specific example. Suppose that in the beginning of the month, using the minimum criterion of mean duration of each job $\overline{F}$ and the minimum value of the maximum tardiness $T_{\max}$ it is required to schedule jobs for a single machine, the data of which is given in Table 4.4.

The idea of the described algorithm is based on theorems 1 and 2 from Sect. 2.3.2. Recall that theorem 1 states that the smallest mean duration of the production cycle $\overline{F}$ is achieved by using the rules of the smallest processing time SPT. Accordingly, theorem 2 states that the minimum value for the maximum deviation of i-th job from the set due date $L_{\max}$ (as well as for the tardiness $T_{\max}$) is achieved by using the EDD rules, in which the priority is given to the job with the nearest contractual due date d_i.

In accordance with these theorems the described algorithm uses the combined priority rule SPT/EDD, and criterion $T_{\max}$ is used as a gradually decreasing constraint of the form

$$T_{\max} \leq \varepsilon, \tag{4.25}$$

and criterion $\overline{F}$ is used as the objective function. Constraint (4.25) means that the maximum possible due dates must be within this neighbourhood, i.e.

$$\hat{d} = d_i + \varepsilon. \tag{4.26}$$

To use SPT rule the data in Table 4.4 is sorted in ascending order of processing time. Figure 4.11 shows the block diagram of the algorithm. At the beginning of the algorithm the initial values of the external cycle are set, which include the value of

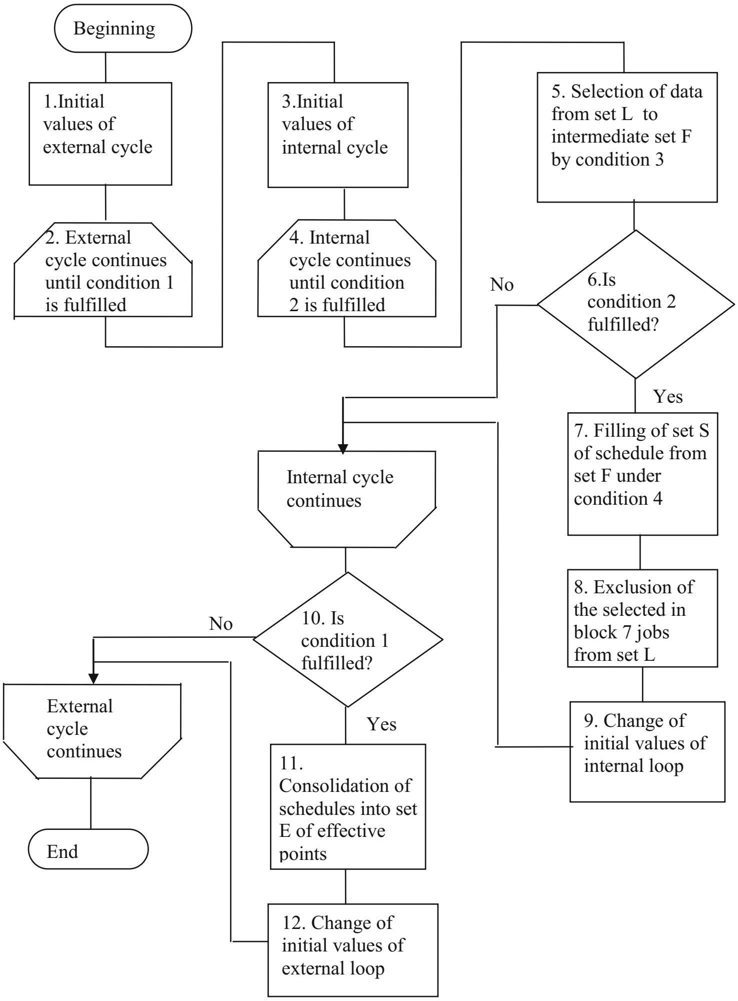

Beginning
1.Initial values of external cycle
2. External cycle continues until condition 1 is fulfilled
3.Initial values of internal cycle
4. Internal cycle continues until condition 2 is fulfilled
5. Selection of data from set L to intermediate set F by condition 3
6.Is condition 2 fulfilled?
No
Yes
7. Filling of set S of schedule from set F under condition 4
8. Exclusion of the selected in block 7 jobs from set L
9. Change of initial values of internal loop
Internal cycle continues
10. Is condition 1 fulfilled?
No
Yes
11. Consolidation of schedules into set E of effective points
12. Change of initial values of external loop
External cycle continues
End

Fig. 4.11 Block diagram of scheduling algorithm

ε-neighbourhood of the due date in this neighbourhood. As the initial value of ε it is advisable to take the full processing time of all the jobs, i.e.

$$\varepsilon_1 = \sum_{i=1}^{N} p_i. \tag{4.27}$$

In the process of work, two nested cycles are performed: external and internal. Possible specific schedule is drawn up during operation of the internal cycle. The work of the outer cycle is to change value ε, which prepares the new job of the inner cycle. At each subsequent step $k + 1$ new value ε-neighbourhood is defined as

$$\varepsilon_{k+1} = T_{\max,k} - 1. \tag{4.28}$$

Criteria values for each of the schedules constructed thereby represent a single point of the trade-off curve.

In this algorithm operation several different sets are used. All planned jobs in accordance with Table 4.4 constitute set T, sorted in ascending order of processing time p_i. Additional set L at the beginning of each inner cycle (block 3) is equal to set T, and then sequentially it excludes the jobs satisfying condition 3, which are placed in set F (block 5). From the set F the jobs are selected in the order determined by condition 4, which creates a new schedule in set S (block 7). Schedules generated this way are accumulated in set E (block 11).

As you can see from Fig. 4.11, the algorithm uses two conditions of cycle operation and two conditions of selection of set members.

Condition 1 is that the outer cycle operates by the flag value which is set in the internal cycle: if the flag Q = 0 the outer cycle operates; if Q = 1 the operation stops. Setting the flag to value of Q = 1 is defined by the absence of members in set F.

Condition 2 is determined by the value of the same flag Q, and the presence of members in set L as well: internal cycle operates, if Q = 0 and L ≠ 0; in violation of these conditions the cycle stops.

Condition 3 determines the selection from set L to set F of job J_i with number i, for which

$$\hat{d}_i \geq \sum_{J_k \in \mathrm{L}} p_k. \tag{4.29}$$

Condition 4 establishes the selection of such element as J_i out of set F for schedule S, for which

$$p_i = \max_{J_k \in \mathrm{F}} (p_k). \tag{4.30}$$

Let us consider the algorithm operation by the example of the data in Table 4.4.

Step 1. The initial values of external cycle (block 1) are defined according to formulas (4.26) and (4.27):

$$\varepsilon_1 = \sum_{i=1}^{N} p_i = 28; \quad \hat{d}_1 = d_1 + \varepsilon_1 = 52; \ \hat{d}_2 = 50; \ \hat{d}_3 = 46; \ \hat{d}_4 = 51; \ \hat{d}_5 = 49.$$

Besides, flag Q = 0 and output set E = 0 are defined.
Step 2. Start of the external cycle.
2.1. Setting the values of the internal cycle (block 3):
Set L = $\{J_1, J_2, J_3, J_4, J_5\}$; the set of first schedule $\mathrm{S}_1 = 0$.
Step 2.2. Start of the internal cycle.

2.2.1. Calculation of the value

$$u = \sum_{J_k \in \mathrm{L}} p_k = 28.$$

2.2.2. Selection (block 5) of members L to F under condition (4.29). Since all $\hat{d}_i$ are defined on Step 0, F $= \{J_1, J_2, J_3, J_4, J_5\}$ meet this condition.
2.2.3. Finding the job with the highest processing time under condition (4.30): $i = 5$.
2.2.4. Transfer of job J_5 (block 7) to set S: $\mathrm{S}_1 = \{J_5\}$.
2.2.5. Exclusion of job J_5 from set L (block 8): L $= \{J_1, J_2, J_3, J_4\}$.
Step 2.3. Continuation of internal cycle

$$u = \sum_{J_k \in \mathrm{L}} p_k = 18;\ \ \mathrm{F} = \{J_1, J_2, J_3, J_4\};\ \ i = 4;\ \ \mathrm{S}_1 = \{J_4, J_5\};\ \ L = \{J_1, J_2, J_3\}.$$

When filling the set S, which is the possible schedule, the new members are inserted in front of the existing ones.
Step 2.4. Continuation of internal cycle

$$u = 11;\ \ \mathrm{F} = \{J_1, J_2, J_3\};\ \ i = 3;\ \ \mathrm{S}_1\{J_3, J_4, J_5\};\ \ L = \{J_1, J_2\}.$$

Step 2.5.

$$u = 6;\ \ \mathrm{F} = \{J_1, J_2\};\ \ i = 2;\ \ \mathrm{S}_1\{J_2, J_3, J_4, J_5\};\ \ L = \{J_1\}.$$

Step 2.6.

$$u = 2;\ \ \mathrm{F} = \{J_1\};\ \ i = 1;\ \ \mathrm{S}_1\{J_1, J_2, J_3, J_4, J_5\};\ \ L = 0.$$

Step 2.7. End of internal cycle
2.7.1. As L $= 0$, then under condition 2 (block 6) the internal cycle stops. Since the flag of end of external cycle is equal to Q $= 0$, the external cycle continues (block 11) and the first possible schedule is transferred to the set of effective points E $= \mathrm{S}_1 = \{J_1, J_2, J_3, J_4, J_5\}$.
2.7.2. The criteria values at Pareto point, corresponding to schedule S_1, are defined:

$$\overline{F} = \frac{F(J_1) + F(J_2) + F(J_3) + F(J_4) + F(J_5)}{N} = \frac{2 + 6 + 11 + 18 + 28}{5}$$
$$= 13\,\text{days}.$$

and
$T_{\max} = \max\ (\max\ (0;\ C_i - d_i)) = \max\ \big(\max(0; 2 - 24),\quad \max\ (0; 6 - 22),$ $\max\ (0; 11 - 18), \max\ (0; 18 - 23); \max\ (0; 28 - 21) = 7$ days.
2.7.3. The initial values of the external cycle are changed (block 12):

$$\varepsilon_2 = T_{\max} - 1 = 7 - 1 = 6;\ \ \hat{d}_1 = d_1 + \varepsilon_2 = 30;\ \ \hat{d}_2 = 28;\ \ \hat{d}_3 = 24;\ \ \hat{d}_4 = 29;\ \ \hat{d}_5 = 27.$$

Step 3. Continuation of the external cycle
3.1. Setting the values of the internal cycle (block 3):
Set L $= \{J_1, J_2, J_3, J_4, J_5\}$; the set of the second schedule $\mathrm{S}_2 = 0$.

3.2. Start of the internal cycle
$u = 28$; when selecting (block 5) members L to F for the jobs with numbers 3 and 5, condition (4.29) is not fulfilled and so $\mathrm{F} = \{J_1, J_2, J_4\}$; $i = 4$; $\mathrm{S}_2\{J_4\}$; $L = \{J_1, J_2, J_3, J_5\}$.
3.3. Continuation of the internal cycle

$$u = 21;\ \ \mathrm{F} = \{J_1, J_2, J_3, J_5\};\ \ i = 5;\ \ \mathrm{S}_2\{J_5, J_4\};\ \ L = \{J_1, J_2, J_3\}.$$

3.4. $u = 11$; F=$\{J_1, J_2, J_3\}$; $i = 3$; $\mathrm{S}_2 = \{J_3, J_5, J_4\}$; L=$\{J_1, J_2\}$.
3.5. $u = 6$; $\mathrm{F} = \{J_1, J_2\}$; $i = 2$; $\mathrm{S}_2\{J_2, J_3, J_5, J_4\}$; $L = \{J_1\}$.
3.6. $u = 2$; $\mathrm{F} = \{J_1\}$; $i = 1$; $\mathrm{S}_2\{J_1, J_2, J_3, J_5, J_4\}$; $L = 0$.
3.7. End of the internal cycle

$$\mathrm{E} = \mathrm{S}_1 + \mathrm{S}_2 = \{(J_1, J_2, J_3, J_4, J_5);\ \ (J_1, J_2, J_3, J_5, J_4)\};$$

$$\overline{F} = \frac{F(J_1) + F(J_2) + F(J_3) + F(J_5) + F(J_4)}{N} = \frac{2 + 6 + 11 + 21 + 28}{5}$$
$$= 13.6\ \text{days};$$

$T_{\max} = \max\big(\max\ (0;\ 2 - 24),\quad \max\ (0; 6 - 22),\quad \max\ (0; 11 - 18),$ $\max\ (0; 28 - 23); \max\ (0; 20 - 21)\big) = 5$ days;
$\varepsilon_3 = 5 - 1 = 4$; $\hat{d}_1 = d_1 + \varepsilon_3 = 28$; $\hat{d}_2 = 26$; $\hat{d}_3 = 22$; $\hat{d}_4 = 27$; $\hat{d}_5 = 25$.

Step 4. Continuation of the external cycle:
4.1. Set L = $\{J_1, J_2, J_3, J_4, J_5\}$; the set of the third schedule $\mathrm{S}_2 = 0$.
4.2. Start of the internal cycle:
$u = 28$; when selecting (block 5) members L to F for the jobs with numbers 2, 3, 4, and 5, condition (4.29) is not fulfilled and so $\mathrm{F} = \{J_1\}$; $i = 1$; $\mathrm{S}_3\{J_1\}$; $L = \{J_2, J_3, J_4, J_5\}$.
4.3. Continuation of the internal cycle:

$$u = 26;\ \ \mathrm{F} = \{J_2, J_4\};\ \ i = 4;\ \ \mathrm{S}_3\{J_4, J_1\};\ \ L = \{J_2, J_3, J_5\}.$$

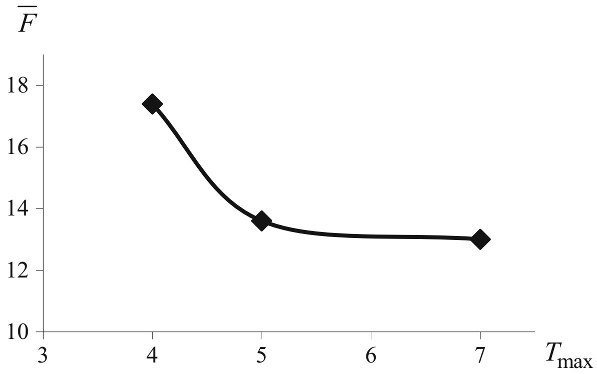
4.4 $u = 19$; $\mathrm{F} = \{J_2, J_3, J_5\}$; $i = 5$; $\mathrm{S}_3\{J_5, J_4, J_1\}$; $L = \{J_2, J_3\}$.
4.5 $u = 19$; $\mathrm{F} = \{J_2, J_3\}$; $i = 3$; $\mathrm{S}_3\{J_3, J_5, J_4, J_1\}$; $L = \{J_2\}$.
4.6 $u = 4$; $\mathrm{F} = \{J_2\}$; $i = 2$; $\mathrm{S}_3\{J_2, J_3, J_5, J_4, J_1\}$; $L = 0$.
4.7. End of the internal cycle:

$$\begin{aligned}\mathrm{E} &= \mathrm{S}_1 + \mathrm{S}_2 + \mathrm{S}_3\\ &= \{(J_1, J_2, J_3, J_4, J_5);\ \ (J_1, J_2, J_3, J_5, J_4);\ \ (J_2, J_3, J_5, J_4, J_1)\};\end{aligned}$$

$$\overline{F} = \frac{F(J_2) + F(J_3) + F(J_5) + F(J_4) + F(J_1)}{N} = \frac{4 + 9 + 19 + 26 + 28}{5}$$
$$= 17.2\ \text{days};$$

$T_{\max} = \max\big(\max\ (0;\ 28 - 24),\quad \max\ (0; 4 - 22),\quad \max\ (0; 9 - 18),$ $\max\ (0; 26 - 23); \max\ (0; 18 - 21)\big) = 4$ days.

Fig. 4.12 Trade-off curve for variants of schedules



$\varepsilon_4 = 4 - 1 = 3$; $\hat{d}_1 = d_1 + \varepsilon_4 = 27$; $\hat{d}_2 = 25$; $\hat{d}_3 = 21$; $\hat{d}_4 = 26$; $\hat{d}_5 = 24$.

Step 5. End of the external cycle:
$u = 28$; when selecting (block 5) members L to F for all jobs, condition (4.29) is not fulfilled, F = 0 and so flag Q = 1 is set.

As a result of the algorithm operation three different schedules with non-dominated and values $\overline{F}$ and $T_{\max}$ are determined. Figure 4.12 shows the trade-off curve based on these values. In T'Kindt and Billaut (2005) it is stated that the computational experiments for this problem with the number of scheduled jobs equal to 50 give up to 29 Pareto points.

4.4 Methods of Decision-Making Theory in Planning Problems

In the problems described in the previous sections of this chapter, it was shown that the result of multi-criteria optimization is building of trade-off curves. However, the known nature of a trade-off curve is insufficient for full solution of these problems, because this curve consists of a number of non-dominated points. Therefore, a trade-off curve is only the basis for decision making, which is done by the planner himself.

4.4.1 Some Information from the Decision Making Theory

The whole set of managerial decisions to be made is divided into two large groups: strictly conditioned and proactive. The former are usually related to the so-called standard decision defined by the regulations and orders. Proactive decisions are the choice of alternative behaviours of several possible, each of which entails a number of positive and negative consequences. In this book, we consider only these decisions.

The decision format will be mostly influenced by the kind of the problem situation. According to the paper of Simon (1959) all the problems can be divided into three classes.

- Well-structured problems are problems, in which the significant relations between parameters may be expressed in a formal way.
- Ill-structured or mixed problems have both qualitative and quantitative elements, and the qualitative, obscure, and uncertain aspects of the problems tend to dominate.
- Unstructured or qualitatively expressed problems contain only the description of the most important resources, attributes and characteristics, the quantitative relationships of which are completely unknown.

The production planning problems are mainly well structured and allow a numerical solution. However, when developing plans for quite long-term prospects the planner faces uncertainty, especially in the assessment of marketing opportunities. In these cases, the problems of planning may be ill-structured.

In addition to classification according to the degree of structuring, the problems, on which it is necessary to make decisions, are characterized by three main parameters. If decisions are made by one person, then the problem is of type J. If the decision involves several persons, the problem relates to type G. If there are several criteria the problem is considered to be of type A. Otherwise, the special designation is not used, and it is assumed by default that there is only one criterion. Similarly, if the problem is quite determined, the special type designation is not provided. If the task is probabilistic, the designation of its type includes parameter S. In this chapter, we consider well-structured problems such as JA, i.e. problems with several criteria, on which the decisions are made by one person.

In the decision-making process several stages can be identified.

- Identification of possible decision area is needed to refer planning decision either to organizational or to the technical or economic areas of activity.
- Determination of the decision type: the standard decision, the usual decision with some modifications, the original decision.
- Description of the possible set of decisions and the limits of their values. The range of possible decisions in the decision theory is called alternatives. Alternatives can be independent and dependent. Independent ones are those alternatives, any actions with which do not affect the quality of the other alternatives. In this book, we consider only such alternatives.
- Analysis of the advantages and disadvantages of possible alternative.
- Assessment of the probability of each alternative.
- Comparison of decisions according to various criteria.

Table 4.5 Matrix of the criteria utility description for problems of JA type

Criteria (goals)	f_1	f_2	...	f_k
Decision alternatives				
Y_1	u_{11}	u_{12}	...	u_{1k}
Y_2	u_{21}	u_{22}	...	u_{2k}
...	...	...	...	...
Y_n	u_{n1}	u_{n2}	...	u_{nk}
Goal priorities	w_1	w_2	...	w_k

The planning decision is usually the organizational decision, and its type, as a rule, has a fairly standard nature with modifications, which depend on the current production situation. As it was mentioned above in Sect. 4.1.3, in this book we use methods for solving multi-criteria problems based on utility theory and the study of criteria space metric.

For making planning decisions using the theory of utility, first of all, the background information should be described, which is a set of possible alternatives and the utility level of each of them. Such information can be convenient to be represented in the form of the matrix, which for problems of JA type is as shown in Table 4.5.

The data in Table 4.5 contain the set of possible variants Y_i. For all the variants, utility values u_{ij} must be defined on each of the criteria f_j. Simultaneously the priorities (weights) values of each of criteria w_j can be set.

One of the most common methods using the criteria space metric is the so-called method of the shifted ideal in feature space. In this method, first the so-called ideal object is formed. As a rule, this object does not really exist but embodies all the best possible properties of different actually existing alternatives. The presence of such an object sets the reference point when comparing real alternatives to the ideal hypothetical option. In this case, the objects of the set of acceptable alternatives Y_i are compared with the ideal object by the criterion of distance from the real object to the ideal one. To calculate the distances the initial information should be presented as shown in Table 4.6.

To compare the diverse criteria it is necessary to transfer to their normalized non-dimensional values. Normalized values in the i-th alternative for each j-th criterion for the criteria, during increasing of which the quality of the alternative improves, are determined using the conversion

$$a_{ij} = \frac{\left(f_j^+ - f_{ij}\right)}{\left(f_j^+ - f_j^-\right)}, \tag{4.31}$$

where f_j^+ is the value of j-th criterion of the ideal object and f_j^- is the value for j-th criterion for the object being the worst by this criterion.

The distance between the objects can be set in different ways. In most cases we use the generalized metric of the form

Table 4.6 Matrix of the criteria description for distance calculation

Decision alternatives	Criteria			
	f_1	f_2	...	f_k
Y_1	f_{11}	f_{12}	...	f_{1k}
Y_2	f_{21}	f_{22}	...	f_{2k}
...	...	...	...	...
Y_n	f_{n1}	f_{n2}	...	f_{nk}

$$L^p = \Big[\sum_{j=1}^{k} \big(w_j a_j\big)^p\Big]^{1/p}, \qquad (4.32)$$

where p is the so-called concentration ratio, which defines the type of metric. The most common metric is Euclidean metric with $p = 2$.

Based on the distance calculation the alternatives are graded according to their proximity to the ideal option and the options quite far from ideal are filtered out.

4.4.2 Example of the Planning Problem Requiring Decision Making

Let us consider the problem of the sizing optimal lot based on several criteria. For such criteria we take the setup time s, the mean cost of work-in-progress $\overline{c}$, and the mean waiting time of processing $\overline{w}$. We continue considering the example (Table 4.3), where three products are produced on the same machine, and take into account that the machine has a limited capacity for each type of product (Table 4.7).

Since in this case the optimal lot values shall be determined on the basis of the minimum of three criteria at once, in this case it is impossible to construct a trade-off curve. Therefore, here we should seek the problem solution by directly comparing different options—alternatives.

First of all, you must determine the number of options under consideration. Psychological research shows that most people can simultaneously learn about seven alternatives (more/less two) Hugo (2006). As these alternatives it is obvious to choose the solutions obtained for similar problems that were solved before, but with a smaller set of criteria.

This problem was solved earlier in Sect. 4.2.2 for criteria s and $\overline{c}$, as well as in Sect. 3.4.2 for criterion $\overline{w}$. In the first case, the solution led to the possibility of constructing effective points in the space of variables of lot sizes Q_i and, in the second case, lot sizes Q_i were calculated according to formula (3.45). Therefore, it makes sense to choose several, for example, three effective points of the first solution and the point in space of variables of lot sizes, reflecting the second solution, as possible alternatives. In addition, apparently, several points of that space should be considered, which are between the points defined by the first and second solutions (Fig. 4.13).

Table 4.7 Products manufactured on the machine

Product no.	Setup time s_i, h	Demand D_i, pcs/day	Product cost c_i, \$/pcs	Machine capacity P_i, pcs/day
1	2	40	80	150
2	1	10	20	60
3	1	20	50	100

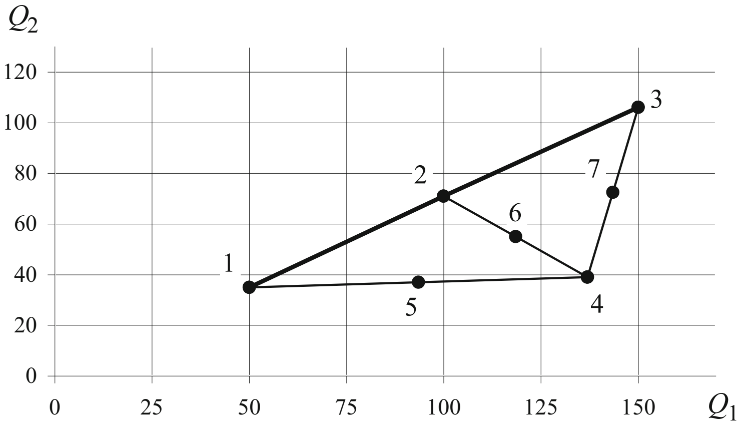


Fig. 4.13 Projection of the possible alternatives of solution to plane Q_1Q_2

Points 1, 2, and 3 in Fig. 4.13 are projections to plane Q_1Q_2 of effective points, defined for three different values of correction factor $\hat{\chi}$ in formula (4.21): 50, 100, and 150. For example, at point 1 $\hat{\chi} = \hat{\chi}_1 = 50$ and the lot of product 2 found by formula (4.21).

$$Q_{21} = \hat{\chi}_1 \sqrt{\frac{s_2 D_2}{C_2}} = 50 \times \sqrt{\frac{1 \times 10}{20}} = 35 \text{ pcs.}$$

Point 4 displays the projection of the point in the space of variables, calculated by formula (3.45). For example, with working day duration $G = 8$ h.

$$Q_{24} = \frac{2P_2}{G\left(1 - \sum_{j=1}^{3} \frac{D_j}{P_j}\right)} \sqrt{s_2 \sum_{j=1}^{3} s_j \frac{D_j}{P_j}} = 39 \text{pcs.}$$

Now we increase the number of possible alternatives to a reasonable value in the range of 5–9. Points 1, 2, and 3, located on the bold straight line, are solutions of the problem of the optimal lot size, which was set out in Sect. 4.2.2, while point 4 is also a solution of the problem of the optimal lot size given in Sect. 3.4.1. Apparently, it is advisable to introduce additional points in the space of variables, located between the points of the first and second solutions, such as midway there between. These points are points 5, 6, and 7, projections of which on plane Q_1Q_2 are shown in Fig. 4.13. For example, for point 6 lot of product 2

$$Q_{26} = Q_{24} + 0.5(Q_{22} - Q_{24}) = 55 \text{ pcs.}$$

To obtain seven alternatives, we shall define the criteria values using relations (4.13), (4.22), and (3.44). For example at point 6

$$s_6 = \sum_{i=1}^{3} s_i \frac{D_i}{Q_{i6}} = \frac{2 \times 40}{118} + \frac{1 \times 10}{55} + \frac{1 \times 20}{64} = 1.17 \ \text{h/day};$$

$$\overline{c}_6 = \frac{1}{2} \sum_{i=1}^{3} c_i Q_{i6} = \$6890.$$

At point 4, value $\overline{w}_4$ is the same regardless, by which of i-th products it is defined. In other products this quality is not observed; certainly, that is why the mean waiting time shall be defined by formula (3.38). For example at point 6 considering the dimension, we obtain

$$\overline{w}_6 = \frac{G \sum_{i=1}^{3} \frac{D_i}{Q_{i6}} \left(\frac{s_i}{G} + \frac{Q_{i6}}{P_i} \right)^2}{2 \left[1 - \sum_{i=1}^{3} \frac{D_i}{Q_{i6}} \left(\frac{s_i}{G} + \frac{Q_{i6}}{P_i} \right) \right]} = 13.5 \ \text{h.}$$

Similar to the projection to planes Q_1Q_2 the projections of the alternatives to plane Q_1Q_3 can be built. The calculation results for point 1–7 are summarized in Table 4.8.

To apply the methods of the decision theory described above in Sect. 4.1.1, we introduce the corresponding designations: $f_1 = s; f_2 = \overline{c}; f_3 = \overline{w}$. Each i-th point in Fig. 4.13 corresponds to set of variables Q_1, Q_2, Q_3. We assume that this set represents alternative Y_i.

Since the criteria have different dimensions, for the purpose of decision theory application, they should be put in a dimensionless form. Furthermore, it is appropriate to normalize the criteria so that the maximum value of utility in the above set of alternatives is equal to 1. For the criteria, the utility of which grows with decreasing, as it is in our case, this can be done with a special conversion. For example, for utility U_{1i} of criterion $f_1 = s$ in alternative Y_i

$$U_{1i} = 1 - \frac{s_i - s_{\min}}{s_{\max}}, \tag{4.33}$$

where $s_{\min}$ is minimal value of s for all alternatives; $s_{\max}$ is maximal value of s for all alternatives.

Table 4.8 Alternatives and criteria values

Option no.	Q_1, pcs	Q_2, pcs	Q_3, pcs	s, h/day	$\overline{c}$, \$	$\overline{w}$, h
1	50	35	32	2.54	3150	40.8
2	100	71	63	1.26	6285	14.5
3	150	106	95	0.84	9435	15.2
4	137	39	65	1.15	7495	13.2
5	93	37	48	1.5	5322	14.4
6	118	55	64	1.17	6890	13.5
7	143	72	80	0.95	8465	13.9

Table 4.9 Utilities of the criteria for various alternatives

Alternative	f_1	f_2	f_3
Y_1	1	0.334	0.322
Y_2	0.667	0.833	0.967
Y_3	0.363	1	0.949
Y_4	0.539	0.877	1
Y_5	0.769	0.721	0.969
Y_6	0.603	0.868	0.991
Y_7	0.437	0.957	0.981

After this conversion, the most utility equalling 1 belongs to the option with the least values of criteria, and the least utility equalling $\frac{s_{\min}}{s_{\max}}$ belongs to the option with the most values of criteria. The results of this conversion are presented in Table 4.9. Table 4.9 is similar to Table 4.5, if the priorities of all the criteria are the same.

As might be expected, the largest utility of criterion f_1 is for alternative Y_1, of criterion f_2 for Y_3, and criterion f_3 for Y_4.

4.4.3 Decision-Making Based on the Guaranteed Result Principle

According to the Guaranteed Result Principle or Max-Min Principle as the most efficient alternative we choose that one which has the most value of the utility function being the most among the least for each option by various criteria. For this purpose, for each option the least values of utility by all criteria are selected and then among the selected values of utility the largest one is defined. This selection strategy is defined by the rule

$$U^* = \max_i \left[\min_j \left(U_{ij}\right)\right]. \tag{4.34}$$

Let us calculate using the rule (4.34) in terms of the data in Table 4.9.

It follows from Table 4.10 that according to the Guaranteed Result Principle the best solution of the problem is alternative Y_5, for which the least possible deterioration in utility of any criteria is provided.

Table 4.10 Determination of the best option according to the Guaranteed Result Principle

Alternative	f_1	f_2	f_3	$\min(U_j)$
Y_1	1	0.334	0.322	0.322
Y_2	0.667	0.833	0.967	0.667
Y_3	0.363	1	0.949	0.363
Y_4	0.539	0.877	1	0.539
Y_5	0.769	0.721	0.969	0.721
Y_6	0.603	0.868	0.991	0.603
Y_7	0.437	0.957	0.981	0.437
$U^* = \max\limits_i \left[\min\limits_j \left(U_{ij}\right)\right]$				0.721

Table 4.11 Determination of the best option according to the Optimism Principle

Alternative	f_1	f_2	f_3	$\max(U_j)$
Y_1	1	0.334	0.322	1
Y_2	0.667	0.833	0.967	0.967
Y_3	0.363	1	0.949	1
Y_4	0.539	0.877	1	1
Y_5	0.769	0.721	0.969	0.969
Y_6	0.603	0.868	0.991	0.991
Y_7	0.437	0.957	0.981	0.981
$U^* = \max\limits_i \left[\max\limits_j \left(U_{ij}\right)\right]$				1

4.4.4 Optimistic Decision-Making

When using the Optimism Principle as the best alternative, then we adopt the alternative having the highest attainable value of the criterion. In contrast to the Guaranteed Result Principle, aimed at selecting the best of the worst options, the Optimism Principle allows obtaining the maximum level of the desired result.

According to the Optimism Principle, the most effective alternative is the one that has the largest value of the utility function out of the largest of all alternatives. This selection strategy is defined by the rule

$$U^* = \max_i \left[\max_j \left(U_{ij}\right)\right]. \tag{4.35}$$

The calculation by the rule (4.35) is presented in Table 4.11.

As can be seen from Table 4.11, application of this principle leads to a controversial decision, because the best alternatives become automatically Y_1, Y_3, and Y_4 with the best values of each of the criteria. Evidently, the optimism principle in pure form can be used only if the criteria are unequal by weight, i.e. some criteria are known to be more important than others.

4.5 Applications of Complex Decision-Making Methods

Principles of selection of alternatives, described in Sects. 4.4.3 and 4.4.4, represent two extreme selection strategies that focus either on a guaranteed, albeit minimal, result or the best possible result, obtaining of which is absolutely not guaranteed and associated with risk. Since the use of these strategies seems to be not rational enough, we will consider a few other more complex strategies.

4.5.1 Hurwitz Principle

The Hurwitz Principle is a combination of the above principles of guaranteed result and optimism. By using the Hurwitz Principle, each of these strategies is attributed with a certain weight according to its importance. The selection rule, which represents the Hurwitz Principle, has the form

$$U^* = \gamma U_1^* + (1-\gamma)U_2^*, \tag{4.36}$$

where γ is the importance coefficient of guaranteed result strategy and

1–γ is the importance coefficient of optimism strategy.

By inserting expressions (4.34) and (4.35) into (4.36) we obtain

$$U^* = \max_i \left[\gamma \min_j \left(U_{ij}\right) + (1-\gamma) \max_j \left(U_{ij}\right)\right]. \tag{4.37}$$

When changing α from 0 to 1 the values of the solution on the Hurwitz Principle vary from the results obtained according to the guaranteed result principle to the values determined on the basis of optimism principle. Thus, to get the solution, firstly, it is necessary to have γ given, then calculate the optimal alternatives according to the above two principles, and use formula (4.37). Let us form the solution of the example proposed in Sect. 4.4.2 according to the Hurwitz Principle with γ = 0.5 (Table 4.12).

It follows from Table 4.12 that with γ = 0.5 we adopt alternative $Y^* = Y_5$ as the best one. By making the similar calculations with other values γ within 0–1, we find the best alternatives with each such value (Table 4.13).

Table 4.13 shows that in this case for the vast majority of values α the best alternative according to the Hurwitz Principle is Y_5.

Table 4.12 Determination of the best option according to the Hurwitz Principle

Alternative	f_1	f_2	f_3	$\min(U_j)$	$\max(U_j)$	$\gamma \min(U_j) + (1-\gamma)\max(U_j)$
Y_1	1	0.334	0.322	0.322	1	0.661
Y_2	0.667	0.833	0.967	0.667	0.967	0.817
Y_3	0.363	1	0.949	0.363	1	0.681
Y_4	0.539	0.877	1	0.539	1	0.769
Y_5	0.769	0.721	0.969	0.721	0.969	0.845
Y_6	0.603	0.868	0.991	0.603	0.991	0.797
Y_7	0.437	0.957	0.981	0.437	0.981	0.709
$U^* = \max\limits_i [\gamma \min\limits_j (U_{ij}) + (1-\gamma)\max\limits_j (U_{ij})]$						0.845

Table 4.13 Best alternatives with various values of coefficient γ

γ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Y*	Y_4	Y_4	Y_5	Y_5	Y_5	Y_5	Y_5	Y_5	Y_5	Y_5	Y_5

4.5.2 Savage Principle

The selection strategy based on the use of the Savage Principle is determined by the tendency to reduce loss (as possible) that would be incurred as a result of non-optimal decision. Therefore, to apply the Savage's method, first of all, the loss functions are defined on the basis of existing utility values. To this end, for each j-th criterion and the set of i-th alternatives the maximum possible values of the utility function are determined

$$U_{\max,j} = \max_i U_{ij}, \tag{4.38}$$

and then the possible values of loss are calculated

$$V_{ij} = U_{\max,j} - U_{ij}. \tag{4.39}$$

The loss assessment shows how close each alternative is to the best possible one of each criterion. According to the matrix of losses the search strategy of the smallest of the maximum possible losses is performed

$$V^* = \min_i [\max_j (V_{ij})]. \tag{4.40}$$

Let us determine the best alternative according to the Savage Principle in terms of the data in Table 4.9. Using Table 4.14 we define the largest values of utility for each criterion.

Using expression (4.39) we obtain the matrix of regrets by which we will search the optimal decision according to formula (4.40). Table 4.15 shows the results of the calculation.

Table 4.14 Criteria utilities for various alternatives

Alternative	f_1	f_2	f_3
Y_1	1	0.334	0.322
Y_2	0.667	0.833	0.967
Y_3	0.363	1	0.949
Y_4	0.539	0.877	1
Y_5	0.769	0.721	0.969
Y_6	0.603	0.868	0.991
Y_7	0.437	0.957	0.981
$U_{\max,j} = \max\limits_i U_{ij}$	1	1	1

Table 4.15 Determination of the best option according to the Savage Principle

Alternative	f_1	f_2	f_3	$\max(V_j)$
Y_1	0	0.666	0.678	0.678
Y_2	0.332	0.167	0.033	0.332
Y_3	0.666	0	0.051	0.666
Y_4	0.461	0.123	0	0.461
Y_5	0.230	0.279	0.031	0.279
Y_6	0.396	0.132	0.009	0.396
Y_7	0.563	0.043	0.019	0.563
$V^* = \min\limits_i [\max\limits_j (V_{ij})]$				0.279

It follows from Table 4.15 that, the same as in use of the Hurwitz Principle, the best solution as a result of calculations according to the Savage Principle is alternative Y_5.

4.5.3 Shifted Ideal Method

In accordance with the sequence of actions in shifted ideal method described in Sect. 4.4.1, first of all, it is necessary to create an "ideal object" that sets a reference point when comparing real alternatives. The elements of the set of alternatives Y_i are compared with the ideal object by the criterion of distance from the current version to the ideal. Based on this comparison the alternatives are graded according to their proximity to the ideal option and those options that are far from ideal are filtered out.

Let us illustrate the use of the shifted ideal method in terms of the above example in Sect. 4.4.2. It is obvious that in this case the ideal object must have the minimum possible value for the options of the criteria values under consideration: $s = 0.84$ h/day, $\overline{c} = \$3150$, and $\overline{w} = 13.2$ h. Since the quality of the alternative improves with the criteria decreasing, then conversion (4.31) has the form

Table 4.16 Rated values of criteria for distance calculation

Alternative	f_1	f_2	f_3
Y_1	1	0	1
Y_2	0.247	0.498	0.047
Y_3	0	1	0.072
Y_4	0.182	0.691	0
Y_5	0.388	0.346	0.043
Y_6	0.194	0.595	0.011
Y_7	0.064	0.846	0.025

$$a_{ij} = \frac{\left(f_{ij} - f_j^-\right)}{\left(f_j^+ - f_j^-\right)}. \tag{4.41}$$

Using conversion (4.41) we rate the initial data in Table 4.8 to define the distances from the ideal object (Table 4.16). For example, for the value of criterion $f_2 = \overline{c}$ in option 5 we have

$$a_{52} = \frac{\left(f_{52} - f_2^-\right)}{\left(f_2^+ - f_2^-\right)} = \frac{(5322 - 3150)}{(9435 - 3150)} = 0.346.$$

According to Table 4.16, we can calculate the distances of different alternatives to the ideal object with different metric of criteria space, which is given by concentration factor p in formula (4.32). If the weights of all the criteria are the same, then for the i-th alternative formula (4.32) has the form

$$L_i^p = \Big(\sum_{j=1}^{k} a_{ij}{}^p\Big)^{1/p}. \tag{4.42}$$

For example, for alternative 5 with concentration factor $p = 2$ we have

$$L_5^2 = \Big(\sum_{j=1}^{3} a_{5j}^2\Big)^{1/2} = 0.521.$$

Table 4.17 shows the distances calculated by formula (4.42) for various p values.

Obviously, the less the alternative's distance from the ideal object, the most preferable it is. Let us grade the preferability of alternatives according to their proximity to the ideal object for different factors p.

For example, for $p = 1$ we have

$$L_5^1 < L_2^1 < L_6^1 < L_4^1 < L_7^1 < L_3^1 < L_1^1$$

and accordingly,

Table 4.17 Distances of alternatives to the ideal object with various p values

Alternative	Concentration p						
	1	2	3	4	5	6	8
Y_1	2	1.414	1.259	1.189	1.148	1.122	1.09
Y_2	0.792	0.558	0.518	0.506	0.501	0.5	0.499
Y_3	1.072	1.002	1	1	1	1	1
Y_4	0.873	0.714	0.695	0.692	0.691	0.691	0.691
Y_5	0.777	0.521	0.463	0.438	0.424	0.415	0.404
Y_6	0.8	0.626	0.601	0.596	0.595	0.595	0.595
Y_7	0.935	0.848	0.845	0.845	0.845	0.845	0.845

$$Y_5 \succ Y_2 \succ Y_6 \succ Y_4 \succ Y_7 \succ Y_3 \succ Y_1,$$

where symbol $\succ$ means that the value on the left from this symbol is more preferable than the value on the right.

Analysis of Table 4.17 shows that in this case the sequence is remained even with all other p. It is, therefore, evident that the smallest deviation from the ideal object is given by alternative Y_5 and it can be accepted as the best. Generally speaking, such situation occurs not always, i.e. there may be cases when with various p the grading of alternatives by proximity to the ideal object is different. In these cases, the graded sequences of alternatives are analysed and the dominated alternatives are identified, i.e. those that are known not to be the best with all p values. After excluding such alternatives from consideration a new ideal object can be formed, and the procedure of grading can be repeated, etc.

In conclusion, we note that for the problem set in Sect. 4.4.2 in most of the methods of decision-making, alternative Y_5 was recognized as the best one, which is likely to be adopted as a solution to this problem.

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Data for Planning 5

5.1 Composition of the Data Used for Planning

Planning is the main and most complex business process determining the production activities of the enterprise, and therefore its implementation requires the greatest amount of diverse information. The entire array of necessary information can be divided into five large groups:

- Archives of design and production documentation
- Archives of orders and contracts
- Reference data
- Databases of transactional information systems
- Databases for decision-making
- Knowledge bases

5.1.1 Archives of Design-Engineering Documentation and Orders

A distinctive feature of the archive information is maintenance of revision history of the documents in the archive. In other words, every document belonging to the archive must be presented in all versions of its consistent development. Availability of archival document in its original form and all its subsequent changes allows analysing the performance of the enterprise later to improve production and interaction with consumers.

The archive of design documentation consists of graphic and text documents, among which the engineering bills have the primary value for planning. The structure of engineering bills (Bill of Materials, BOM), as is known, is hierarchical, where each BOM, on one hand, is included into the supreme bill, and, on the other hand, includes other bills and individual objects. For the purpose of planning the BOMs are subject to explosion process, the result of which is to determine the amount of each design object in the unit of finished production.

Y. Mauergauz, *Advanced Planning and Scheduling in Manufacturing and Supply Chains*, DOI 10.1007/978-3-319-27523-9_5

The archive of production documentation, in contrast to the design archive, is used almost to the full extent during planning. In planning, the most essential is the following: content of operations and their sequence, job allocation between the workshops, norms of material expenditure, equipment and standard time of operations, personnel requirements and their qualifications, the need for special-purpose tools, etc.

In the references on planning, the design documentation is usually regarded as a given set of data, which may even be related to the reference data. In fact, nowadays, both design and production documentation in a number of industries are subject to very rapid changes. This is due primarily to market fluctuations and keen competition, which generates a need for frequent changes in the parameters of the manufactured products. Furthermore, to meet the market requirements it is necessary to increase the speed of innovation development, which, of course, leads to the need for subsequent refinement of products during the manufacturing process. As a result, the frequency of changes in production in some cases (especially in machinery engineering) is increased to an extent that we can talk of a continuous stream of changes at a certain speed.

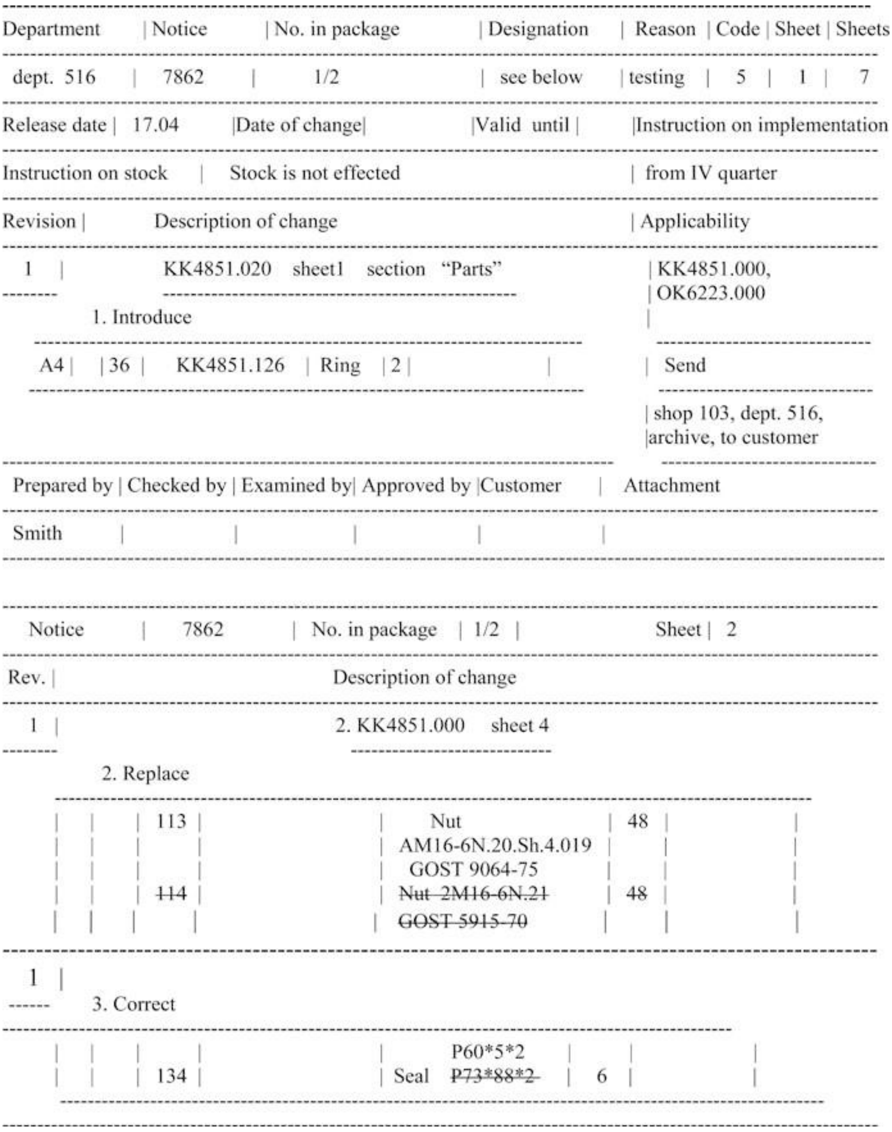
Each change in design and production documentation comes into effect in accordance with the document, which is called a change notice. In different systems of standardization, this document may have a different form. In view of the special role of this document in the archive management, we shall consider the work using the example developed in the Russian system of design documents (Table 5.1). The change notice can consist of several sheets. This example shows two first sheets of the notice consisting of seven sheets.

As can be seen from Table 5.1, on the first sheet of the change notice there is the header of the notice and instructions regarding which products these changes shall be made and to whom they shall be distributed. If the corrected documents are used in different items and are used by different subscribers, the instruction on the applicability and distribution must be recorded for each of the modified documents separately in the field “Description of change”.

The header indicates the author of the notice, its serial number, reason for change and its code, release date, and the total number of sheets. If the design notice entails the need to issue the production notice, both of these notices constitute a single package with a common number. In this case, the design notice has number 1, and production one—number 2. Attachments to the notice may be new BOM sheets, drawings, etc.

Table 5.1 presents various cases of changes in engineering bills and drawings. For example, in bill KK4851.000, a new part “Products manufactured on the machine KK4851.126 Ring” is entered, “Nut 2M16-6N.21” is replaced by “Nut AM16-6N.20”, and designation “P73*88*2” is corrected to “P60*5*2”. Upon replacement or correction, the old records and drawings are recorded in the notice with strikethrough, as shown in Table 5.1. In the bills the sheets can be cancelled and new sheets can be introduced; in the parts drawing, dimensions or graphics can be changed, etc. Serial number of changes in field “Revision” indicates the number of changes made over the specified line of the bill or part.

Table 5.1 Change notice



```
------------------------------------------------------------------------------------------
Department     | Notice     | No. in package     | Designation    | Reason  | Code | Sheet | Sheets
------------------------------------------------------------------------------------------
 dept. 516     |   7862     |      1/2           |  see below     | testing |  5   |  1   |  7
------------------------------------------------------------------------------------------
Release date |  17.04     |Date of change|            |Valid until |    |Instruction on implementation
------------------------------------------------------------------------------------------
Instruction on stock   |  Stock is not effected                   | from IV quarter
------------------------------------------------------------------------------------------
Revision |        Description of change                          | Applicability
------------------------------------------------------------------------------------------
  1   |          KK4851.020   sheet1   section  "Parts"           | KK4851.000,
--------         -----------------------------------              | OK6223.000
         1. Introduce                                             |
   ---------------------------------------------------------       -----------------------
    A4 |   | 36 |    KK4851.126   | Ring   | 2 |            |      |  Send
   ---------------------------------------------------------       -----------------------
                                                                  | shop 103, dept. 516,
                                                                  |archive, to customer
------------------------------------------------------------       -----------------------
 Prepared by | Checked by | Examined by| Approved by |Customer   |  Attachment
------------------------------------------------------------------------------------------
 Smith       |            |            |             |           |
------------------------------------------------------------------------------------------

------------------------------------------------------------------------------------------
  Notice       |    7862       |  No. in package  |  1/2  |              Sheet |  2
------------------------------------------------------------------------------------------
Rev. |                         Description of change
------------------------------------------------------------------------------------------
  1  |                         2. KK4851.000    sheet 4
--------                          ----------------------
         2. Replace
     ----------------------------------------------------------------------------------
     |   |    | 113 |                  |    Nut                  | 48 |             |
     |   |    |     |                  | AM16-6N.20.Sh.4.019     |    |             |
     |   |    |     |                  |   GOST 9064-75          |    |             |
     |   |    | ~~114~~ |              | ~~Nut 2M16-6N.21~~      | 48 |             |
     |   |    |     |                  | ~~GOST 5915-70~~        |    |             |
------------------------------------------------------------------------------------------
  1  |
------   3. Correct
--------------------------------------------------------------------------
     |   |    |     |                  |      P60*5*2      |     |             |
     |   |    | 134 |                  | Seal ~~P73*88*2~~ |  6  |             |
     ----------------------------------------------------------------------------------
------------------------------------------------------------------------------------------
```

All of these changes should be displayed in the corresponding archive. However, the immediate change of the archive lines would lead to the situation when the information about its previous state was lost immediately. Therefore, the old line should remain along with the new ones. The parallel existence of the data describing the same object is possible if only each of these options is identified somehow.

Usually, every change notice is associated with its own version of the product or process. This version, as a rule, should include all previous changes that occurred with this object, i.e. cover the previous change notices. Since the object to be changed may enter into various kinds of finished product, as shown in Table 5.1, then even one change can lead to the emergence of new versions of several types of finished products.

Of course, it does not make sense to keep such a large number of different versions of the finished product. Therefore, the necessary change notices should be kept and accounted in explosion, but in this case, it brings the logic question as to which version should be employed. It is generally believed that only one version is valid at the current moment and it is called active.

The active version of the finished product material specification and the corresponding active version of the technological process should be created automatically by the information systems based on primary documentation and the current notices of change. The latter, however, is not always obvious.

If we look at Table 5.1, we can see that it has two positions determining the initiation of the notice. The main position is “Instruction on implementation” that determines the moment of implementation (activation) of the notice. This moment can be specified by the date, by the number of finished product set, or by such even not a very specific phrase as “upon wear-out of the mould”. In addition, in Table 5.1 there is position “Instruction on stock”, which may contain information about the future of previously manufactured objects subject to these changes.

The difficulty of the instruction performance regarding the implementation and use of the stock greatly complicates the possibility of automatic detection of activation of a change notice. Therefore, in many cases, if the automatic detection of the activity of the notice fails, it is necessary to identify information on its activity manually.

Arrangement of active versions of specifications for products and process is carried out during preparation for planning of customer order fulfilment according to the bills of these orders. The order bills are stored in the archives of contracts and orders and provide data on the amount and time of the shipments of finished products and other objects to the customers. Each order bill can contain multiple items (lines) differing in the type of product, date of shipment, special requirements for products, etc. Preparation of planning should be done if a decision is made on the performance of a specific position of a specific order. In this case, the specific active versions of the product and process can be determined based on the customer’s requirements.

As a result, the so-called Production Process Model (PPM) is created for each line of the customer order bill. The PPM contains full data on all manufactured objects, their required quantity, and applied processes. Using the PPM allows associating each instance of the product with the appropriate version of the design documentation and thus tracing the influence of this document version on the quality of the product performance within its lifecycle. The use of production process model is described in detail below in Chap. 12.

5.1.2 Reference Data and Standards

The reference data used in planning can be divided into three components:

- Information about the departments of the enterprise
- Item master file
- Scheduling and planning standards

Each industrial enterprise has a historically established set of production departments. In most cases, these are shops and sites arranged to perform either one or a set of similar processes, or for producing a particular product or a group of similar products. Modern literature on planning points out so-called work centers within these production departments.

"*Work center*" (APICS Dictionary 2008) refers to a specific production area, which consists of one or more people and/or units of equipment with identical features, which can be considered as one unit for planning capacity requirement and detailed scheduling. In fact, this definition of a work center coincides with the widespread notion of the equipment group, which consists of equipment units with the same processing capabilities. However, if we consider the work center as a unit for planning, this concept can be interpreted more broadly including any number of equipment and personnel within the unit into a work center. In fact, the size of the work center can vary from one equipment unit to a shop, or even the whole production plant. However, in most cases, the working center does refer only to a small set of equipment with one or more operators.

For planning purposes, the data on the departments should contain information on all work centers, processing capabilities of equipment units included in these centers, and the number and qualifications of staff.

The performance of the enterprise is directly related to updating of item master file, i.e. to systematic adjustment of the file according to the information coming from the manufacturers, standard organizations, etc.

Scheduling and planning standards are the basis of calculations carried out in planning. The role of the scheduling and planning standards is generally considered fundamental in the production planning system according to some sources of literature. For example, in Grachiova et al. (2005), it is stated that the system of operational planning is characterized by the kind of a planning item, the composition of scheduling and planning standards, and the handling procedure of planning and accounting records. Therefore, we will consider the purpose and composition of the scheduling and planning standards in more detail.

With automated planning the composition of the scheduling and planning standards is not strictly fixed. First of all, the use of a particular planning value as standard or calculated depends on the stage of planning. For example, the production cycle duration for a specific type of product can be considered to be a planned predetermined value in planning sales and operations but in master planning this duration is determined during the planning process.

Besides, the system can automatically change the use of calculated standards by the rules of their priorities established in the system. For example, if the database has a standard for the shifts in a specific shop, this value is used for calculation, otherwise for the same shop the shifts in the enterprise in general are as the shift standard.

In planning different scheduling and planning standards of six types can be used:

- Performance standards for operation of the enterprise as a whole
- Standards relating to the organizational group (shop)
- Standards of a work center (equipment group)
- Standards for each engineering object
- Standards of process version for the object in general
- Standards for the process operations

The entire set of the standards of the first type (whole enterprise) can be divided into two groups. The first group includes the standards describing the enterprise activity as one whole production entity. The use of these standards does not depend on the enterprise structure parameters. These standards, for example, are as follows:

- Planning-accounting period (year, half year, quarter, month, 10 days, week, day)
- Method of determining the reference point for accounting of the lot numbers of product (from the beginning of production, the current year, quarter, month, 10 days, week, day)
- Standard of time for the purchase of materials and components to provide the production program in days
- Time for development of resource limit cards in days
- Standard of time for obtaining the materials from warehouses in hours
- Standard of time for transferring the schedule data to the workshops in hours

The rest of the standards of the first group in the whole enterprise mainly include indices governing the time spent on the transfer of information across the enterprise’s departments, as well as the averaged data on cooperation with suppliers and customers.

The second group of standards of the whole enterprise includes the following standards affecting the activity of the departments:

- Number of work shifts
- Time of the workshop planning-accounting period (month, 10 days, week, day)
- Duration of the transfer of the lot from the operation to the operation inside the workshop in hours taking into account the mean waiting time for the equipment to be free in hours
- Duration of the transfer back and forth of the lots to outsourcing enterprises for processing in days

For each department these standards may be duplicated, and in that case they will have priority over the standards that are in the general list of the enterprise. At the same time, for many departments it is often sufficient to use the standards of the second group of the enterprise as workshop standards.

The main standard, set for each unit of equipment, is the fund of working time. In the absence of data on each unit of equipment the standard fund of time can be set for a work center, located in one shop. If the data on the fund time for all units of equipment in the work center are available, then the fund time turns from the work center standard into a calculated value.

For each type of finished goods, as well as for each type of assembly unit or part, several planning standards can be used in the process of planning. For example, for large parts and assembly units the highest number of objects in the lot can be set as a standard. For workpieces, the standard amounts of stock are often set, and sometimes the sizes of the lots—for example, when there is a certain number of seats for moulding. However, as a rule, the modern planning involves not the standard but the calculated sizing of the lots.

5.1.3 Databases of Transactional IT Systems

In the information system, each event of a business process corresponds to a transaction. A transaction is a sequence of operations, which is a logical unit of work with data. The transaction may be either performed completely and successfully maintaining data integrity and independently from other parallel transactions or failed completely and then should not produce any effect. Transactions are processed by the transactional information systems and in this process, a transaction history is created.

A classic example of a transaction is a transfer of a certain amount from one account to another. In this case, it is required to read the account balance of the sender, reduce the balance by the transfer amount, and save the new balance value. Then it is necessary to read the account balance of the recipient, add the transfer amount, and save the new value. All these procedures represent one logical unit and cannot be recorded or processed separately, since this may cause the loss of the transferred amount. In many cases, for example, when accounting the receipt or release of the goods, accounting the production operations, etc., a transaction consists of a single operation.

As a rule, transactions are recorded in the transactional database immediately upon receipt and this process is called on-line transaction process (OLTP). If the transaction is found to be erroneous, then the database must allow the user to cancel or correct this transaction, and it should be possible not only immediately after the transaction, but after certain and sometimes even long time. In the transactional database, the incoming source information usually does not undergo any modification. These data are not systematized upon receipt and stored in the same format as received. Some systematization of these data is possible during report preparation.

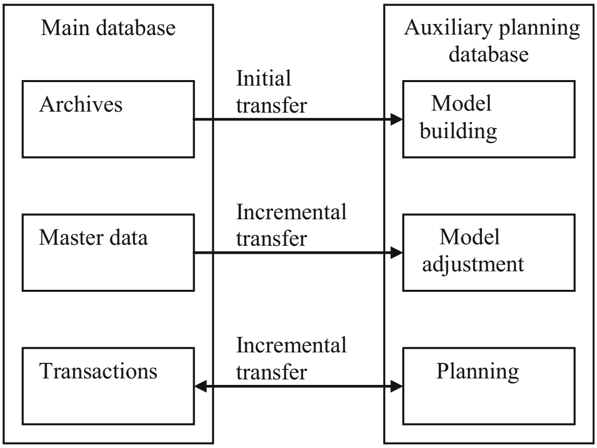


Fig. 5.1 Data transfer between the main transactional database and the auxiliary planning database [based on Stadtler and Kilger (2008)]

When implementing the "advanced" planning AP&S, except for the online transactional database, a special planning database is used (Stadtler and Kilger 2008). This database is also transactional, but its data is updated much less frequently than in the online database. There is regular exchange of data (its diagram is shown in Fig. 5.1) between these two bases in the planning process.

The diagram in Fig. 5.1 shows that initially upon startup of the system or subsystem of advanced planning the necessary information from the archives of the main transactional database are transferred to its own database in order to build computational models of planning. As the work advances, in the transactional database archives, the changes accumulate and they are periodically transmitted to the planning database. The solutions developed in the planning system in the form of plan targets can be transferred to the main base, for example, ERP-system. At the same time, new transactions necessary for the planning system can be periodically transmitted to the planning system. Some data needed for planning can be unavailable in the main database, and in this case, be entered directly into the planning database.

5.1.4 Decision Support Databases

According to Shapiro (2001), the decision-support databases are quite different from the regular databases discussed in Sect. 5.1.3. First of all these databases include the information on the commodity lines aggregated by the manufactured products. In Shapiro (2001) it is found empirically that there should be not more than 200 of these lines in the supply chains, but usually there are much less lines. Aggregation rules depend on the industry specifics. For example, it is possible to aggregate in the so-called phantom groups, which do not actually exist in the design specifications. An example of such aggregation is the grouping of beverages of various brands by packages of the same standard size. Grouping is also possible according to customers, suppliers, geographical distribution, etc.

The second direction of organizing data in bases in decision-support databases is accumulation of information about the location of companies within the supply chain, about the transport network among these companies, and their available stocks of aggregated products. The third type of data in these databases includes information on suppliers of materials, raw materials and components, on the prices of these goods, and the delivery cost. Finally, the decision-support databases can record information about the constraints concerning available equipment resources, finance, personnel, etc.

Decision-support databases are used in analytical information systems. Information analysis systems operate using the operational data obtained online from operational systems that automate the main activities of the company, as well as other available data sources that may be needed for various decisions. Information analysis systems are usually an add-on of already functioning data applications at the enterprise. The systems do not have much influence on their functioning or require their replacement.

The database, created and used by an analytical information system, in contrast to conventional transactional databases, is needed not to store the information about the events but to plan future actions. For this purpose, the information in a transactional-based is systematized and studied from different perspectives. In some cases, this information may be insufficient and then additional data may be entered directly into the decision-support database.

In the paper of Zaratuychenko (1998), the transactional and analytical systems including their created databases (Table 5.2) were compared. Though this study was conducted for the banking system, it is fully reflects a similar situation and for industrial systems.

Analysis of the figures in Table 5.2 indicates that the decision-support database is a necessary link connecting transactional data with analytical methods of decision-making. In such a basis the data from different areas of administrative activity: accounting, planning, operational management, organization of transport can be collected. The use of the decision-support database for analytical data processing is discussed below in Sect. 5.2.7. The decision-support database can serve as a basis for the use of optimization planning models.

5.1.5 Knowledge Bases

Knowledge base is a database of a special kind which is intended to manage knowledge, i.e. to collect, search, and retrieve the knowledge for use. Knowledge bases and methods of work with them are studied by the section of Artificial Intellect (AI) theory, which is called knowledge engineering. The knowledge base contains various facts and rules of their retrieval from the totality allowing for logical processing of the information received. For development of knowledge bases, logic programming languages are mostly used, such as Prolog.

Table 5.2 Comparative indices of transactional and analytical systems [based on Zaratuychenko (1998)]

Index	Transactional system	Analytical system
System purpose	Accounting, storage, and timely processing of constantly incoming data	Receipt and storage of aggregated data on the enterprise state and possibility to provide selected information in a convenient form for decision-making
Data nomenclature	Detailed information on inventory items flow	Summary data for a long period of time, obtained on the basis of detailed information stored in the transactional base
Type of data	Detailed data	Summary data
Update frequency	Data are updated constantly but in small portions	The system works with a fairly rarely updated source data
Usage pattern of the system	Automation of main operations set for accounting	Based on the stored, getting indicators that determine patterns of development of the enterprise and its performance
Presentation of the work results	Make up a certain set of report forms	Preparation of a large number of different reports based on aggregated data

Knowledge bases can have very different scales—worldwide, such as Wikipedia, to the base, which is used only by one specialist. Corporate knowledge bases store systematized key documents of the company: rules and orders, guidelines, standards, etc. Systematization of documents is mainly to establish the hierarchy of documents.

The main purpose of knowledge bases is to transfer the experience of previous generations to new staff and users. Knowledge bases provide the basis for operation of so-called expert systems used for solving various specialized management problems. Expert systems in the form of software applications allow analysing the occurring situation and generating recommendations according to their orientation.

Knowledge bases cannot represent a congealed set of facts and should reflect the dynamic nature of diverse human activities. This process of constant change and updating is called knowledge base evolution. In the paper of Hinkelmann et al. (1994), there is an example of the knowledge base evolution intended for operations planning at the enterprise producing car seats. In fact, in this example, the changes in design and manufacturing techniques are traced similarly to the method mentioned in Sect. 5.1.1 above, but with using a dynamic knowledge base.

Let us describe the construction of the corporate knowledge base in the sequence proposed in Smirnov (2004). First of all, we assume that the knowledge base is a well managed centralized electronic archive of applications, documents, directories, and classifiers of the company. For this archive, it is necessary to develop the basic catalogue structure and create a tool for information retrieval from the knowledge base by several different parameters. In this example, it is demonstrated how the entire knowledge base can be developed using only MS Excel.

In the directory structure, not more than three hierarchical levels should be used, although it is usually advisable to confine to two levels. For each independent project of the company, it is recommended to create a separate directory, in which technical and organizational documents should be separated by subdirectories. Separate directories are made for regulatory documents, for partners, catalogues of products, etc. The directories are coded with reference numbers.

Into the reference directories, it is advisable to place such reference files as the organizational structure of the company, main provisions and regulations, staff catalogue with phone numbers, customer and partner catalogues, etc. The main directories of the knowledge base should be filled gradually by sending documents within the company through the knowledge base. With the course of time, the knowledge base will be sufficiently complete, and most of the current activity of the company can be done by referring to the knowledge base and retrieving information from this database.

As an example of the use of the knowledge base the intelligent system (Kargin and Mironenko 2008) can be used to automate accounting and planning of production (metal cutting) for blank production shops of Mariupol Heavy Machinery Plant (Ukraine), shown by diagram in Fig. 5.2.

As can be seen from Fig. 5.2, the data from different user groups (supervisors, engineers, technical departments, site foreman, planners, etc.) get into the accounting and planning system. These data are then stored in the transactional database of the system and also get into the intelligent planning system. In block of problematic situation analysis, the most appropriate (similar) use case (or use cases) for the current situation is searched and derived from the knowledge base. For this purpose, the algorithm of the current situation identification is used on the basis of the algorithm for determining the similarity level between the current situation and that of the knowledge base. The planner uses the received data for scheduling.

Another example of the knowledge can be the so-called bank of analytical models proposed in Novitsky (2010). This article deals with the planning of work processes with grain when it is stored in elevators and used by flour and feed mills. The bank of models is a set of mathematical models (usually in the form of formulas) in the process of selection and allocation of resources in raw materials procurement, production, sales, and shipment.

For example, such models are the calculated relationships for the cost of raw materials, sales, profit margins, etc. These calculated values can serve as the optimization criteria (objective functions) of grain treatment processes. Other models, such as evaluating the quality of the grain, can be used as constraints in the calculations. The relevant set of functions from the bank of models allows performing various business processes: planning grain trade, definition of grinding mixtures recipes, calculation of feed recipes, etc.

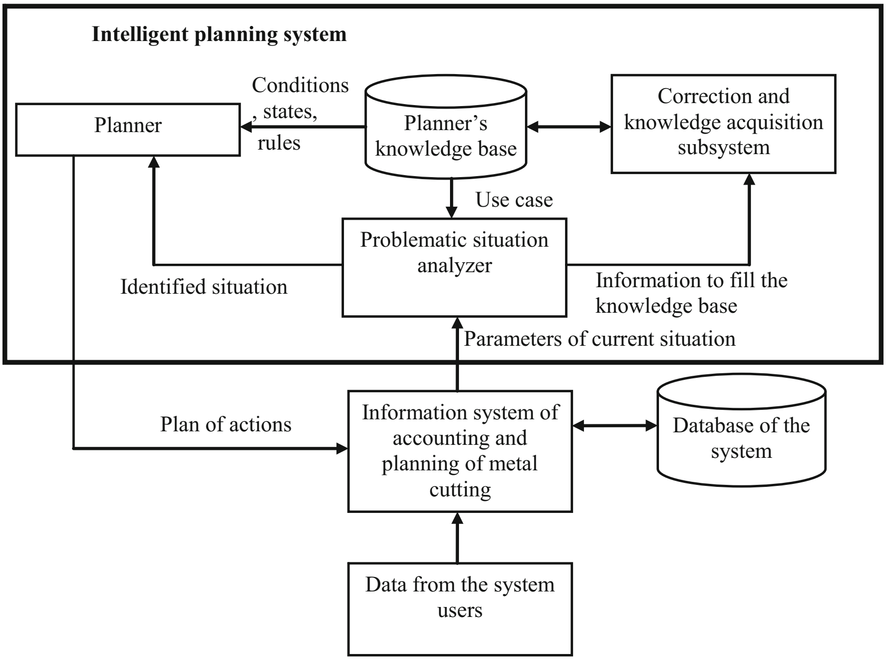

Intelligent planning system
Planner
Conditions, states, rules
Planner's knowledge base
Correction and knowledge acquisition subsystem
Use case
Problematic situation analyzer
Identified situation
Information to fill the knowledge base
Parameters of current situation
Plan of actions
Information system of accounting and planning of metal cutting
Database of the system
Data from the system users

Fig. 5.2 The structure of planning system for metal cutting [based on Kargin and Mironenko (2008)]

5.2 Data Storage and Management

This section deals with the issues of entering, storage, deletion, retrieval, systematization, and presentation of data, i.e. methods of data management.

5.2.1 Relational Databases

For the purpose of data storage in modern information systems, the so-called relational databases are basically used. All data in a relational database are presented in the form of rectangular tables of data values, and all database operations are reduced to the table handling. The table has a name that is unique within the database and consists of columns (fields) and lines (records). The data on a set of objects of the same type (one entity) is recorded in the table; each line relates to one particular object.

Each column in the table has a unique name and is represented by a set of values of a specific attribute (property) of the object. The values are selected from the set of all possible values of the object's attribute, which is called domain. In the simplest case, a domain is defined as the admissible potential set of values of the same type. For example, a set of dates of birth of all employees contains "domain of dates of birth" and the names of all employees are "domain names of employees". Domain of dates of birth has the data type that allows you to store information about moments of time, and the domain of employee names must have the character data type.

Each table should have a primary key—a field or set of fields, content of which unambiguously defines the record in the table and sets it apart from others.

Communication (foreign key) between two tables is usually formed when added to the first table field containing the primary key value of the second table. Tables should be organized in such a way to eliminate duplication of data in the tables and ensure their consistency. The process of this organization is called relation normalization between tables.

Let us consider the process of normalization in the example with suppliers and supply of goods. Suppose that we need to store information about the names of the suppliers, the names and quantity of the goods supplied, at that each supplier can supply several products of various kinds, and each item of one type can be supplied by several suppliers. Obviously, there is a relation between suppliers and goods that determines what goods can be delivered by each supplier. This relationship is rather complicated, because it is a connection between the two sets. This type of connection is called a "many-to-many" and cannot be properly reflected in the relational database directly.

In the relational databases, "one-to-many" interrelations are basic and "many-to-many" interrelations implemented by using several "one-to-many" interrelations. In this case, for example, there is "one-to-many" interrelation between the supplier and the supplier's deliveries since one supplier can make several deliveries, in each of which the products of any possible type can be supplied. Similarly, the same relation also occurs between the product of one type and deliveries, since one product can be supplied many times and by different suppliers.

The table being part of the relationship from side "one" (e.g. "Suppliers") is called a parent. The table being part of the relationship from side "many" (e.g. "Shipments") is called a child. Mechanism for implementing relationship "one-to-many" is that the child table is added with attributes, which are references to the key attributes of the parent table. Thus, in the above example there should be two parent tables (Tables 5.3 and 5.4) and one child table (Table 5.5).

In Table 5.5, attributes "Supplier code" and "Goods Code" are references to the primary key attributes in Tables 5.3 and 5.4 and therefore, are foreign keys. Using foreign keys, you can connect Tables 5.3 and 5.4 when retrieving information from Table 5.5. Real relational tables that describe the mechanism of shipments accounting, of course, include significantly more information than given in Table 5.5. The management of these tables will be discussed below in Sect. 5.2.3.

Table 5.3 Suppliers

Supplier code	Supplier name
1	Base “Building”
2	Construction materials store
3	Construction market
4	Plant of lighting engineering

Table 5.4 Goods

Goods code	Goods name
1	Paint
2	Cement
3	Linoleum
4	Lighting fixture

Table 5.5 Shipments

Supplier code	Goods code	Quantity	Measurement unit
1	1	100	kg
1	2	2	ton
1	3	12	roll
2	1	150	kg
2	2	1.5	ton
3	3	10	roll
4	4	15	pcs.

5.2.2 Concept of Object-Oriented Databases

In addition to relational databases, in recent years, the use of so-called object-oriented databases based on the object concept of information management has begun. Object-oriented approach is based on the following concepts:

- Objects and object identifiers
- Attributes and methods
- Classes
- Hierarchy and inheritance of classes

Term “object” means the combination of “data” and “program”, representing a certain real-world essence. The data consist of the components of arbitrary type called “attributes”. Each program in the software is called “method”. In the object concept, it is considered that the object “encapsulates” the data and program. In other words, users cannot see the inside of the object, but they may use it by referring to the part of its software.

All objects that have the same attributes (properties), and the same methods are collectively known as a class and same object should correspond to only one class. As an example of a class, we can name a set of mobile telephones. In this case, the object is any unidentified element of this set with all the characteristic properties

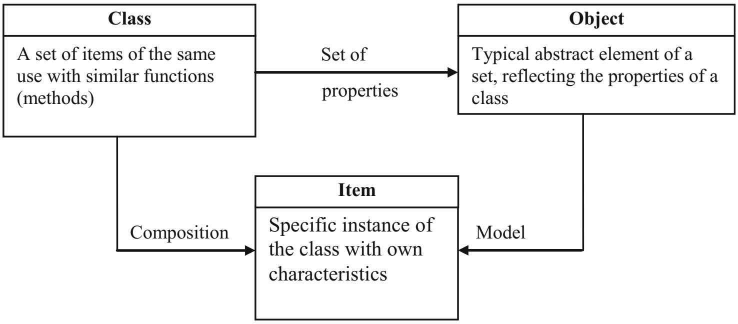

Class
A set of items of the same use with similar functions (methods)
Set of
properties
Object
Typical abstract element of a set, reflecting the properties of a class
Composition
Item
Specific instance of the class with own characteristics
Model

Fig. 5.3 Relationship of classes, objects, and instances

(size, colour, functions, etc.). The description of the object (encapsulation) may include the program of some actions with the object (on, off, set of messages, etc.). Each physical element (object) of this set is called an instance. Figure 5.3 shows the relationship of the class, corresponding object and instance.

Some classes may inherit attributes and methods from other classes. In general, the class can inherit properties from one or more existing classes and thereby forming structure of inheritance is called “hierarchy of classes”.

Since the inheritance makes it possible to share the various classes of one set of attributes and methods, the same program can work with the objects belonging to all these classes. For example, the object-oriented user interface of software is based on this. The same set of programs (open, close, new, delete, move, etc.) can be used to manage different types of data (image, text, graphics, directory, etc.).

In general, the use of object-oriented databases can lead to significant improvement of operational properties of databases with large amounts of information: increased search speed, possibility of using not only two-dimensional but multidimensional tables, etc. However, nowadays the use of these databases is limited.

5.2.3 Database Management Systems

The software designed to work with databases is Database Management System (DBMS). DBMS is a shell, which is used to generate a certain database in process of table structure organization and filling the tables with data. DBMS is used for the well-ordered storage and processing large amounts of data so that it is convenient to view, fill, modify and look for the necessary information, draw any samples and sort in any order.

Relational database management system is based on the relational data model. In a relational data model, as mentioned above in Sect. 5.2.1, any representation of the data is reduced to a set of relational tables.

DBMS must meet the requirements such as high performance, easiness of updating, ability to multi-user use, security, and others. The ability to meet these requirements is largely provided by the basic database language, which is Structured Query Language (SQL).

SQL statements look like normal English sentences, which facilitates their learning and understanding. SQL is the language of interactive queries to a relational database, which provides users with immediate access to data. With SQL, the user can interactively get answers to the most complex queries in a split second.

Despite the fact that there is a wide variety of databases, in practice only a few well-known databases (Oracle, Microsoft SQL, Informix, Interbase, Sybase and others) are used in manufacturing information systems. Above all, this situation is due to the huge cost of development of high-quality DBMS, as well as difficulties in its implementation, support, etc.

Database management system MS Access takes a special place among all DBMS. Generally speaking, this DBMS cannot be used to construct a complete information system because it does not provide a reliable multi-user usage of data. On the other hand, the wide availability of the DBMS allows ordinary users to build their databases for personal use. Therefore, a number of information systems enable users to upload data from a powerful network DBMS into MS Access and then work with the obtained data on a local computer.

5.2.4 Tiered Data Storage

In most such cases, the arrival rate of transactions is such that we can speak of a certain intensity of the transactions flow. If the intensity is high, then over time the amount of data in the transactional base can become very large. In this case, a logic question arises as to the retention period of the data on transactions. It is usually assumed that since it is usually not possible to determine the necessary retention period then it is necessary to arrange storage of information in several tiers (Fig. 5.4).

Figure 5.4 shows that manufacturing data flow enters the pre-set database. These data are analysed and presented in the form of reports and sent to the short-term storage archive. This archive data are sorted to the data to be sent to the long-term storage and the data of no value to store. The first group of data is transferred to the long-term storage archive, and the data of the second group are deleted. In the archives of long-term storage, the data are compressed into special CSV-files to reduce the amount of required memory. The files are text files in which the data are separated by commas (Comma Separated Values, CSV).

Feasibility of replacement of a relational database by a plurality of CSV files is due to the fact that each database field is assigned to a certain number of characters, although the actual data always occupy only part of the space allocated to them. In the CSV-file, each data value occupies the memory volume exactly corresponding to the value, which significantly reduces the need for total memory.

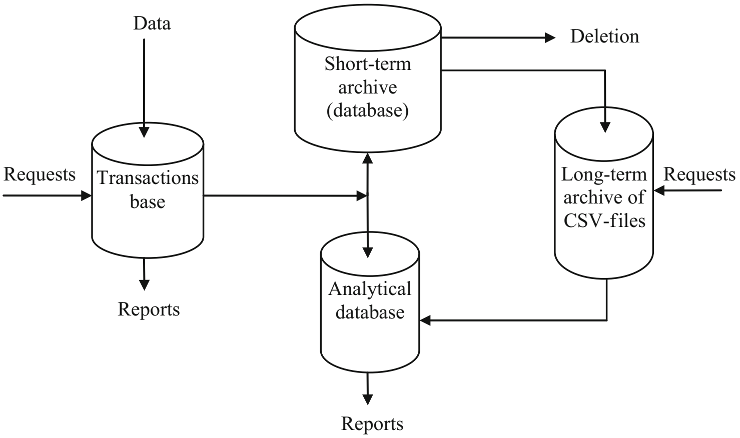


Fig. 5.4 Tiered data storage

5.2.5 Distributed Databases

A distributed database is a set of logically integrated databases residing on computers of different enterprise services, or even different enterprises. The computers are connected by data communications network with a structure called architecture. Currently, client–server architecture is used as a rule.

In this architecture, one of the computers having the largest memory and high performance become major in the network and is called a server. The server contains the data intended for collective use by different users. Client (workstation) is any other computer in the network, which has smaller technical capabilities and uses the resource provided by the server. The client has a variety of applications, as well as dictionaries and references for local use.

The powerful BDMS, listed above in Sect. 5.2.3, were developed readily for use in the client–server architecture. In contrast, the MS Access DBMS cannot be used in this architecture, and therefore it is necessary to duplicate the data on all network computers for its use. In this network architecture, called file-server, different users need to exchange stored information regularly, which dramatically increases the load on the network and reduces the ability of the current information exchange significantly.

In the client–server architecture, two possible structures can be distinguished. The first one uses the above one-tier structure, in which client applications (programs) are located either on users' computers entirely or some part of them is passed to the server, but the main part still remains on the client machine. This client is called "thick".

In the second structure, almost all software applications are passed to the server and the client is used only for output of the results of calculations and their analysis. This client is called "thin". The reason for using "thin" clients is first of all very significant decrease in requirements to the technical capabilities of the client

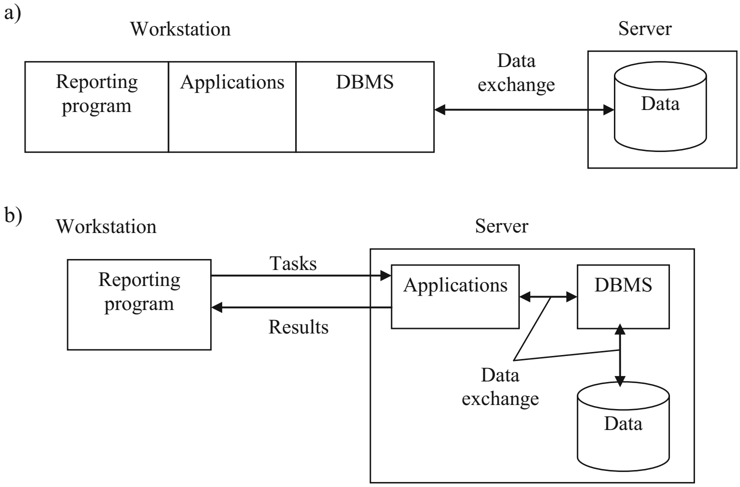


Fig. 5.5 (**a**) Thick client–server; (**b**) Thin client–server

computers and, consequently, reduction of the whole network cost. In addition, the data processing speed increases significantly, because there is no need to transfer them over the network from the server to the client. Let us explain this difference by using Fig. 5.5.

In general, it can be assumed that the software of the information system consists of three components: DBMS, applications, and reporting programs of these applications. If all of the software, as shown in Fig. 5.5a, is focused on the workstation, such client is considered to be "thick". If the workstation contains only a reporting program (Fig. 5.5b), then, as mentioned above, this workstation is a "thin" client.

As seen from Fig. 5.5b, in this case the data exchange occurs in the server, and between the workstation and the server, only the tasks and results of their performance are transferred. The network architecture shown in Fig. 5.5b is two-tiered as it consists of two components—server and workstations. If the network is constructed in such a way that the execution of applications is not assigned to the server with data and DBMS but to a dedicated stand-alone server, then this is a three-tiered architecture, and this server is called an application server.

Individual application functions of the application server can take the form of stand-alone programs that provide services to all other programs. Such functions are called services. With a large number of different services on the network, multiple application servers can simultaneously operate.

For cooperation, the servers and workstations of a company are united in the so-called corporate network. The main objective of the corporate network is to ensure the transmission of information between different applications used in the company. An application refers to the software that is directly require for the user, such as accounting software, word processing program, email, etc. A corporate network provides for interaction of the applications, located in different geographical areas, and access for remote users. The compulsory component of the corporate network is local networks connected to each other.

To maintain the corporate network the regular network administration is required. The network administration should provide variety of functions. The objectives are implementation of various changes in the structure of the network, such as connection of new workstations, configuration of network communication protocols, setup of network access services, troubleshooting, etc. An important function of administration is to ensure data security and protection against unauthorized access. Data protection includes a number of different tasks: backup and data recovery, development and change of passwords, wireless security, etc.

5.2.6 Service Oriented Architecture of IT Systems

Since modern software applications can rarely operate in isolation, the application usually cannot do anything significant without interaction with other applications. Currently, the construction of enterprise information systems increasingly involves the so-called service-oriented architecture (SOA). Service-oriented architecture integrates applications for collaboration and accelerates their operation by breaking an application into parts that can be combined with each other.

Term SOA refers to the modular approach to software development using services (utilities) with standardized interfaces. A service refers to an independent software component that performs a specific task, such as "check credit card", which does not require some mandatory software technology to be used by customers.

SOA is characterized by three important properties

- Each service implements a single business-function that is logically distinct, repetitive task being an integral part of an enterprise business process.
- Services are components of the information system that interact with other components only through their interfaces due to the use of open, widely used standards. Service interfaces are independent of platforms, programming language, operating system, and other technical features of implementation; services interact with each other and support other services.
- Services in the systems based on SOA can be performed irrespective of other system services, it is only necessary to know the interface of the used services.

The setup of a set of services for business processes is called "orchestration". The SOA usage requires a special technology for integration of services, which, in contrast to software, is called middleware.

This technology is the basis for the environment of distributed information processing. Middleware is a class of programs located between the application layer and system network layer and managing the interaction of applications on different computing platforms. Most packages of the middleware support high-level programming interface and a set of services types (such as authentication, directory and names processing) required for the operation of the distributed computing environment and its operation management. The software of this class allows programmers to avoid the subtleties of network architecture and operating system and focus on creation and improvement of applications.

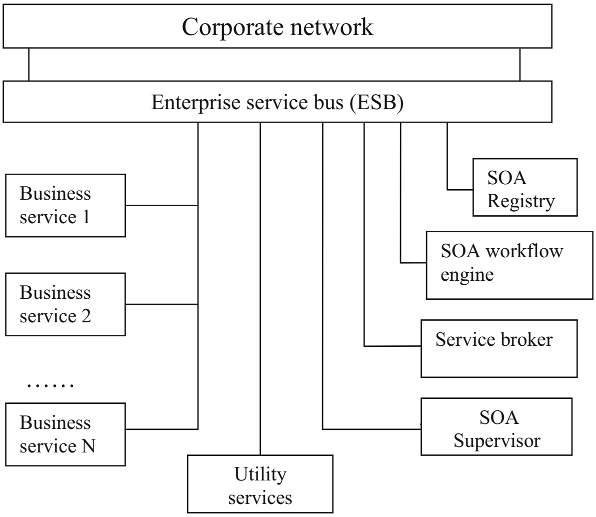
Usually, the middleware include those software components that provide applications interaction with the operating system and each other. This, in particular, remote call procedure mechanisms, transmission and processing of messages, access to the remote data sources, application servers, and monitors of transaction processing. Some programs of the same type can belong both to software and middleware. For example, a standalone DBMS, with which the user works directly, is an application program and a server DBMS is rather an intermediate program.

Middleware technology includes a middleware engine to determine the information route, its transformation, security, etc. The combination of these tools is called enterprise service bus (ESB). Figure 5.6 shows the major components of a perfect SOA system.

The main components of the diagram in Fig. 5.6 are enterprise service bus (ESB), SOA registry, SOA workflow engine, service broker, and SOA supervisor. All of these play own role in the system by interacting with each other. Here business services are specific applications; utility services are used to control the service bus.

The SOA registry is an electronic directory, which stores information about each component composing the corporate information system and about the interfaces, which are used by these components to communicate with each other. The SOA workflow engine is a software product that drives the entire business process in the enterprise information system from start to completion. The SOA workflow engine consistently triggers the workflow according to the reference model of the business process. The service broker, by receiving information on services in the system

Fig. 5.6 Diagram of corporate network using SOA



from the SOA registry, coordinates their work with the SOA workflow engine. The SOA supervisor monitors the operation of various components within the system, evaluates the correctness of their functioning, and tracks the requests sent to external systems.

As can be seen from Fig. 5.6, all software of SOA is connected to the service bus. Interaction of services via the bus is only possible if certain service level agreements (SLA) are performed. The quality of services communication can be evaluated by a set of key performance indicators (KPI) similar to indicators used in business.

The diagram of group services in Fig. 5.6, presented a chain, can be extended to the concept of "manufacturing chain" that provided transmission of information between related production facilities. For example, in the article of Biennier et al. (2007) ESB, which takes into account a number of production constraints, is manufacturing service bus (MSB). In the latter, the manufacturing services are combined with dynamic monitoring system and transfer information between manufacturing control elements.

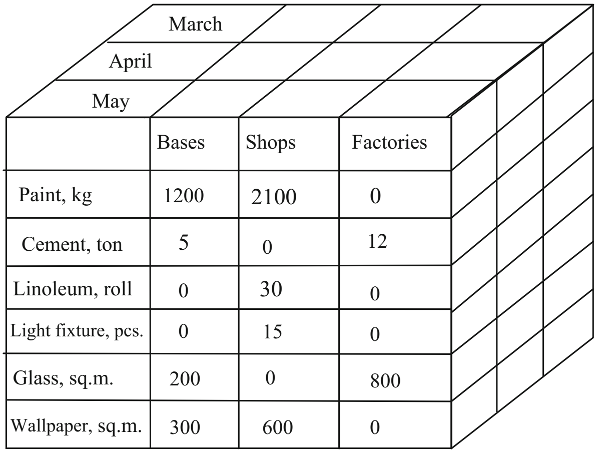
5.2.7 On-Line Analytical Processing

Section 5.2.3 described the queries to the relational database, which is the basic method of information retrieval from the database. If the query is quite simple, it is performed within seconds. However, in case of complex queries with many conditions the performance time increases dramatically. Moreover, in analysis of data, it becomes necessary to perform several (sometimes many), different complex queries that also require considerable time.

To speed up the analysis of large databases it is possible to prepare the existing database that allows you to pre-aggregate data in different directions (dimensions). In fact, in process of this preparation the relational database is transformed into a sort of database for decision-making described above in Sect. 5.1.4. For analytical work with such database the special technology was developed. It is called on-line analytical processing (OLAP), in which the base itself is called "data warehouse".

While the conventional databases are subject to constant changes in process of users' work, the data warehouse is relatively stable: its data are usually updated according to the schedule (e.g. weekly, daily, or hourly—as needed). Ideally, the process of filling is simply the addition of new data of a certain period without changing the previous information already stored in the data warehouse.

Unlike the relational database consisting of flat tables, the OLAP data warehouse is a data model organized in the form of multidimensional cubes. The axes of the multidimensional coordinate system are the main attributes of the business process under consideration. For example, for sales it may be goods, area, types of customer. Time can be used as one of measurements. At the intersections of the axes (dimensions) there are the data (measures) characterizing the process quantitatively. These data may be represented by sales volume in pieces or in monetary terms, stock balance, costs, etc. During the analysis the user can receive



	Bases	Shops	Factories
Paint, kg	1200	2100	0
Cement, ton	5	0	12
Linoleum, roll	0	30	0
Light fixture, pcs.	0	15	0
Glass, sq.m.	200	0	800
Wallpaper, sq.m.	300	600	0

Fig. 5.7 Three-dimensional cube of supplies

summarized (e.g. annual) or on the contrary detailed (weekly) information by "cutting" the cube in different directions.

Figure 5.7 shows a three-dimensional cube for the supply of construction materials described in Tables 5.3–5.5. The amount of supplied goods serves as a measure in this cube, but as a dimension—time, goods and suppliers. For each of the dimensions grouping can be performed: goods are grouped into categories, suppliers—by type of organization (base, shops, factories), and data on the time of the transaction—by months.

Besides measures in physical volumes of supplies, the analysis by the supplied goods value is also carried out almost always. In order to carry out two such analyses in parallel, it can be assumed that these two measures form another dimension. In this case, the supply cube should have four dimensions, i.e. become multidimensional. Although it is not possible to depict this cube in the three-dimensional, it can be stored in a multidimensional data array, and for visualization of the data stored in the cube, it is possible to use the conventional two-dimensional, i.e. tabular, presentation.

The two-dimensional presentation of the cube can be obtained by "cutting" it across one or more axes (dimensions). If in this section we fix the values of all measurements, except for two, we obtain a conventional table. In this case, the column headings of the table represent one dimension and the row headers—the other dimension and the cells of the table contain values of the measures. If the table uses multiple measures at the same time, then one of the table axes will belong to the measure names, and the other—the values of the single "uncut" dimension.

The two-dimensional presentation of the cube is also possible when more than two dimensions remain "uncut". In Table 5.6 in the column headers two dimensions are used—periods and measures, and in the row headers there is one dimension—the names of the goods with their measuring units. At the same time, in this case the values of physical quantities and cost values are summed up in the direction of supplier dimension, shown in Fig. 5.7.

Table 5.6 Example of two-dimension table presenting three dimensions

	March		April		May	
Goods	Quantity	Cost	Quantity	Cost	Quantity	Cost
Paint, kg	500	6000	1100	13,800	3300	34,000
Cement, ton	10	12,000	8	98,000	17	163,000
Linoleum, roll	0	0	20	1580	30	2300
Light fixture, pcs.	0	0	0	0	15	1900

Values plotted along the dimensions are known as marks. For example, period marks may be words “March”, “April”, “May”. Marks are used for “cutting” the cube and for the restriction (filtering) of the selected data as well if for analysis instead of all values their subset is used, for example, 3 months of several tens. The marks values are displayed in the two-dimensional presentation of the cube as row and column headers. The marks may join into hierarchies, consisting of one or several levels. For example, the marks of dimension “Shops” naturally join into the hierarchy with levels:

Worldwide
Country
Area
City
Shop

The data can be aggregated by each of the levels.

Multidimensionality in OLAP application can be divided into three levels (Alperovich 2001).

- Multidimensional data presentation is means for the end user. It provides multidimensional visualization and manipulation of data. At that the multidimensional presentation layer is abstracted from the physical structure of data and accepts data as multidimensional.
- Multidimensional processing is means (language) for multidimensional queries formulation. Since this language is different from relational language SQL, you need a processor which can process and perform a multidimensional query.
- Multidimensional storage is means of physical organization of data to ensure the effective performance of multidimensional queries.

All OLAP tools, for example Pivot Table Service from Microsoft provide first two levels of the multidimensional presentations. The third level although being widely spread is optional as the data for the multidimensional presentation can be retrieved from regular relational data structures as well. Multidimensional storage allows treating the data as a multidimensional array, and due to this the equally quick calculations of summarized indices and various multidimensional conversions of any dimension are provided. However, for the purpose of the multidimensional storage there should be special multidimensional databases, which dramatically increase the required volume for storage and the system cost.

5.3 Information Exchange

Currently, the efficient manufacturing activity of an enterprise is possible only if there is close and most important quick cooperation with other companies, which create the so-called enterprise network together. As companies has been able to involve resources and manufacturing capabilities of practically any enterprise around the world for their own business, the enterprise network can be created for short-term cooperation as well and have a variable composition.

The enterprise network includes three different flows—material, financial, and informational and the latter always accompanies the rest. Therefore, it is clear that for effective collaboration in the network the enterprises must have a set of methods and related tools providing complete exchange of necessary information.

5.3.1 Internal Data Communication

The issues of data exchange between the information systems available at the enterprise have been covered quite in details above in Chap. 1. In particular, Table 1.8 showed five information exchange levels and for the top three layers the corresponding standards have been described in Sect. 1.5. Here we will discuss the most common current standard of information processing for level 1, i.e. for the transmission level of raw information received from various sensors on the equipment to the supervisory system SCADA.

OPC (OLE for Process Control) is the industrial standard developed by a consortium of world-famous producers of hardware and software with the participation of Microsoft. This standard describes the communication interface between the process control devices. The main purpose of the standard was to provide developers of the dispatch systems some independence from a particular type of controller. Acronym OLE (Object linking and embedding) is used to denote data transfer technology in the systems developed by Microsoft, for example, to transfer data from MS Word to MS Excel and back.

As this name OPC implies, this technology is a way of distributing OLE technology to processing of the information used for industrial process control. Development of OPC standard is carried out by OPC Foundation. The OPC standard clearly delineates responsibilities among hardware manufacturers and developers of data transfer drivers and this allows you to collect data from various sources and send them to any client application regardless of the type of equipment used. As a result, modification of the equipment or production of new items on this equipment does not require changing the raw information processing programs. In addition, there is a wide choice of equipment suppliers, as well as possibility to integrate this equipment in the enterprise information system.

The OPC standard includes a number of so-called OPC specifications, the most common of which are Data Access specifications providing access to data online. These specifications are used to obtain the following types of data:

- Data from online sensors (temperature, pressure, etc.)
- Control actions (open, close, start, stop, etc.);
- Information about the current status of equipment and executive programs

The OPC technology always involves two computer programs: OPC server and OPC client. The first provides its functionality through user interfaces and the second requests and receives the information. The OPC server may be provided for a controller, I/O card, field bus adapter, conversion program, random number generator, i.e. any device, transmitting or receiving data. The OPC client is usually included in the scope of software packages SCADA.

There are three main ways for the OPC client to obtain data from the OPC server: synchronous reading, asynchronous reading, and subscription. When reading synchronously the client sends a request to the server with a list of the manufacturing process parameters required at the moment and waits for the server to execute it. During asynchronous reading, the client sends a request to the server and continues to work. When the server fulfils the request, the client receives a notification. In the case of a subscription, the client sends the server a list of manufacturing process parameters in interest, and the server then sends the information of the changed variables from this list to the client regularly.

5.3.2 Data Transfer Between Enterprises

Information exchange in a production network is defined by two factors—the extent of information transfer and its intensity. Information extent is determined by the number of stages of information transfer from the place of its origin to the last in the enterprise information chain in the production network. Let us consider once more the commonly used example of the diaper supply chain by Proctor&Gamble for large retail network Metro, which has been shown above in Fig. 1.2.

As in the case in Fig. 1.2, the production has three stages of supplies and the sales have two stages, the extent of information transfer from the sales towards the production—up the supply chain—is three and the extent from the production towards the sales—down the chain—is two. It should be noted that in reality any enterprise participates in a number of supply chains, in which the extent of information transfer can be quite different.

The information intensity describes the amount of information per time unit transferred via the network. For the supply chain, the entire amount of information can be classified into six categories (Huang et al. 2003): products, processes, resources, stocks, orders, and plans (Table 5.7).

The amount of transmitted data, its content, and even the structure of the production network may change over time. The life cycle of the network can be

Table 5.7 Classification of production information [based on Huang et al. (2003)]

Category	Production information	Category	Production information
Product	Product structure	Resource	Capacity
Process	Material lead time		Capacity variance
	Lead time variance	Order	Demand
	Order transfer lead time		Demand variance
	Process cost		Order batch size
	Quality		Order due date
	Shipment		Demand correlation
	Setup cost	Planning	Demand forecast
Inventory	Inventory level		Order schedule
	Holding cost		Forecasting model
	Backlog cost		Time fence
	Service level		

divided into three periods—its creation, maintenance, and transformation. Let us consider the characteristics of each of these periods, starting with creation of a network.

As a rule, the partners used to be selected on the stage of organization of production, when the bill of materials (BOM) was already established, and evaluation of the partner quality selection was carried out using the key quality indicators (KPI), described above Sect. 1.6.3. At the same time at other stages of the product life cycle, the requirements for the partners may differ significantly. The opportunity to attract partners in the production network at the stage of product development greatly increases the possibility of creating a better product by bringing the experience and capabilities of all potential partners.

Since a new product is usually created not from scratch, but on some basis, it uses a variety of existing components. The process of creating a consortium with enterprises producing the components can be divided into two stages—search for partners and their selection. Each phase of the life cycle—development, production, launch to market, support in operation, end of service life—requires relevant partners. At each successive stage of the life cycle of the freedom to choose a partner is reduced compared to the previous one. Evaluation of partner selection can be done from three perspectives: cost of its services, quality, or suitability for the developed manufacturing process.

In contrast to the usual practice of partner selection by tradition or personal relations, now it is advisable to use databases of these businesses that are developed by the chambers of commerce. These databases have administrative data of enterprises and detailed information about their products. It is possible to compare selected partners by determining their KPIs.

Partners can be selected both by comparing the companies producing similar products of the same level in BOM and by expanding the list of enterprises, including the enterprises, producing the components of products of that level. The first approach is called horizontal enlargement of the list; the second is vertical enlargement. This process is performed by the method of successive approximations and in the result the product tree is correlated to the list of partners and participants at all stages of the life cycle.

At the stage of the operation maintenance of production network, the structure of the transferred information is usually quite stable. At the same time, the transfer rate may vary because there may be considerable fluctuations in the network operation, in particular relating to the so-called bullwhip effect. This effect is that small deviations in demand and stocks at the beginning of the production network grow in proportion to the information transfer by the steps deeper into the network and become more significant. More details about this effect will be given below in Chap. 9.

The transformation of the production network has to be carried out when it is set up for the new products. The process is particularly difficult for the network of small- and medium-sized enterprises, where it is necessary to coordinate the relevant changes in the sequence of the manufacture of various products. The inclusion of a new product into production is connected to two main problems: postponement problem, which is necessary to determine the features of the new product and its inclusion into the production chain and order-fulfilment problem to ensure the lowest possible losses for the network of enterprises.

The inclusion of a new product in the production network is carried out in four stages. At the first stage the need for a new product is defined. The second stage classifies the product to the corresponding group of products in a hierarchical classification. At the third stage, the sequence of production of all products is determined including the required volume of the new product. At the last stage, the new product is included into the production plans of enterprises based on their capacities load.

5.3.3 Information Exchange in Different Types of Cooperation

Cooperation of enterprises in the production network is not limited to the supply chain. It is also possible to single out the following areas of activity: (a) cooperation of developers and manufacturers, (b) joint work of various geographically distant offices of one firm, (c) cooperation of manufacturers and enterprises for products sale, repair, and maintenance, and (d) joint development of proposals for the products manufacturing with customers, including individual consumers. Let us consider the specific distinctions of information exchange in these cases.

As an example of such exchange during new product development the cooperation between the Swiss developer in Zurich and Chinese manufacturers in Shanghai (Zhu et al. 2007) can be mentioned. In this project, it is assumed that the design process comprises the steps of design itself, detailed specification and control, and the production comprises the steps of manufacturing a prototype, testing, production planning, and full-scale production. To speed up commercial manufacture it is proposed to provide the manufacturer's participation directly in the design process, starting with a detailed specification. Herewith, it is possible to control the quality earlier, 70–80 % of which depends on the design level according to the studies.

With this approach, it is possible to reduce processing flaws of a product, which often cause loss because of further maintenance, etc. It turns out that most of the product flaws stems from ignoring the so-called noise factors—various uncertainties (incidents) that appear during operation and it is possible to reduce their impact by controlling them. This method is called "robust design".

From the perspective of performance analysis of the enterprise and its geographically distant branches, the experience of Toyota on establishment of electronic interaction system of various members of production and sales of automobiles (Yamaji and Amasaka 2007) is of great interest. In western companies, the emerging problems are expected to be solved by the people of respective competence. The strong point of the Japanese approach is that to solve the problems other employees of the corporation are involved as well. Based on this approach, the company has created "global intellectual partnership" model. In this system, the information on management policies, business plans, and other similar tasks is distributed among many departments, and the trends of these tasks have numerical rating.

The importance of collaboration of a number of departments within the company and its subsidiaries, including overseas, is demonstrated by the example of the process of improving the car body paintwork. In this process, both the type of paint and the way of its application were selected taking into account multiple requirements of different consumers. As a result of long-term collaboration of all members the anticorrosive properties of the coating increased 10 times, while reducing the total cost at all stages of coating by 30 %.

After-sales service is an extremely important activity. Suffice it to say that the total income from products sales taking into account its after-sales service can be up to three times more than the actual price of the product. In terms of after-sales service all products can be assigned to one of four segments of service (Lele 1997). The first segment is the so-called disposable, which includes low-value products, usually not repairable. The "repairable" segment includes the products to be repaired, but not requiring its urgency. The segment of "rapid response" includes products that require immediate repair after damage. There is also "never fail" segment, the products in which shall never be faulty, for example, airplanes, medical equipment, etc.

After-sales service processes can be classified according to the level of consumer involvement in the process. If the consumer can perform service totally independently, this method is called "indirect support". If the user identifies the fault with the help of an expert and then removes them him/herself, this is "remote support" method. The method of returning the defective product to the manufacturer for repair is called "off-site support". A repair on site is called "on-site support". Each of these methods requires different information support.

To standardize the service related to business in the manufacturing sector, in 2006–2008, the European project "Innovation, Coordination, Collaboration in Service Driven Manufacturing Supply Chains" (InCoCo-S) was developed. In terms of the information exchange between the production and service, the experience of usage of the reference model of these relations is of interest. The model was established by SKF (Swedish bearing manufacturer) based on InCoCo-S (Osadsky et al. 2007). This model provides collaboration among 81 manufacturers and 81 companies providing service with the help of computer technology and contains five main fields of service: support, packaging, logistics, quality control, and upgrading, on the basis of which it is possible to analyse the cost of service.

Best-Practice and Performance Metrics data are entered into the reference model. The first data were based on the top 100 results achieved in service. To make the metric system the evaluation of performance including nearly 300 quality indicators was developed. As a result of using this model, superfluous operations were deleted, necessary ones were added, and the flowchart was improved. The quality of order processing was evaluated on the basis of the indicators (metric) of quality.

Cooperation of manufacturers and retailers with customers is a form of information exchange in the production network. In the course of this cooperation there is always an attempt of customization of the individual consumer to some product without reducing the potential for mass production of this product. This process includes (Etgar 2008) five different steps: (a) creating the conditions for information exchange, (b) establishment of motivations that encourage consumers to cooperate, (c) calculation of the benefits of cooperation, (d) involvement of consumers in the active cooperation, and (e) result output and its evaluation. Obviously, the described cooperation is possible only when buying some rather complex products and also in case of corresponding skills of the buyer.

From the manufacturer's point of view, the principal way of achieving "customization" is to expand the variety of products; herewith to reduce the costs they use certain postponement, which can defer some of production operations to the right moment, for example until a relevant order. This delay is determined by the position of the so-called decoupling point described above in Sect. 1.3.2 and determines the depth of the decoupling with the production process. The smaller this depth is, the faster and cheaper the requirements of the consumer can be met.

5.3.4 Information Exchange Automation

Currently, besides the Internet and e-mail only the methods of information exchange within the production network developed by such industry giants as Microsoft, Oracle, and SAP are widely used. As stated in the advertisement of Oracle's AutoVue Mobile module, "manufacturers around the world can cooperate effectively in the supply chains by optimizing their information exchange and creating a flexible and responsive production network". Similarly, SAP encourages to use its product SAP Supplier Network Collaboration, promising to integrate its systems to the products of other companies and documents in MS Excel. However, apparently, the capabilities of these software tools are limited for full exchange of information in the network, as evidenced, for example, by a special project for collaboration in the supply chain of such systems as Microsoft and i2 Technologies.

Typically, users believe that the use of software for the exchange in a network is reasonable only after the introduction of ERP system at the enterprise in order to use data of this system for transmission to other companies. However, probably (Wailgum 2008) even in the absence of complete ERP system, the use of special tools for exchange in the network can have a serious effect. From this viewpoint, it is advisable to pay attention to software systems specifically designed for use in production networks, such as, for example, cX Collaborative Supply Solution from Persistent System.

Such systems tend to operate in the network of e-business RosettaNet, where business processes among the partners are carried out by means of a document called Partner Interface Process (PIP). Rosettanet Standards 2004 are used in exchange of information through PIP, providing relevant data formats. In particular, in the mentioned system cX, partner interface process visually represents the so-called action center. In the process of working with the action center, the user can operate the products of all entities involved in the network, set specific targets for each participant, schedule events, and send messages.

Some very large companies develop their own systems for the information exchange among remote offices. These firms include Toyota mentioned above, as well as, for example, Norwegian company Mustad specializing in the supply of fishing equipment worldwide. The data on stocks and sales, production, plans and forecasts, equipment, and raw materials supply are collected in its control center. The powerful data processing system and data visualization allow making decisions in online mode (Dreyer et al. 2007).

The use of radio frequency identification (RFID) technology opens new opportunities for the information exchange. One of the first examples of the use of the technology was the supply chain shown above in Fig. 1.2. Proctor&Gamble company for the purpose of exchanging the data on supply to the retailer’s distribution warehouse provides the boxes of products with special labels or “tags”, the so-called RFID tag, represented by microchips equipped with antennas. Microchips can record information on product (barcodes) and about suppliers, lots of goods, date of receipt, etc. Reading this information is performed remotely by consignee’s device at a distance usually not greater than 5 m.

Currently, RFID technology has gained a wide variety of applications. An interesting example is a system of continuous daily monitoring of the individual samples condition of household appliances during its operation. The system is developed under PROMISE project of Indesit at the Polytechnic Institute of Milan (Cassina et al. 2007). The purpose of development of this system is a systematic preventive maintenance support to enhance the competitive properties of goods. The system is also used for monitoring the life cycle of vehicles, railway facilities, electronics, etc. and consists of a number of devices, software packages, and extended infrastructure.

The basis of the system is devices embedded in a controlled product (Product Embedded Information Devices, PEID), which help to collect data on operating parameters of the latter. System testing was carried out on household refrigerators using a control device that is defined in the system as a “smart adapter”. This device is installed on the power cord right at the plug, the adapter and remote monitoring center communicate by radio, the range of which is much greater than the range of RFID tags.

The operating parameters of the refrigerator are recorded in the memory of the adapter and then read from it by product monitoring and support program. These adapters can be used both in the home monitoring system of household appliances and in the center that process the results of PROMISE project. The adapter can be used for a wide variety of products regardless of their specific set of parameters, and the use of these data makes it possible to predict failure and do preventive maintenance reasonable in advance. It should be noted that the cost of both the adapter itself and the subsequent data processing is low, which makes this technique quite promising.

Another example of the use of RFID technology is the production system architecture (developed at the University of Kobe, Japan) that provides control when the production equipment is geographically distributed, for example, on construction sites (Tsumaya et al. 2007). In this system, the tag is rigidly connected with the product and automatically transfers the data to the computer network when the product arrives at the input of the factory, construction site, vehicle, warehouse, etc. Comparison of these data with the predetermined plan allows evaluating their difference and, if necessary, developing a new plan. The described dynamic production management architecture is called “technology of standardized parts and packages”.

Two examples of the use of such technology are known. The first example describes such automated system consisting of five pieces of equipment, two robotic vehicles, and a warehouse. In this system, the sequence of processing a variety of products is hard-coded, and the sequence of processing of incoming lots is accepted according to “least processing time” approach. Available tags allow constant monitoring of the lot in the processing queue on each piece of equipment and, if necessary, recalculating work assignments. The second example relates to the construction site, where three contractors perform the three kinds of jobs simultaneously. The system automatically controls the possibility of interference with each other, checks the supply of materials, and recalculates work schedules.

Nowadays, a very significant part of products of different types has a uniform numerical code (Global Trade Item Number, GTIN). This code consists of 14 characters and is registered by international organization GS1 (www.gs1.org). GTIN code is only a part of the so-called electronic product code (EPC), developed by organization GS1. EPC has four components or fields: EPC version code, EPC development organization code, class of objects, which a particular product belongs to, and GTIN code.

The use of EPC codes enabled organization GS1 to develop EPC Global Network. In this system, RFID tag flow is monitored by radio signals from the transmitter of the information system of the user-organization dealing with operation or processing of the product. Additional information about the product can be obtained by the information system from the system of ONS addresses of organization GS1, as well as from the manufacturer of each product. Planning of the products flow is carried out in the systems of distribution and logistics, and the corresponding data are transmitted to the user-company.

5.3.5 Use of Cloud Environment

Cloud environment enables consumers to enjoy a variety of software installed on other computers via the Internet. The US National Institute of Standards and Technology (NIST) provides the following definition for computing (NIST SP 800-145):

"Cloud computing is a model for enabling convenient, on-demand network access to a shared pool of configurable computing resources (for example, networks, servers, storage, applications, and services) that can be rapidly provisioned and released with minimal management effort or service provider interaction."

Thus, in the cloud computing (online) it is possible to input data, set up and run programs, display the results, and transfer them to other users.

The architecture of the cloud environment conditionally may be divided into two parts, each of which comprises a large number of separate components—Front End and Back End. Front End is a client part of cloud computing. This part contains the interfaces and applications that provide access to cloud computing platform, for example, Web Browser. Back End is actually the cloud, where computing takes place. The structure of Back End consists of huge data storage, virtual machines, security mechanism, services, deployment models, servers, etc. Front End and Back End are connected via the Internet.

Here is the list of many advantages of cloud computing:

- Users do not need to buy expensive computers with large memory and disks to use the program through the web interface. There is also no need for CD and DVD drives, as all the information and programs remain in the cloud. In this case, the user's machine is actually a thin client (Sect. 5.2.5) of the cloud environment. The costs for equipment and its maintenance may be reduced down to 50 % in some cases.
- Instead of purchasing software packages for each local user, companies buy the necessary programs in the cloud. These programs will only be used by those users who need these programs for work. Moreover, the cost of the programs, oriented towards access through the Internet, is much lower than their analogues for personal computers. If the programs are not often used, they can be simply rented hourly. The cost of software upgrades and support in working condition at all workplaces is reduced to zero at all.
- Any time when a user runs a remote program, he can be sure that this program has the latest version—no need to reinstall anything or pay for the upgrade.

- In comparison with PC the computing power available to the user of cloud computers is practically limited only by the size of the cloud that is the total number of remote servers. Users can run more complex tasks with a lot of the necessary memory, space for data, when necessary. In other words, users can, if desired, easily and cheaply work with a supercomputer without any actual acquisitions.
- In Cloud Computing the operating systems cut no figure. Unix users can share documents with users of Microsoft Windows and vice versa without any problems. The programs and virtual machines are accessed via a web browser or other means of access to be installed on any personal computer with any operating system.
- Unlike backup to another PC or other storage media such as DVD drives or flash drives, when working in the cloud it is not necessary to monitor and perform this backup yourself—it is automatic and extremely reliable.

The main disadvantage of cloud computing is the need of permanent connection to the Internet, which, moreover, should provide high data transfer capacity. Working in the cloud often does not provide sufficient protection of the stored data from unauthorized access.

The cloud computing uses reference models of three types (Fig. 5.8). The most fundamental is model Infrastructure as a Service (IaaS). In this model, the user can use the cloud infrastructure to manage the resources of processing, storage, networks, and other fundamental computing resources oneself. The consumer can install and run arbitrary software, which may include operating systems, platform, and application software, in these resources.

Platform-as-a-Service (PaaS) is the model, where the user is able to use the cloud infrastructure to accommodate the basic software that provides operation of new or existing applications. The composition of such platforms includes design tools, test tools, and app software run tools: database management systems, middleware, programming languages runtime.

Software-as-a-Service (SaaS) enables to use application software of the provider operating in the cloud and accessible from various client devices.

Monitoring and control of the basic physical and virtual cloud infrastructure including network, servers, operating systems, and storage devices is performed by the cloud provider except for the developed or installed applications, as well as configuration of environment (platform), if possible.

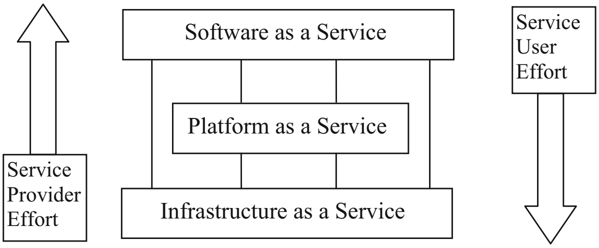


Fig. 5.8 Cloud service model layering

As follows from Fig. 5.8, higher layering of the use of cloud models requires more efforts of the cloud provider but the work of users becomes easier at the same time. On the contrary, lower layering of the model leads to lower requirements for the provider, but the user's work becomes more complicated.

From the point of view of the users of information systems for production planning, the work with such a system in the cloud is extremely useful when selecting the system to be most suitable for a particular company. In this case, without investing heavily in purchasing the system, the customer can work with it on lease for the time enough to explore it properly.

If the decision is made on implementation of the system at the enterprise, then the practicability of further work in the cloud depends on the scale of the enterprise. If the scale is significant, it makes sense to gradually abandon the cloud services provider, as the computational capabilities of the enterprise increase. For small businesses, the work in the cloud is probably the only opportunity to use modern powerful information systems. An example of operation in the cloud with one of such systems is shown in Appendix F.

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Demand Forecasting

6

6.1 Demand Modelling Based on Time Series Analysis

Routine observation of the product quantity demanded results in recording a time-ordered sequence of values of this demand. This sequence is called a time series. Usually, observations are performed at regular intervals and numbered sequentially. As the demand values vary randomly then the corresponding time series is a random variable. The main purpose of the time series study is to forecast, i.e. to make an attempt to predict the future values of the time series based on its present and past values.

Numerous observations of different time series show that the current value of random variable in question D at time t is dependent on four factors

$$D_t = f(I_t, b_t, C_t, e_t), \tag{6.1}$$

where I_t is the so-called seasonality index reflecting the seasonal effect; b_t is the trend, i.e. systematic motion; C_t is more or less regular fluctuations against trend; and e_t is the random value.

In the simplest case demand D is the value falling under random deviations from a certain constant value a, i.e.

$$D_t = a + e_t. \tag{6.2}$$

Dependence (Eq. 6.2) describes the demand for the products on the market in stable development stage described above in Sect. 1.6.1. The example of these products can be toothpastes, spare parts, some tools, etc.

If a product belongs to any other market quadrant shown above in Fig. 1.10, then the demand for the product increases (new or growing market) or decreases (mature market). In this case, the demand, for the most part, can be approximately described by the following dependence:

Y. Mauergauz, *Advanced Planning and Scheduling in Manufacturing and Supply Chains*, DOI 10.1007/978-3-319-27523-9_6

$$D_t = a + bt + e_t, \tag{6.3}$$

where b is the trend coefficient, which can be either positive or negative.

Taking into account the seasonality index, expression (6.3) can be written as

$$D_t = (a + bt)I_t + e_t, \tag{6.4}$$

reflecting the so-called multiplicative seasonality model. In terms of this model it is assumed that if 1 year includes T of various seasonal periods, then for any of these subsequent periods the following expression is valid:

$$\sum_{k=1}^{T} I_{t+k} = T, \tag{6.5}$$

i.e. the average seasonality index is equal to 1.

With multiplicative seasonality, the value of seasonal fluctuations depends on the overall level of the time series values. Unlike this type of seasonality, the so-called additive seasonality can be observed, where the demand of the form (Eq. 6.2) is added with some periodic component with period l, i.e.

$$D_t = a + I_t + e_t, \tag{6.6}$$

and the following condition is observed:

$$I_t = I_{t+l}. \tag{6.7}$$

An example of multiplicative seasonality can be the market of toys, sales of which are growing in the pre-holiday period. The more popular toy in ordinary days is, the more the demand for it grows during holidays. The additive seasonality is true for the goods, consumption of which is concentrated in certain periods and depends little on the consumption in other periods. These products include, for example, ice cream, soft drinks, fur products, etc.

The use of more complex models of time series does not necessarily lead to better results in forecasting (Axseter 2006). The reason for this is the significant influence of random component e_t in the above dependencies. With large scatter of e_t the simpler models can lead to even better results than more complex ones.

Any forecast is usually prepared in advance for several specific forecast periods. The time interval, for which the forecast is prepared, is called a forecast interval. The number of the periods, for which a one-time forecast is made, is a forecasting horizon. For example, if the forecast interval is 1 month, the forecasting horizon is three, and if the forecast is made at the end of December, the forecast period is January, February, and March of the next year.

At the end of forecast period t, for each forecast it is possible to define its deviation value M_t from the actual demand value for this period, which can be evaluated more conveniently in percentage of the actual demand

$$M_t = 100\frac{D_t^{'} - D_t}{D_t}\%, \tag{6.8}$$

where $D_t^{'}$ is the forecast value and D_t is the actual demand value.

The quality of forecasting is determined by two main indices—mean forecast error for n periods

$$\overline{M} = 100\frac{\sum_{t=1}^{n}\left(D_t^{'} - D_t\right)/D_t}{n}\% \tag{6.9}$$

and mean absolute forecast error

$$\overline{M}_a = 100\frac{\sum_{t=1}^{n}\left|D_t^{'} - D_t\right|/D_t}{n}\%. \tag{6.10}$$

Assume that in April it appears that forecast errors for January, February, and March have made −20 %, +15 % and −7 %, accordingly. In this case $\overline{M} = -4\%$ and $\overline{M}_a = -14\%$. Mean absolute forecast error $\overline{M}_a$ describes the range of possible deviations from the demand while mean error $\overline{M}$ defines how much the forecast trend itself deviates from the demand trend.

6.2 Main Methods of Forecasting

All of the following methods are based on the use of historical data on supply and extrapolate the found tendencies of the demand to a relatively small forecasting horizon (up to a year).

6.2.1 Moving Average Method

In the cases where the available time series data due to the large random or periodic fluctuations make it difficult to identify the process trend, it is possible to use the moving-average method to determine the trend. This method involves the replacement of the actual time series values with the calculated values, which have much smaller fluctuations than the original. In this method, we calculate average values of the variable data based on groups of data for a certain period of time, and each subsequent group shifts against the previous one by one period (month, year, etc.). When forecasting, it is assumed that the demand value in the coming accounting period will be equal to the average demand calculated for the last time interval elapsed.

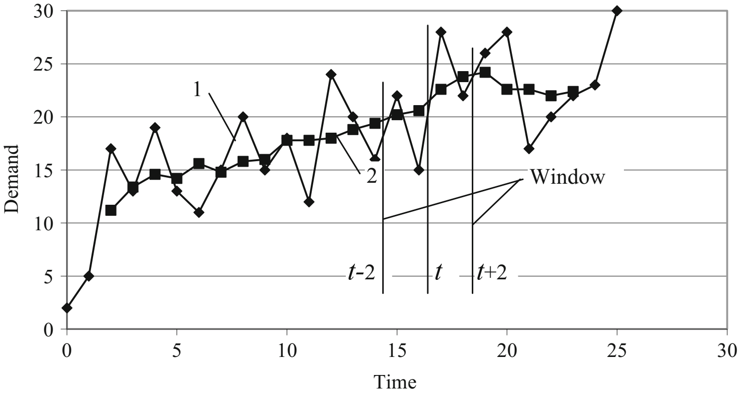

Fig. 6.1 Graph of demand trend in time. 1—actual demand; 2—demand calculated by moving-average method

The time interval, within which the demand values are averaged, is called a "window". To construct the estimate of the demand value at time *t* we set the width of the window determined by quantity *m* of the considered time intervals towards each side of time *t* (Fig. 6.1).

Graph 1 in Fig. 6.1 displays the variance of actual demand D_t within 25 days and the interval of each observation was 1 day. Based on these data the averaged values of the demand are determined by formula

$$D_t^{'} = \frac{1}{2m+1}\sum_{i=t-m}^{t+m} D_i, \tag{6.11}$$

where it was accepted that $m = 2$ of the observation interval. Thus, the observation window covered five values of demand within interval $t - 2$ to $t + 2$. As can be seen from the graphs, the application of the moving-average method leads to strong smoothing of fluctuations in demand and reveals the trend of demand. In this case, it is obvious that the trend is the gradual increase in demand.

The described method has a significant disadvantage in that it cannot be directly used to calculate the demand values at the extreme points of the observation interval. Therefore, in this example, for the points up to the third interval and the last two intervals the averaged values were not calculated. At the same time, since the purpose of any forecasting method is to define the probable demand in the near future, it is necessary to define the method of calculating the expected demand value at the time of last observation.

For this purpose, formula (6.11) applies the *m* wide asymmetric calculation window, i.e.

$$D'_{t+1} = \frac{1}{m+1} \sum_{i=t-m}^{t} D_i. \tag{6.12}$$

For example, in this case, with $m = 4$ and $t = 25$

$$D'_{26} = \frac{(D_{21} + D_{22} + D_{23} + D_{24} + D_{25})}{4+1} = 22.4.$$

6.2.2 Exponentially Smoothing Forecasting

In forecasting, the influence of actual demand data on the forecast value should obviously decrease with increasing distance of the data from the current time. One of simple but widely used methods that take into account this data "ageing" is the method of exponential smoothing. In this method, exponential average D'_{t+1} is constructed similarly to asymmetric moving average, but takes into account the data ageing degree. Moreover, each new forecast is based on the account of the previous forecast and its deviation from the actual value, namely

$$D'_{t+1} = \alpha D_t + (1-\alpha) D'_t, \quad 0 \le \alpha \le 1, \tag{6.13}$$

where α is the smoothing parameter defining the weight of the last actual observation D_t when preparing the forecast for the next time interval. The more influence the last observed value has in the forecasting model, the higher parameter α should be selected.

Value D'_t in expression (6.13) is the value of estimated forecast, which is made at the previous stage. Therefore, to use formula (6.13), it is necessary to find value D'_t at some initial point of calculation. Usually, in capacity of this initial value D'_t the actual demand value at the first point of the period in question is taken.

For example, pursuant to the graph of actual demand in Fig. 6.1, at time $t = 0$ $D'_0 = D_0 = 2$. Then

$$\begin{gathered} D'_1 = 0.3D_0 + (1-0.3)D'_0 = 2; \\ D'_2 = 0.3D_1 + (1-0.3)D'_1 = 2.9; \\ \ldots. \\ D'_{26} = 0.3D_{25} + (1-0.3)D'_{25} = 24.4. \end{gathered}$$

Figure 6.2 presents the graph of actual variation of the demand coinciding with the graph in Fig. 6.1, as well as two graphs built by formula (6.13) with two different values of α.

From the graphs in Fig. 6.2, it is clear that the forecast for time $t = 26$ according to the exponential smoothing method with $\alpha = 0.3$ is 24.4, while with $\alpha = 0.6$ is 26.9. Both of these forecasts have error probability, which can be estimated by

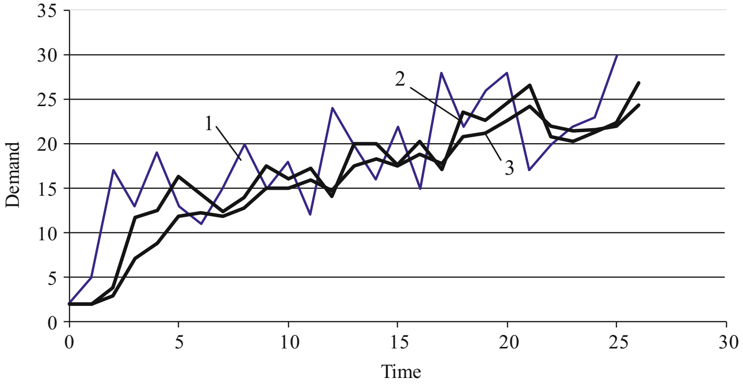

Fig. 6.2 Graph of demand trend in time. 1—actual demand; 2—smoothed demand with $\alpha = 0.6$; 3—smoothed demand with $\alpha = 0.3$

various criteria. For example, Lukinsky (2007) described the error estimation method in detail for such a forecast using the so-called Student's test.

6.2.3 Trend Adjusted Exponential Smoothing

Practice shows that the exponential smoothing method gives a significant forecast error in the case of monotone increasing or decreasing in demand. Indeed, if we compare graphs 2 and 3 of smoothed demand with graph 1 of actual demand in Fig. 6.2, we can easily notice that the smoothed values lag behind in time significantly from the demand trend. To address this disadvantage, forecast value D'_{t+1} can be divided into two parts, the first of which a_t reflects the smoothed values of demand fluctuations, and the second b_t reflects the influence of the trend. This technique is called Holt method and is described by the following equations:

$$a_t = \alpha D_t + (1 - \alpha)(a_{t-1} + b_{t-1}), \quad 0 \leq \alpha \leq 1; \tag{6.14}$$

$$b_t = \beta(a_t - a_{t-1}) + (1 - \beta)b_{t-1}, \quad 0 \leq \beta \leq 1; \tag{6.15}$$

$$D'_{t+1} = a_t + b_t \qquad \text{at } t \leq t_e \tag{6.16}$$

and

$$D'_{t_e+k} = a_{t_e} + b_{t_e}k \qquad \text{at } k \geq 1, \tag{6.17}$$

where t_e is the period of the last actual observation and k is the number of the forecasted period after period t_e.

Since the Holt's method explicitly determines the value of the demand trend b_t, then, in contrast to the method of exponential smoothing, here there is a possibility to make forecast not only for one time period but also for the next few periods. Pursuant to formula (6.17) for the period with number k after the last period t_e, of which there is actual observation, the value of forecasted demand D'_{t_e+k} changes at the rate determined by trend b_t. At the same time, when the smoothed values of demand before period t_e are calculated it may be possible to determine D'_{t+1} based on current values a_t and b_t (Eq. 6.16).

Smoothing parameter values α and β determine the response rate of the smoothed curve of demand to actual demand trend. The larger values of these parameters are, the faster the Holt's method responds to demand deviations. In order to use the equations (6.14–6.17), it is necessary to set the values of parameters at some initial point of calculation, similar to exponential smoothing method. It is easier to assume that initial value a_t is equal to the actual value of demand at the first point of the period in question and initial value $b_t = 0$. For example, according to the actual demand graph's data in Fig. 6.1, at time $t = 0$ $a_0 = D_0 = 2$ and $b_0 = 0$ and, accordingly, $D'_1 = 2$. Then with $\alpha = 0.6$ and $\beta = 0.3$
$a_1 = 0.6D_1 + (1 - 0.6)(a_0 + b_0) = 3.8;$ $b_1 = 0.3(a_1 - a_0) + (1 - 0.3)b_0 = 0.54$ and according to Eq. (6.16):

$$D'_2 = a_1 + b_1 = 4.34.$$
$$\ldots$$

$a_{25} = 0.6D_{25} + (1 - 0.6)(a_{24} + b_{24}) = 26.84;$ $b_{25} = 0.3(a_{25} - a_{24}) + (1 - 0.3)b_{24} = 1.44$ and the first forecasted value by formula (6.16):

$$D'_{26} = a_{25} + b_{25} = 28.28.$$
$$\ldots$$

The last (3 days later) forecasted value by formula (6.17)

$$D'_{28} = a_{25} + b_{25} \times 3 = 31.17.$$

Figure 6.3 shows graph 1 of actual demand trend coinciding with the graph in Fig. 6.1 and graph 2 built by formulas (6.14–6.17).

From the comparison of the graphs in Figs. 6.2 and 6.3, it follows that the use of the Holt method gives smoothing results much closer to actual than the simple exponential smoothing method. In addition, the Holt method allowed forecasting at several points after the last observation, which took place at $t_e = 25$. The technique that allows to assign the smoothing parameters optimally, as well as to evaluate the forecast error in the Holt's method is described in detail in Lukinsky (2007).

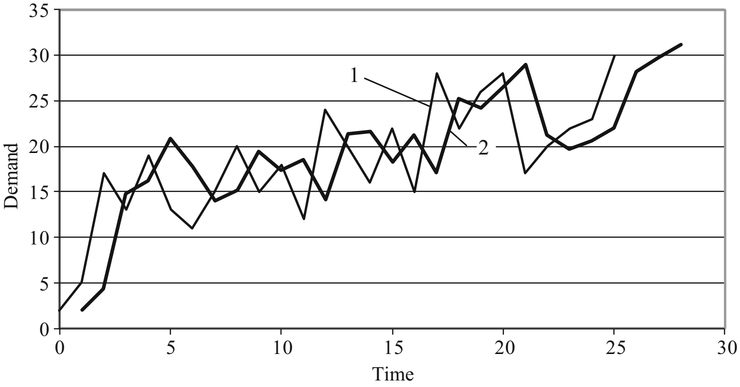

Fig. 6.3 Graph of demand trend in time. 1—actual demand; 2—demand calculated by the Holt's method

6.2.4 Trend and Seasonality Adjusted Exponential Smoothing

If actual data of demand observations reveal systematic fluctuations of the demand, then they indicate the presence of a seasonal component. The annual fluctuations with 12-month observation period repeat most often. Weekly fluctuations are also frequent. They give peaks of demand, such as on weekends.

In the examples of Figs. 6.1–6.3 regular fluctuations with period l equalling about 5 days of observation can be noticed. We assume that in capacity of observation days here only weekdays are used. In this case, we can believe that the demand curve here has a weekly seasonality with the peak falling within the middle of the week.

The most common method of forecasting taking into account the trend and seasonality is now considered to the method that uses the so-called the Winters model. The Winters model actually expands the Holt method to the case when there is seasonality. For this purpose, besides parameters α and β the third smoothing parameter γ is introduced in addition, and accordingly the amount of calculation equations increases:

$$a_t = \alpha \frac{D_t}{I_{t-l}} + (1-\alpha)(a_{t-1} + b_{t-1}), \quad 0 \le \alpha \le 1; \tag{6.18}$$

$$b_t = \beta(a_t - a_{t-1}) + (1-\beta)b_{t-1}, \quad 0 \le \beta \le 1; \tag{6.19}$$

$$I_t = I_0 \quad \text{at} \ \ t < l \tag{6.20}$$

and

$$I_t = \gamma \frac{D_t}{a_t} + (1-\gamma)I_{t-l} \text{ at } t \geq l, \quad 0 \leq \gamma \leq 1; \tag{6.21}$$

$$D'_{t+1} = (a_t + b_t)I_{t-l} \quad \text{at } t \leq t_e \tag{6.22}$$

and

$$D'_{t_e+k} = (a_{t_e} + b_{t_e}k)I_{t_e-l+k} \quad \text{at } k \geq 1. \tag{6.23}$$

To calculate, similarly to the previous paragraph, we assume that initial value a_t is equal to the demand value at the first point of the period in question, and initial value $b_t = 0$, i.e. in accordance with the data of the actual demand in Fig. 6.1 at time $t = 0$ $a_0 = D_0 = 2$ and $b_0 = 0$. Besides, we assume that initial value of seasonality index $I_0 = 1$ and accordingly from Eq. (6.22) we obtain $D'_1 = 2$.

Further with $\alpha = 0.6$, $\beta = 0.4$ and $\gamma = 0.3$ we have

$$a_1 = 0.6\frac{D_1}{1} + (1-0.6)(a_0 + b_0) = 3.8; \; b_1 = 0.4(a_1 - a_0) + (1-0.4)b_0 = 0.72.$$

Pursuant to Eq. (6.20) the first value $I_1 = 1$ and smoothed demand value $D'_2 = (a_1 + b_1)I_0 = 4.52$

$$\ldots$$
$$a_{25} = 0.6\frac{D_{25}}{I_{20}} + (1-0.6)(a_{24} + b_{24}) = 25.42;$$
$$b_{25} = 0.4(a_{25} - a_{24}) + (1-0.4)b_{24} = 1.85;$$

$I_{25} = 0.3\frac{D_{25}}{a_{25}} + (1-0.3)I_{20} = 1.09$ and first forecasted value according to Eq. (6.22):

$$D'_{26} = (a_{25} + b_{25})I_{20} = 27.06.$$
$$\ldots$$

The last (3 days later) forecasted value of demand in accordance with Eq. (6.23):

$$D'_{28} = (a_{25} + b_{25} \times 3)I_{23} = 31.9.$$

Figure 6.4 shows graph 1 of actual demand trend coinciding with the graph in Fig. 6.1, and graph 2, built by formulas (6.18–6.23).

Let us compare the graphs in Figs. 6.2–6.4. In this case, it turns out that the Winters method gives smoothing results quite close to the original actual data. It follows that the probability of correct forecast can increase significantly.

In the paper of Makridakis and Hibon (2000), the quality of forecasting for 21 different models was studied, and forecasts were tested on approximately 1000 time series. The results showed clearly that the use of more complex models has

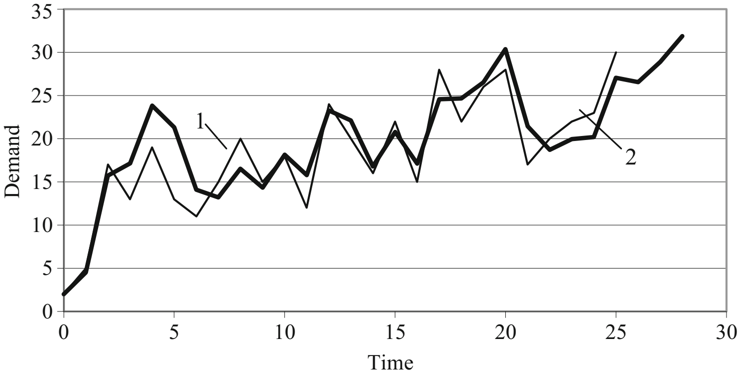

Fig. 6.4 Graph of demand trend in time. 1—actual demand; 2—demand calculated by the Winters' method

almost no advantage in forecasting and one can confine oneself to simple methods described above for most situations in planning.

6.3 Demand Aggregation

In different types of medium-term planning, for example, when planning sales and operations the forecast of demand for different commodity items aggregates (Table 1.3). Aggregation can be carried out according to the equipment, on which the items are manufactured, consumer groups, point of production or consumption, etc. As a rule, the forecast errors are significantly reduced with aggregation because, for example, the decreased demand for one of the items can be compensated by the increased demand for other items from the aggregation group.

The forecast, made by any of the methods described above, for an aggregate group of several items may differ from the sum of forecasts made using the same (or other) methods for each of the items. At the same time, at the same enterprise, it is desirable to have a line-up of various levels for commodity items and their aggregation groups. In order to achieve such consistency, the so-called pyramidal forecasting and planning technique was developed in the paper of Newberry and Bhame (1981).

Figure 6.5 shows the example of the aggregation pyramid that has three levels. Assume in the case that there are two production lines X and Z, on the first of which products X_1 and X_2 are manufactured, and on the second four products Z_1, Z_2, Z_3, Z_4. On the bottom, the third level, the forecasts are made for each of the manufactured products. The aggregation of the forecast values for each product taking into account its price defines the aggregate forecast of the total products and average price on each line. In particular, for line X the aggregate forecast makes up 12,000 units, and the price is $21.7.

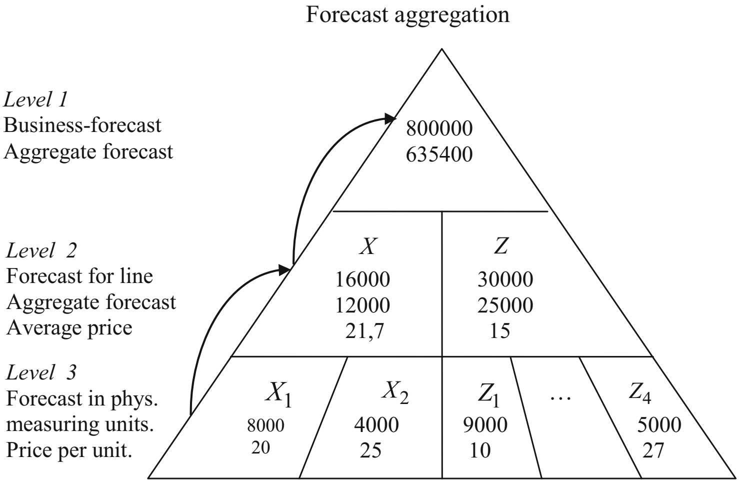

Fig. 6.5 Forecast aggregation pyramid bottom–up

Although the forecast based on the results of the production analysis on this line for the previous periods and equal to 16,000 units may differ from the aggregate forecast for all product of the lines, the technique of Newberry and Bhame (1981) recommend to use aggregated data of forecasts for further calculations for each product, as they are more accurate. The data of the aggregate forecast for all production lines in terms of value cost are brought to Level 1. These data are compared with the business forecast determined on the basis of market research and production development plans, which in this example has value 800,000.

Data analysis of the business forecast and the aggregate forecast for sales of each product allows to set the control point of the production plan in terms of value cost. As can be seen from Fig. 6.6, this value is accepted in amount of 720,000.

At the next stage of planning, the value is planned in physical terms and distributed between the production lines in proportion to their forecasted contribution, namely:

$$\begin{aligned}&\text{for line } X: && \tfrac{720000}{635400} \times 12000 = 13600 \text{ units;}\\ &\text{for line } Z: && \tfrac{720000}{635400} \times 25000 = 28320 \text{ units.}\end{aligned}$$

For each of the products the plan is also set in physical terms, for example:

$$\text{for product } X_1: \quad \frac{13600}{12000} \times 8000 = 9070 \text{ units}$$

$$\dots$$

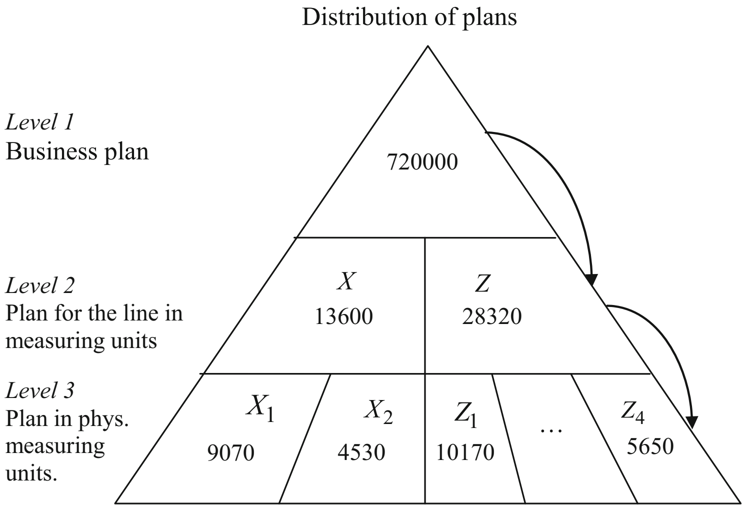

Fig. 6.6 Plan distribution pyramid top–down

. . .

for product Z_4 : $\frac{28320}{25000} \times 5000 = 5650$ units

6.4 Aggregated Demand Forecasting

As mentioned above, the aggregation of demand allows to improve accuracy of forecasting as it relates both to product groups aggregation and to aggregation in time. In the first case, the forecast errors reduce because different products of one group are frequently found in various phases of the life cycle—quadrants in Fig. 1.10, and the demand for them varies in different directions therefore. In this situation, the demand trends for different products can be mutually compensated, which facilitates forecasting groupwise.

Increasing of the forecast interval duration smoothens random fluctuations of demand significantly and simplifies forecasting as well. When forecasting with a sufficiently large forecast interval, it is possible, especially for groups of products, first consistently to single out seasonality and trend from historically accumulated sales data, and to make a forecast for subsequent periods based on this information.

Let us consider this forecasting technique (Piasecki 2009) by the example of the data set on the sales of a certain group products within 3 years. Table 6.1 shows initial data on demand D_{ti} by months with number t, the aggregate values for three i-th years in question and calculated seasonality indices of each month.

The value of seasonality index in month t for this case is defined from expression

Table 6.1 Calculation of seasonality indices

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Year 1	830	780	800	970	1200	1050	1010	990	940	890	840	760
Year 2	940	800	860	980	1330	1100	1300	1250	1010	950	890	820
Year 3	920	830	870	990	1420	1210	1420	1360	1120	1020	950	870
Total	2690	2410	2530	2940	3950	3360	3730	3600	3070	2860	2680	2450
Index	0.89	0.8	0.84	0.97	1.31	1.11	1.23	1.19	1.02	0.95	0.89	0.81

Table 6.2 Calculation of demand trend

Month	Demand	Seasonality index	Normalized demand	Smoothed normalized demand	Trend	Smoothed trend
January	920	0.89	1034			
February	830	0.8	1038	1034		
March	870	0.84	1036	1035	1	1
April	990	0.97	1021	1036	1	0.5
May	1420	1.31	1084	1030	−6	−3
June	1210	1.11	1090	1051	21	12
July	1420	1.23	1154	1067	16	14
August	1360	1.19	1143	1102	35	27
September	1120	1.02	1098	1118	16	20
October	1020	0.95	1074	1110	−8	3.3
November	950	0.89	1067	1096	−14	−7
December	870	0.81	1074	1084	−12	−10
January	940	0.89	1056	1080	−4	−6
February	880	0.8	1100	1071	−9	−7.8
March		0.84		1082	11	3.7

$$I_t = 12\sum_{i=1}^{3} D_{ti} / \sum_{i=1}^{3}\sum_{t=1}^{12} D_{ti}, \tag{6.24}$$

i.e. as the ration of the aggregate value of the 3-year demand falling within the relevant month to the average of all-months value of total demand for 3 years. For example, in April $t = 4$ and $I_4 = 12 \times 2940/36270 = 0.97$.

Note, then in accordance with condition (Eq. 6.5)

$$\sum_{t=1}^{12} I_t = 0.89 + 0.8 + 0.84 + 0.97 + 1.31 + 1.11 + 1.23 + 1.19 + 1.02 + 0.95 + 0.89 + 0.81 = 12.$$

Let forecast to be made for March of year 4 based on results of the current demand, including the data for January and February of this year. To determine the current trend let us consider the trend of demand over the past year 3 and over the past 2 months of this year (Table 6.2).

The value of the normalized demand in Table 6.2 is determined by dividing the demand by appropriate seasonality index. For example, in January of this year, the normalized demand is 940/0.89 = 1056.

We proceed to making the forecast in March of this year. To do this, first of all, we assume that the seasonality index in the current year will be maintained at the level of the previous year and, accordingly, will be 0.84. To determine the trend the random fluctuations of normalized demand are removed by the method of

Table 6.3 Demand forecasting for 3 months

Month	Demand	Seasonality index	Smoothed normalized demand	Smoothed trend	Forecast ignoring seasonality	Forecast
January	920	0.89				
February	830	0.8	1034			
March	870	0.84	1035	1	1038	872
April	990	0.97	1036	0.5	1037	1006
May	1420	1.31	1030	−3	1021	1338
June	1210	1.11	1051	12	1081	1199
July	1420	1.23	1067	14	1102	1355
August	1360	1.19	1102	27	1168	1390
September	1120	1.02	1118	20	1169	1193
October	1020	0.95	1110	3.3	1119	1063
November	950	0.89	1096	−7	1077	959
December	870	0.81	1084	−10	1060	859
January	940	0.89	1080	−6	1064	947
February	880	0.8	1071	−7.8	1050	840
March		0.84	1082	3.7	1092	917
April		0.97		3.7	1096	1063
May		1.31		3.7	1100	1440

exponential smoothing with smoothing factor equal to $\alpha = 0.4$. In this case, for example, the value of the smoothed normalized demand for March of the current year according to formula (6.13) is $\widetilde{D}_3 = 1100 \times 0.4 + 1071 \times (1 - 0.4) = 1082$.

The value of the trend for each forecast period b_t is defined as the difference between the values of the smoothed demand in the forecast and previous periods. Thus, in March $b_3 = 1082 - 1071 = 11$. In the book of Piasecki (2009), it is proposed to smoothen with smoothing factor equal to $\alpha_1 = 0.6$ the obtained values of the trend. For this purpose formula (6.13) is also used, which uses the current trend value and the previous value of smoothed trend to calculate the value of the smoothed trend, and at the beginning of the calculation, the value of the smoothed trend is considered to be equal to the value of the trend.

In this case, the first calculated value of the smoothed trend is taken in March of last year and it is equal to the trend, i.e. 1. In the subsequent periods, the smoothed trend value is calculated as mentioned above—for example, in March of this year it amounts to $\widetilde{b}_3 = 11 \times 0.6 + (-7.8) \times (1 - 0.6) = 3.7$

Based on the results shown in Table 6.2, we make a demand forecast for March of this year as well as a preliminary forecast for April and May (Table 6.3). We will take seasonality indices for April and May the same as for the corresponding period last year, while we keep the value of the smoothed trend in April and May the same as in March.

Forecast value ignoring seasonality D_t^* in each past month t, as well as in upcoming March of this year, is defined by expression (Piasecki 2009):

$$D_t^* = \widetilde{D}_t + \widetilde{b}_t/\alpha, \tag{6.25}$$

and the total seasonality-adjusted forecast is

$$D_t^{'} = D_t^* I_t, \tag{6.26}$$

where $\widetilde{D}_t$ as above is the smoothed normalized demand, $\widetilde{b}_t$ is the smoothed trend, I_t is the seasonality index, and α is the smoothing coefficient of normalized demand. In this case for March of this year, we have $D_3^* = 1082 + 3.7/0.4 = 1092$ and total forecast value $D_3^{'} = 1092 \times 0.84 = 917$.

For an approximate forecast for the subsequent months, it is proposed in Piasecki (2009) to define a new forecast without taking into account seasonality by simply adding the trend, namely:

$$D_t^* = D_{t-1}^* + \widetilde{b}_t. \tag{6.27}$$

For example, for April we will obtain $D_4^* = 1092 + 3.7 \approx 1096$ and the total seasonality-adjusted forecast is $D_4^{'} = 1096 \times 0.97 = 1063$.

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Examples of Advanced Planning Models 7

7.1 Joint Operation Model of APS System and ERP System from SAP R/3

The main joint operation of various APS systems and R/3 system is the diagram of Stadtler and Kilger (2008) shown in Fig. 7.1. This diagram reflects planning tasks in supply chains (SC) and has the form of so-called Operations—Time horizons matrix.

Pursuant to Fig. 7.1 planning modules shall provide elaboration of long-term tasks, mid-term tasks and short-term schedules for the four operations—procurement, production, distribution, and sales. It is necessary to note the difference in understanding the master planning in the diagram in Fig. 7.1 and the common notion of master plan in ERP systems. In fact, the master planning in Fig. 7.1 refers to development of sales and operations plan of ERP systems (Table 1.3). Accordingly, the notion of production planning in Fig. 7.1 is equivalent to the master plan and the material requirement plan in the ERP system, and the scheduling is equivalent to operational planning.

This matrix does not show directly which components of the planning should be performed within the ERP system and which should be assigned to the APS system or any other system. As a rule, the developers of various APS systems select some part from this diagram, develop the software for it and interface it with the basic ERP system. In this case, it is inevitable that there is duplication of functions from one side and planning objectives may be unachieved from the other side.

Typically, APS systems of this type are designed to work with SAP R/3 system. The considerable experience of joint operation of such complex systems in various industries has led to the widely spread belief that it is this way of advanced planning that is at least the core way, if not essential.

Above in Sect. 1.4, the place of advanced planning in the enterprise information system was discussed in detail, from which it follows that the modern planning can be applied in a variety of options including the model considered in this section. The reason for this widespread application of this method, in our opinion, is that the

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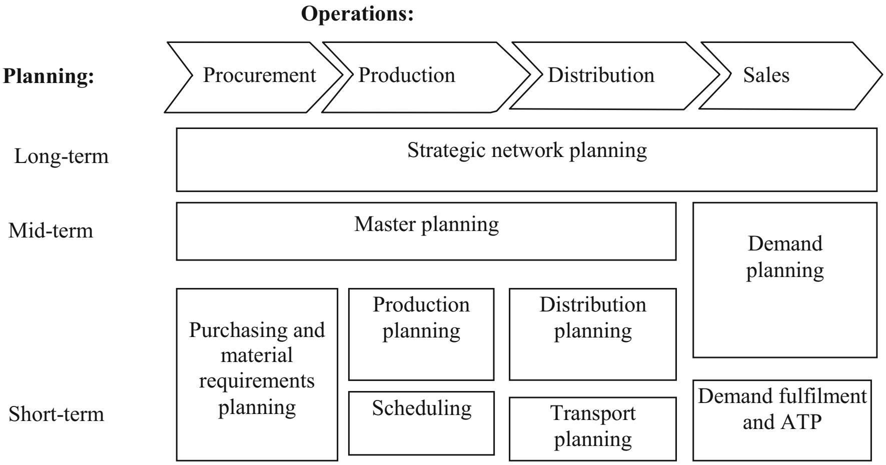

Fig. 7.1 Planning modules for SC matrix [based on Stadtler and Kilger (2008)]

developers need to apply the new methods of planning somehow without significant modifications in the old ERP systems. Let us consider three examples of implementation of such models.

7.1.1 Main Business Process Attributes in Various Industries

Earlier in Sect. 1.2.1, it was noted that every business process of SC could be characterized by a set of functional and structural attributes. Table 7.1 shows the values of the functional attributes for enterprises of three different industries (Stadtler and Kilger 2008).

Comparison of the functional attributes of enterprises in Table 7.1 reveals significant differences in the nature of their activities. First of all, it refers to the nomenclature (attribute 1) and shelf life of incoming materials and components (attribute 3). The computer build company uses a much larger nomenclature of incoming components than the other two companies do in Table 7.1, and at the same time, the shelf life of these components is much shorter.

In addition, the computer build company is much more dependent on customers. Indeed, the dispatch (attribute 9) in this case is performed when the orders are processed, and thus the production lot size (attribute 5) can vary. Besides, there is an opportunity here to vary products' configuration according to the customer's special requirements (attribute 15). As a result, the life cycle of products of this company (attribute 13) is much shorter than for other comparable companies.

The computer build company has the configurable flow-line production (attribute 4) of type 3b (Sect. 1.2.1), focused on the production of core products. In addition, the individual computer units are assembled at cell manufacturing sites

Table 7.1 Functional attributes influencing the business process of planning

	Attribute value		
Type of attribute	Pharmaceutical company	Computer build company	Oil refinery
1. Nomenclature of incoming materials and raw materials	Small (up to 50)	Large (over 1000)	Medium (up to 200 types)
2. Duration and reliability of delivery	Long, reliable	Long, low reliability	Long, low reliability
3. Shelf life of materials and components	Long	Short	Long
4. Type of production	Flow-line	Flow-line and cell manufacturing	Flow-line
5. Size of lot	Fixed	Variable	Fixed
6. Production bottle-neck	Known	Non	Known
7. Variation of labour time	Widely used	Rarely used	Rarely used
8. Number of processing stages	2	2–3	2–3
9. Frequency of dispatch	Constant frequency	By orders	Mixed
10. Transportation facilities	Selected according to the lowest cost	Set in orders	Standard
11. Sales calculation	By forecasting	By forecasting and orders	By forecasting and contracts
12. Demand fluctuations	Seasonal for medicines	Days of week	Quarters
13. Duration of stable output	Several years	Several months	Years
14. Number of standard modifications (package types)	Several	Many	Few
15. Availability of custom special properties	No	Yes	No
16. Material flow	Separation of raw materials and blending	Components assembly	Separation of raw materials and blending

(type 4). The other two companies operate with configurable flow-line production of type 3c, where it is possible to produce by-products in parallel with the core products. This is a large-lot production (Table 1.2), i.e. it has a quite large scale.

With these scales, the optimization of Sales and Operations Plan by the maximum profit criterion (Sect. 2.2.3) is of prime importance. In addition, at the level of production planning it is important to achieve the best possible values on basic criteria of the process. According to Table 2.6, the determining criterion of the master plan optimization for production type 3b is K1 criterion of minimal direct costs (Table 2.3), and for production type 3c it is K2 criterion of efficient use of raw materials. The use of APS systems for all enterprises in this example allows optimizing plans according to these criteria.

In Sect. 1.2.1, it is mentioned that the reference model of planning is influenced also by the so-called structural attributes in addition to functional ones. These attributes define the structure of the network, its globalization indices, bottleneck positions in the network, the capacity balance of the individual components in the chain, types of information in the interchange, etc. For all three companies under consideration, these figures differ little. Suffice it to say that each of the companies has a quite well-balanced production capacity, and their products are distributed world-wide.

7.1.2 Software Modules for Planning Solutions

For each of the three companies under consideration, a specific version of the software in the form of a set of software modules was selected. The modules allow solving the planning problems in a particular work situation (Table 7.2). Since the problem of perspective planning in the described systems was not addressed, there are no modules for long-term planning in Table 7.2.

Pursuant to Table 7.2 in all three cases, the special APS system is used for the actual planning (the first three lines of Table 7.2), but of different type in each case.

The simplest case of the considered type model is a combination of SAP R/3 with APS system from the same company SAP APO, which is used for planning in the pharmaceutical company. In this case, three modules of SAP APO are used: Supply Network Planning (SNP), Production Planning/Detailed Scheduling (PP/DS), and Demand Planning (DP). The rest of the planning functions are left for R/3 system.

Table 7.2 Software modules for planning

	Module type		
Module function	Pharmaceutical company	Computer build company	Oil refinery
Master planning (sales and operations plan)	SAP APO SNP	i2 SCP	ASPEN XPIMS
Production planning	SAP APO PP/DS	i2 FP	ASPEN PPIMS
Detailed scheduling	SAP APO PP/DS	i2 FP	ASPEN PIMS-SX
Purchasing and material requirements planning	SAP R/3	SAP R/3	ASPEN CSS
Products distribution planning	SAP R/3	SAP R/3	SAP APO TP/VS
Transport planning	SAP R/3	SAP R/3	SAP APO TLB
Demand planning	SAP APO DP	i2 DP	SAP APO DP
Demand fulfilment and available-to-promise	SAP R/3	i2 DF	SAP R/3

A slightly more complicated version is used at the computer build company. Here the functions of production and demand planning are performed by the following modules: Supply Chain Planner (SCP), Factory Planner (FP), and Demand Planning (DP). In addition, the APS system type i2 Technologies is assigned with the function of Demand Fulfilment (DF).

Application of i2 Technologies system is associated here with a very large scope of production planning and the need for frequent changes in planning. Indeed, due to the considerable amount of incoming components and their rapid obsolescence, the need to take into account the needs of different customers, it requires high-speed operation of the planning system, which is provided by i2 Technologies system due to so-called special fast platform (i2 Agile Business Process Platform) provided.

The most complex system is used in the example of planning for the oil refinery. In this case, for the purpose of production planning PIMS (Process Industry Modelling System) software from ASPEN is used. It is designed specially for the oil industry. This software allows optimizing the raw materials and semi-finished products cleaning and blending processes, maintaining their balance and optimizing costs. The optimization is carried out in the framework of both linear and non-linear programming with powerful computing programs ILOG CPLEX from IBM and XPRESS from Dash Optimization.

For periodic planning (scheduling), PPIMS-SX version of PIMS software is used. It allows for scheduled payments with the horizon of 36 weeks. XPIMS product is an add-on of PIMS, providing planning optimization of joint operation of several enterprises that are located geographically in different places. CSS (Crude Scheduling System) module based on PIMS module is used for crude oil procurement planning.

In this example, for the distribution and transportation planning, as well as demand planning SAP APO modules are used, despite the fact that the relevant modules are also available in dedicated system ASPEN. Perhaps, this is due to the history of the information system implementation at the enterprise. The role of SAP R/3 system in the planning is reduced to the demand (order) fulfilment in this case.

7.1.3 Planning Modules Interaction

Each cycle of planning in any of the above variant of planning starts with demand planning. The activity on the market of each company in this example has its own peculiarities. In particular, the pharmaceutical company operates mainly by contracts with a lead-time horizon of 9–12 weeks in average. The level of demand for oil refinery products and supplies of crude oil for it are determined monthly within some possible limits. In addition, the price limits for raw materials and finished products are forecasted. The demand at the computer build company is distinguished by rapid changes resulting in market analysis to be carried out either weekly or even twice a week.

Based on the forecasts made for sales, the so-called master planning is performed, which is actually the development of sales and operations plan. This plan is the basis for all further planning process and for the enterprises of different industries, the sales plan horizon differs significantly. For example, at the computer build company, the master planning is performed every week with the planning horizon equalling the longest expected delivery time for components. At the oil refinery, the sales plan can be developed simultaneously for different horizons, 3–36 months. In this case, the basic long-term plan is adjusted periodically by drawing up short-term plans. The pharmaceutical company draws up sales plans with the 24-months horizon. This company extensively uses vendor managed inventory (VMI), which allows fulfilling almost all their possible demand.

Based on the sales plan, optimal (including price, speed of delivery, reliability of supply, etc.) plans for raw material orders are developed. For example, the computer build company makes or corrects this plan every business day and the set of ordered parts and materials meet the demand rather strictly.

The pharmaceutical company has the ability to obtain raw materials from a number of suppliers in different countries. This allows optimizing the cost of raw materials taking into account the peculiarities of the tax legislation of these countries. In addition, it is possible to vary the supply of raw material kinds and consequently to achieve contractual product composition by adjusting production processes.

A similar situation occurs with crude oil supplies. Furthermore, in this case the choice of oil grade, the possibility of its transportation within the ordering period and the cost of such transportation is of great importance. The possible differences in finished product costs in different geographical markets, etc., must also be borne in mind. The fundamental specificity of operation with oil led to the necessity to develop the above-mentioned software CSS, which takes into account most of the factors that affect the optimal planning of crude oil procurement.

Production planning is based on the control-aggregated volumes of sales plan. First of all, the planning requires specifying of sales plan indicators for specific types of products, and secondly, availability of complete data on products and related processes. These data are transmitted from SAP R/3 system to production planning modules in the form of so-called Production Process Model (PPM) (see Sect. 5.1.1).

For the pharmaceutical company, this model is a master recipe of the corresponding version and the composition of manufactured products. Since the process does not change during computer building, the production process model is simplified here and contains only the configuration of a product of each standard size. In oil processing, on the contrary, the composition of manufactured products varies little, but for each kind of raw material the relevant parameters are set in the production process model.

At the stage of finished products dispatch to consumers, optimization issues do usually not arise. Therefore, both the pharmaceutical company and the computer company use mainly the modules of basic ERP system R/3 for this purpose. However, for the oil refinery the optimization issues of dispatch are as important as of obtaining raw materials. Well-formed lots according to their destination can significantly reduce the cost of delivery. From Table 7.2 we see that for this purpose the optimal planning modules of SAP APO system are used, which are well coordinated with the basic R/3 system. In any case, the paperwork for dispatch is done by means of the ERP system.

7.2 Reference Model of Production Planning for Instrument Engineering Plant

This section describes the planning model developed in 2008–2010 by Moscow company Infosoft with the author's participation.

7.2.1 Initial Planning Status Analysis

The sequence of the reference model development primarily corresponded to the methodology outlined above in Sect. 1.2.2. According to this methodology, first the current state of the planning system (state AS-IS) was studied and the tool (BPwin system) was selected to develop the model. With the help of this tool a number of documents describing the information flows and decision flows in production planning were developed.

It was found that in the planning of core production, for most of the products (Production No. 1) the assembly-to-order planning method is used. Simultaneously, some products (Production No. 2) are produced as part of manufacturing strategy "make-to-stock". All kinds of products are made in the framework of push-type production. Production of the core products is characterized by high instability, where the flow of changes in design and technological documentation, made by the change notices (Sect. 5.1.1), amounts up to several tens per week.

Within the planning system, three production plans are developed: sales and operation plan, master plan and production schedules. The purpose of the first of them is to define the annual production volumes with an outlook for 1–2 years, as well as verification of capacity utilization for a long period broken down by months.

The master plan is generated annually on monthly basis according to the output plan. In this case, from the output plan the orders, which to be fulfilled in the planning year, and the orders, which to be launched in the planning year, are selected. The due dates of works in the master plan are calculated from the date of output in the sales and operation plan back in time to the determination of launch dates.

Based on the master plan, the production site schedules are made with breakdown by day. The schedules are prepared from the launch date forward in time.

Under this model, similarly with Table 7.1, a table of functional attributes (Table 7.3) was made. Due to the fact that at the same enterprise there are actually two different types of production for the products for different applications with different scales and production strategies, the attributes are described individually for each of these productions.

Table 7.3 Functional attributes of the production sites of the instrument engineering plant

	Attribute value	
Attribute type	Production No. 1 (assembly-to-order)	Production No. 2 (make-to-stock)
1. Nomenclature of incoming materials and raw materials	Large (over 1000)	Small (less than 100)
2. Duration and reliability of delivery	Long, low reliability	Long, reliable
3. Shelf life of materials and components	Short	Long
4. Type of production	Multipurpose and cell manufacturing	Cell manufacturing
5. Size of lot	Variable	Fixed
6. Production bottle-neck	Known	Non
7. Documentation stability	Instable	Stable
8. Number of processing stages	7–10	2–3
9. Frequency of dispatch	By orders	By orders
10. Sales calculation	By orders	By forecast
11. Seasonal demand fluctuations	Non	By quarters
12. Duration of stable output	Several years	Several years
13. Number of standard modifications	Several	Many
14. Availability of custom special properties	Yes	No
15. Material flow	Processing and assembly	Processing and assembly

Let us compare the values of functional attributes for two existing production sites of this enterprise, as well as the values of these attributes with the similar attributes of the previous example in Sect. 7.1.1. Obviously, for both productions and especially for production No. 2 in Table 7.3, the closest example is the computer build company in Table 7.1. The values of the attributes of the materials used and the product life cycle performance are practically the same.

Production No. 2 is a set of cell manufacturing sites (type 4 in Sect. 1.2.1), similar to sub-assemblies at the computer build company. At the same time in production No. 1, the cell manufacturing is used only at the assembly stages and the parts machining is performed by multipurpose machines at the production sites of specific jobs.

In general, production No. 2 differs from the computer build company of the previous example only by much smaller production scale, which, however, is still serial. Production No. 1 distinguishes itself from production No. 2 not only by small-serial but also by high instability (attribute 7) caused by various factors and principally by frequent changes in documentation.

When making the description of AS-IS state, as recommended by methodology for a reference model (Sect. 1.2.2), the nature of information flow at the enterprise was studied, in particular its aspects such as the list of main inputs of a business process, transformation of information in business process, etc. It was found that the enterprise applied the ERP system of its own design, which provided logging of design documentation, accounting and data storage of material procurement, sales, accounts and finance departments. Archiving of orders in the system was reduced mainly to their registration.

The input information for scheduling, besides orders, is also the so-called card file that contains a list of products manufactured in the shop for current "active" version. Recall (Sect. 5.1.1) that active current version is the design documentation with regard to the change notices in effect at that moment. The card file also indicates the availability of products manufactured by various versions. The card file is corrected based on the design and technological change notices, and the results are used in performance of operations.

In accordance with the developed planning schedules and processes for each manufactured item of the plan, the route sheets are generated that accompany the products at all stages of production from the receipt of materials or workpieces until submitting to the quality control department. After delivery of good products the relevant checkoffs are made in the schedules, and in case of defective products the defect certificate is drawn specifying the reasons therefore. Good products are delivered to the warehouse and transferred against an invoice. The situation in the shop is regularly monitored and regulated by the planning and operating bureau (POB).

Analysis of the AS-IS state revealed a number of drawbacks in business process of production planning. First of all, the existence of a significant number of scattered throughout the enterprise, unrelated and poorly controlled orders leads to data uncertainty about the real state of production. In the master plan based on the annual output plan the job tasks for the shops have too aggregative nature, which causes their failure to be completed in time through many items. The production scheduling directly by the shops foremen is often subjective and does not contribute to high performance.

7.2.2 Decision Support Database

The results of the AS-IS state analysis within the reference model led to development of a number of suggestions and documents describing the planning business process for the TO-BE state. For the purpose of order management it was suggested to create a special planning authority allowing concentration of all the external and internal orders and ensure their registration and recording of actual fulfilment. It was suggested to establish plan types that are standard for ERP systems—long-term plan (sales and operations), master plan, material requirement plan and operational plan.

When developing the planning model for the TO-BE state both the enterprise's wishes and the basic principles of advanced planning were taken into account, namely (Sect. 1.4): use of flexible horizon, compulsory analysis of available information in order to support and optimize the decisions made, as well as consideration of various constraints, especially in equipment capacities, directly in the planning process.

In particular, it was proposed to make the master plan not once a year but review it regularly with some interval (a month) and corresponding shift in phase of accomplishment. In preparation of the material requirement plan it was suggested to consider the requirements of MRP as well as the intensity of the order in accordance with the definition of intensity in Sect. 2.4.1 in sequence of individual order fulfilment. Further, simultaneously the utilization ratio of individual, the most loaded work centers were tested according to the procedure described in Mauergauz (2007).

For planning with regard to its optimization elements, it was decided to use the opportunity to create a decision-support database (Sect. 5.1.4) directly within one active ERP system at the enterprise. This organization of the information system allowed not developing a special APS system but simply extending the functionality of the existing system.

A decision-support database is, as described above in Sect. 5.1.4, analytical basis of any information system. This basis, as has been said, can be arranged separately from the main transactional database and can be combined or even aligned with the latter. In this example, the database of the analysis system is designed as a special set of tables under the control of the same DBMS, which controls the main transactional database. Figure 7.2 shows the databases tables for decision-making in planning and their interaction pattern.

The central table of the database is table "Orders", the records of which can be of two types—agreements and production kits. In the first case, the orders are fulfilled for specific external consumers, in the second case the production kits are ordered for "make-to-stock" production and the subsequent sale. Each order can have a number of items, i.e. lines that contain information about a product, its quantity, composition, cost and so on, reflected in table "Order composition". The sequence of supplies for each order item is specified in table "Delivery schedule".

The individual configuration (composition) for each order item is selected or created from a set of options recorded in table "Order configuration". The objects of this configuration are the parts and assembly units (PAU), which may be included into other assembly units by different ways.

Each PAU object has its own design and technological history. Table "History of PAU" contains a list of various versions of design, and table "History of SPR" contains a list of versions of the summary process routes for each PAU object. Source documents (drawings, processes, change notices, etc.) related to each of the versions contained in the main (transactional) database are not shown here.

A set of PAU objects and codes of their current (active) versions for each configuration of the order is placed in a separate Production Process Model (PPM) as described above in Sect. 5.1.1. Each line of the delivery schedule has its own PPM.

After preparing the PPM and delivery schedules, it is possible to make planning decisions that are executed in the form of master plan. In this plan, each type of product (PAU code) corresponds to a certain number of planned items, each of

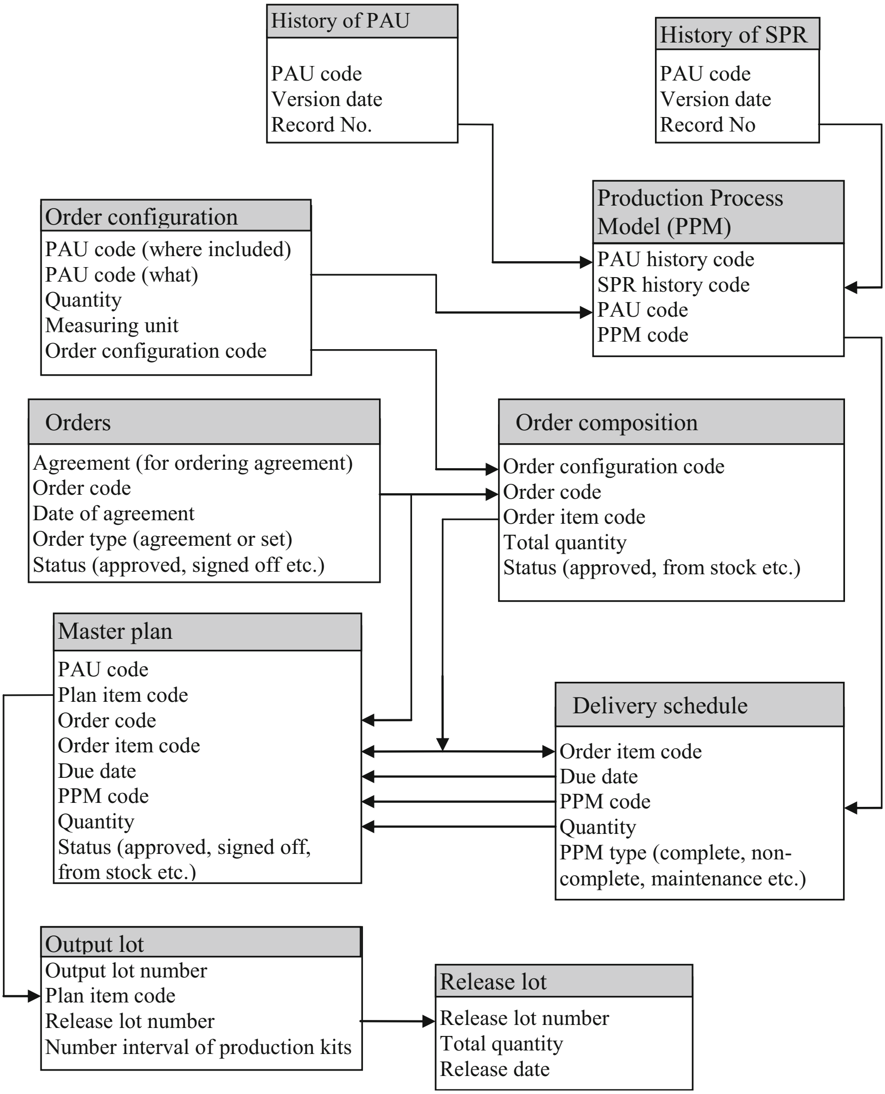

Fig. 7.2 Tables of the database for planning decision-making

which is designed to fulfil the orders according to the delivery schedule. The information about the PPM, due date and number of products received from the delivery schedule during development of the schedule is referenced and can be updated. During implementing the schedule the status of a scheduled item may vary depending on the production situation.

Several planning items usually with the same date of fulfilment are grouped into an output lot. In table "Output lot" each plan item is assigned with number interval of PAU sets, which then allows tracing the life cycle of each instance of the product.

The release of a new lot of finished product into production often makes sense to be carried out simultaneously for several output lots. This allows you to increase the size of production lots for many types of PAU, which are common components of different orders for finished products, including the processed ones according to different PPM. Therefore, in this example, special table "Release lot" is provided. This approach in planning database organization is described in more detail in Mauergauz (2007).

7.3 Mathematical Model in Chemical Industry

The described model was proposed in the article of Kondili et al. (1993) and until present has been the focus of attention of many studies by various authors, for example Ierapetritou and Floudas (1998) and others.

7.3.1 Analytical Structure of Model

Let us consider the structure of production shown in Fig. 7.3 representing the simplest variant of chemical production. According to the classification in Sect. 1.3.1, this structure is type 3c—repetitive serial production line that produces core products and by-products. In this model, there is a special piece of equipment for each manufacturing operation, and upon completion of an operation the product is discharged into an appropriate buffer tank. For different orders the line can be set up and manufacture products with different properties.

In accordance with the above terminology we will call each necessary process operation as a job or task with number *i*, and each process installation—a machine or unit with number *j*. In the example in Fig. 7.3, each unit is designed to perform only one type of tasks. During the manufacturing process the raw material fed to the line input is first converted into semi-product with various degree of finishing and then into the core or by-products. To perform each operation, a machine is loaded with some quantity of raw material or semi-products, and after the operation the obtained product lot is discharged into the relevant tank. We further assume that the

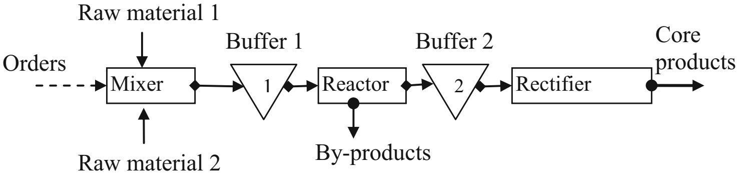

Fig. 7.3 The simplest structure of chemical production

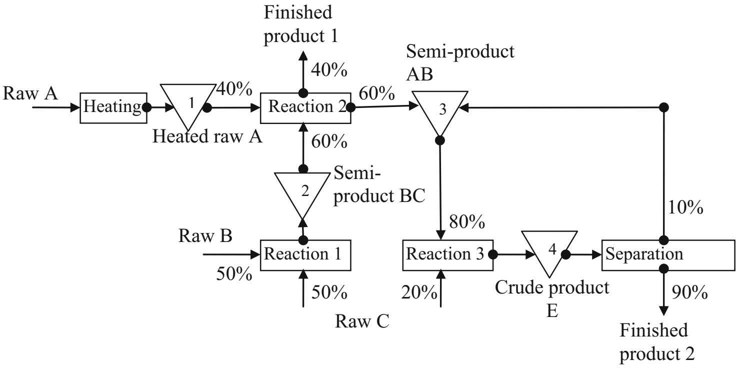

Fig. 7.4 Analytical structure of STN-model [based on Ierapetritou and Floudas (1998)]

initial raw materials, semi-finished products, stored in buffer tanks, as well as finished products represent a state of the processed product.

In the paper of Kondili et al. (1993), it was suggested to name the structure like shown in Fig. 7.3 the production state-task network (STN). Using the diagram of this structure it is not necessary to specify particular manufacturing equipment directly but just to enumerate all the states of the products undergoing the processing, as well as the jobs for transferring these products from the current state to the subsequent one. Figure 7.4 shows an example of an analytical structure of this type of model (Ierapetritou and Floudas 1998).

As can be seen from Fig. 7.4, in the result of manufacture two finished products should be received of three types of raw materials, and product 2 is a core product (received at the end of the manufacturing process), and product 1 is a by-product. For manufacturing operations, which involve several types of raw materials or semi-finished products, Fig. 7.4 shows the data on their percentage in an operation. After operation “reaction 2” some part of the resulting product is transferred for further processing and some part is the product ready for immediate sale. The wastes resulting from the separation are included in the reprocessing during “reaction 3”.

The main difference of the diagram in Fig. 7.4 from the previous diagram in Fig. 7.3 is that in the first one the number of processing units (machines) is not necessarily equal to the number of process operations, but may be not only greater than the number of operations but less than this number. It is obvious that if the number of machines for a particular operation is greater than one, it does not change the type of production 3b of the entire repetitive serial production line.

If the total number of machines of the production line is less than the number of operations, this means that one or more machines must provide performance of different production operations, i.e. according to the terminology introduced above, perform multiple tasks. In this example, each of three reactions in Fig. 7.4 can be performed on any of the two available reactor-machines, while for heating and

Table 7.4 Technological capabilities of the machines for model in Fig. 7.4 [based on Ierapetritou and Floudas (1998)]

Machine	Capacity	Operations	Process time
1. Heater	100	Heating	1.0
2. Reactor 1	50	Reaction 1, 2, 3	2.0; 2.0; 1.0
3. Reactor 2	80	Reaction 1, 2, 3	2.0; 2.0; 1.0
4. Separator	200	Separation	1.0—for product 2, 2.0—for recovery to semi-product AB

Table 7.5 Task variants

Operation	Machine	Task number
1. Heating	1. Heater	1
2. Reaction 1	2. Reactor 1	2
	3. Reactor 2	3
3. Reaction 2	2. Reactor 1	4
	3. Reactor 2	5
4. Reaction 3	2. Reactor 1	6
	3. Reactor 2	7
5. Separation	4. Separator	8

separation special units are used. The data about capabilities of these machines are shown in Table 7.4. The duration of process operations in Table 7.4 can vary up to 33 % from the adjusted mean value depending on the number of raw materials or semi-products loaded into the unit.

Obviously, in this situation, operations in reactors 1 and 2 must change. It is possible, for example, to select reactor 1 with the smaller volume for only one operation “reaction 1”, while for the other two reactions to reserve reactor 2 with larger volume, but in any case, at least one of the reactors have to be reconfigured to perform various operations. To solve this problem it is proposed that every possible variant of operation on a particular machine match an individual task (Table 7.5).

In the analytical structure of the model it is assumed that within the entire range of planning from the moment of performance start to the time horizon a limited number of event points is possible, at which each task event can occur. It is obvious that to start a particular task one of the machines should be started on which this task can be performed. Since the number of the tasks suitable for processing on a certain machine, in general is not equal to 1, at the time of unit event the various tasks events can start, and therefore these issues should be distinguished.

If task event i starts at possible point event n, the function of this event $w(i,n)=1$, and at other times this function equals 0. Similarly, if the machine with number j starts at possible event point n, then the function of this event $y(j,n)=1$, and at other times this function equals 0. The objective of the advanced planning in this case is first of all to calculate the sizes of lots $Q(s,n)$ for each product in state s at event point n and secondly to determine functions $w(i, n)$ and $y(j, n)$, with which the maximal productivity or profitability is achieved.

Given that functions $w(i, n)$ and $y(j, n)$ can take values equal only to 1 or 0, this model is an example of integer linear optimization with binary variables discussed above in Sect. 2.5.2.

7.3.2 Objective Function and Constraints

For the described model, the maximization of the overall value of production in all its states for the period from the beginning of planning the set horizon is usually taken for an objective function. In this model, the definition of the overall value for this period is replaced by summation of the value at all possible points of events, namely,

$$c = \sum_{s} \sum_{n} c(s) q(s, n), \tag{7.1}$$

where $c(s)$ is the unit price of the product in state s, and $q(s, n)$ is the quantity of the product to be put on sale in state s, at event point n.

Table 7.6 below shows the data on storage capacities for products in various states and relevant prices.

As shown in Table 7.6, in this model only finished products are considered as the value, consequently, in expression (7.1) there are two components for states 8 and 9 defined for all events.

The first constraint of the model is the constraint of performance of a variety of tasks on one machine

$$\sum_{i \in I_j} w(i, n) = y(j, n) \text{ for all } j \in J \text{ and all } n \in N. \tag{7.2}$$

In the left-hand side of Eq. (7.2) there is the sum of binary functions $w(i, n)$ for all jobs i possible to perform on machine j at time n. The equality sign in formula (7.2) means that at each time n for each machine j can be performed only one task i of set of tasks I_j that are possible for machine j.

Condition (7.2) should be met for all machines with number j, belonging to set J of this model.

Capacity constraints for each machine has the form:

$$V_{ij}^{\min} w(i, n) \leq Q(i, j, n) \leq V_{ij}^{\max} w(i, n) \text{ for all } i \in I, j \in J_i \text{ and } n \in N. \tag{7.3}$$

In expression (7.3) $V_{ij}^{\min}$ is the minimal required quantity of material (raw material, semi-product) for beginning of task i on machine j, $V_{ij}^{\max}$ is the maximal quantity of this material which can be loaded into machine j, $Q(i, j, n)$ is the lot size

Table 7.6 Product state parameters [based on Ierapetritou and Floudas (1998)]

State	Capacity	Initial quantity	Price
1. Raw material A	Unlimited	Unlimited	0.0
2. Raw material B	Unlimited	Unlimited	0.0
3. Raw material C	Unlimited	Unlimited	0.0
4. Heated raw material A	100	0.0	0.0
5. Semi-product AB	200	0.0	0.0
6. Semi-product BC	150	0.0	0.0
7. Crude product E	200	0.0	0.0
8. Finished product 1	Unlimited	0.0	10.0
9. Finished product 2	Unlimited	0.0	10.0

of materials loaded into machine j to fulfil task i at the moment of event n, I is the set of tasks, and J_i is the set of machines, suitable for fulfilling task i.

Values $Q(i,j,n)$, like functions $w(i,n)$, are defined when modelling. If $w(i,n) = 0$, then values $Q(i,j,n)$ also become equal to 0.

The following condition requires that the current quantity of product $B(s,n)$ in any state s does not exceed the limited capacity of buffer $B(s)^{\max}$, namely

$$B(s,n) \leq B(s)^{\max} \text{ for all } s \in S \text{ and } n \in N. \tag{7.4}$$

In expression (7.4), S is the set of possible states of product (Table 7.6).

The most important condition is to meet the mass balance

$$\begin{aligned} B(s,n) = B(s,n-1) - D(s,n) + \sum_{i \in I_s} \rho_{si}^{p} \sum_{j \in J_i} Q(i,j,n-1) \\ + \sum_{i \in I_s} \rho_{si}^{c} \sum_{j \in J_i} Q(i,j,n) \end{aligned} \tag{7.5}$$

for all $s \in S$ and $n \in N$.

In the condition of the mass balance, coefficient ρ_{si}^{p} is the product share in state s from any lot of the product, manufactured on all machines j, allowing to fulfil task i, at the moment of event $n - 1$. Similarly ρ_{si}^{c} is the product share in state s from any other lot of the product on all machines j, allowing fulfilment of task i, at the moment of event n.

The following constraint is that it is necessary to fulfil orders $D(s)$ for the products, delivered to state s

$$\sum_{n \in N} q(s,n) \geq D(s) \text{ for all } s \in S. \tag{7.6}$$

The end moment of the production operations depends on the duration of the process and equals

$$S_{ijn} + \hat{p}_{ij} w(i,n) + \tilde{p}_{ij} Q(i,j,n) \text{ for all } i \in I, j \in J_i \text{ and } n \in N. \tag{7.7}$$

In expression (7.7), S_{ijn} is the time of fulfilment start, $\hat{p}_{ij}$ is the constant component of task i duration on machine j, and $\tilde{p}_{ij}$ is the coefficient defining the increase in duration of this task depending on the size of lot $Q(i,j,n)$, launched at event point n.

Overall duration of task i on machine j $p_{ij} = \hat{p}_{ij} w(i,n) + \tilde{p}_{ij} Q(i,j,n)$. Note that pursuant to condition (7.3) value $Q(i,j,n)$ with $w(i,n) = 0$ is also equal to 0.

The important element of this model is constraints of possible sequence of events. These constraints link the beginning and the end of the tasks and binary variables $w(i,n)$ and $y(j,n)$. These constraints have different characters depending on the sequence of various kinds of tasks on each machine. In general, we can distinguish four kinds of such sequences:

- Repetitive task on the same machine
- Different tasks on the same machine
- Different tasks on different machines
- Task performed upon completion of all previous tasks for a particular machine

In the simplest case of repetitive tasks on the same machine, there are constraints

$$S_{ij(n+1)} \geq S_{ijn} + p_{ij} - h \times [2 - w(i,n) - y(j,n)] \tag{7.8}$$

for all $i \in I$, $j \in J_i$, $n \in N$, and $n \neq N$.

Designations in Eq. (7.8) have the same meaning as in the previous expression, h is the planning horizon. If at the time of event n task i is processed on machine j, then $w(i,n) = y(j,n) = 1$ and condition (7.8) simply states that the subsequent task must start after the end of the previous one. In general, condition (7.8) reduces the size of the planning area to the size set by the horizon.

If one machine processes various tasks, conditions (7.8) take the form:

$$S_{ij(n+1)} \geq S_{i'jn} + p_{i'j} - h \times [2 - w(i',n) - y(j,n)] \tag{7.9}$$

for all $i \in I_j$, $i \neq i'$, $i' \in I_j$, $j \in J$, $n \in N$, and $n \neq N$.

In this case, each constraint (7.9) is imposed on the tasks that can be performed on machine j.

If tasks are different and performed on different machines, the constraints in question take the form:

$$S_{ij(n+1)} \geq S_{i'j'n} + p_{i'j'} - h \times [2 - w(i',n) - y(j',n)] \tag{7.10}$$

for all $i \in I_j$, $i \neq i'$, $i' \in I_j$, $j \in J$, $n \in N$, $j' \in J$, and $n \neq N$.

Here, each task i refers to a single set I_j, which can be performed on machine j.

Provided that the task can be performed only after completion of all preceding tasks on machine j, the constraint has the form

$$S_{ij(n+1)} \geq \sum_{n' \in N, n' \leq n} \sum_{i' \in I_j} p_{i'j'n'} \tag{7.11}$$

for all $i \in I$, $j \in J_i$, $n \in N$, and $n \neq N$.

Another condition imposed on the model is the requirement that both the end and the beginning of any task does not go beyond the horizon, i.e.

$$S_{ijn} + p_{ij} \leq h \text{ for all } i \in I, j \in J_i, n \neq N. \tag{7.12}$$

As an example of recorded constraints, we give the constraints for the possible transitions from performing one task to another on machine 2 (Table 7.5). According to this table, the machine 2 (reactor 1) can perform tasks 2, 4 and 6 (reactions 1, 2, 3). Since we are considering constraints for performing different tasks on the same machine, we should use such constraints in the form (7.9).

For example, with possible quantity of event points $N = 5$ for transition to task 4 from task 2 constraints (7.9) have the form:

$$S_{4,2,n+1} \geq S_{2,2,n} + p_{2,2} - h \times [2 - w(2,n) - y(2,n)] \text{ for } n = n_0, \ldots, n_4.$$

If the transition to task 4 takes place after task 6 then constraints (7.9) are written as:

$$S_{4,2,n+1} \geq S_{6,2,n} + p_{6,2} - h \times [2 - w(6,n) - y(6,n)] \text{ for } n = n_0, \ldots, n_4 \text{etc.}$$

In addition to general constraints (7.2–7.12), it is necessary to establish constraints describing the sequence of process steps. For example, in accordance with process chart in Fig. 7.5 reaction 2 can be carried out only after heating raw material A and after completion of reaction 1. According to the terms of tasks in Table 7.5, this means that tasks 4 and 5 can be performed only after task1 and at least one of tasks 2 or 3. In particular, for fulfilment of task 4 on machine 2 (reaction 2 in reactor 1) the constraints for carrying out this task after task 1 on machine 1 has the form:

$$S_{4,2,n+1} \geq S_{1,1,n} + p_{1,1} - h \times [2 - w(1,n) - y(1,n)] \text{for} n = n_0, \ldots, n_4.$$

Similarly, the constraints for carrying out task 4 after completion of task 3 on machine 3 are written as:

$$S_{4,2,n+1} \geq S_{3,3,n} + p_{3,3} - h \times [2 - w(3,n) - y(3,n)] \text{for} n = n_0, \ldots, n_4.$$

Summing up the construction of the model in question, it should be noted that here production type 3b (Sect. 1.3.1) has the objective to calculate the optimal operational plan. The classification formula of the model according to Sect. 2.2.2 can be represented in the form (Appendix C):

$$F\left|prec, batch, p_i \in \left[\underline{p}_i : \overline{p}_i\right]\right| f_{\max}. \tag{7.13}$$

In expression (7.13) according to the designations in Appendix C: F is the flow shop, *prec*—presence of constraints on process operations sequence, *batch*—using the manufacture lots for fulfilment of tasks, $p_i \in \left[\underline{p}_i : \overline{p}_i\right]$ means that duration p_i of each task i can be in the range from $\underline{p}_i$ to $\overline{p}_i$ and $f_{\max}$—maximization of the objective function of value cost.

7.3.3 Some Results of Modelling

In modelling, as a starting parameter the number of possible event points N is set, which may vary and lead to somewhat different results of optimization. In the paper of Ierapetritou and Floudas (1998), the results for $N = 5$ and $N = 6$ are given, the differences between them are small. The volume of computational work is characterized by a number of independent variables of the problem and constraints. In the described model, there are 260 continuous variables, 40 binary variables and 374 constraints inequalities.

The results of modelling in the form of Gantt diagram are shown in Fig. 7.5. In this figure, the numbers of tasks and amount of product in the relevant state produced on each of four machines are presented. The duration of horizon h is accepted to be 8 h. As can be seen from Fig. 7.5, the optimal plan provides the start of both reactors at the beginning of the shift to perform reaction 1 (tasks 2 and 3). By the end of the processes in the reactors it is planned to heat raw material A in the heater and then run the reactors for sequential performance of reaction 2 (tasks 4, 5) and reaction 3 (tasks 6, 7). The resulting crude product E (state 7) is fed to the separator for cleaning (task 8). The presence of buffer capacities for semi-product BC (state 6) allows processing the semi-finished product in reaction of 2 (tasks 4, 5) in two stages, as shown in Fig. 7.5.

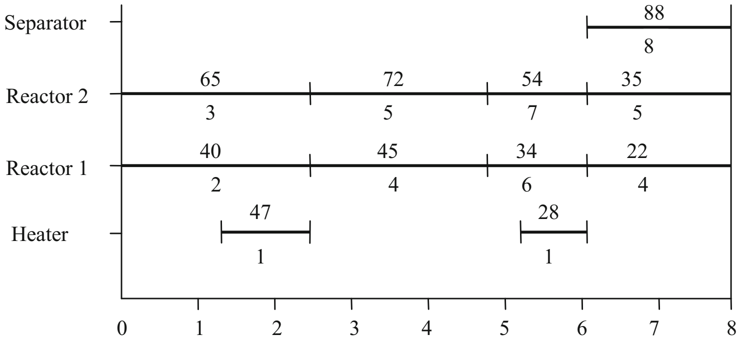

Fig. 7.5 Gantt diagram of subsequent task performance [based on Ierapetritou and Floudas (1998)]

Obviously, the work plan for the subsequent shift should be different from the plan in Fig. 7.5 because the initial amount of the product in various states will differ from the values in Table 7.6, which were taken as 0. Generally speaking, the shift task should be linked to a production task assigned for longer duration, for example, with the master plan.

In the paper by Li and Ierapetritou (2009), the schedule model for the described example expands considering its collaboration with the mathematical model of the schedule, which is valid at least for a few shifts. In this case, the objective function schedule optimization is different from the function of shift schedule.

It should be noted that most of the studies on production scheduling theory published in recent years, in contrast to earlier studies, usually regard planning as a process consisting of two steps. This is due to the understanding of the fact that at different stages the planning criteria differ (Tables 2.5–2.7). In the article by Li and Ierapetritou (2009) the objective function for the schedule is to minimize the integrated storage cost, loss of profits due to the inability to fulfil orders and immediate production costs. When preparing the shift task the cost of production is actually minimized.

Unlike the conventional optimization of each stage in Li and Ierapetritou (2009) attempts are made to harmonize the optimal calendar plan and all schedules for the corresponding period by planning in several (30–40) iterations. The obtained results show that the proposed model can provide a good match between the demand for core product (finished product 2) and its production.

An interesting result of this study is determination of the relationship between the volume of by-products (finished product 1) output and core product 2 output (Fig. 7.6). From the chart in Fig. 7.6, it follows that at first with the increase in production of product 1 the yield of core product 2 also increases. If the value of product 1 volume is about 55 units, the yield of product 2 reaches the maximum and then decreases gradually. When the volume of product 1 reaches 70 units the yield of product 2 drops dramatically and if the volume of product 1 is equal to 87 units, it tends to zero.

This type of chart is easily explained from the perspective of equipment utilization. In the left part of the chart, the equipment is not fully loaded and its capacity is sufficient for simultaneous output growth for both products. With the volume of product 1 of 55 units, the load reaches the possible maximum, and further growth of this product is only possible by reducing the output of product 2. If the volume of product 1 tends to 87 units, all production capacity is entirely used only for this product and the further increase is impossible.

7.4 Rapid Supply Chain Reference Model in Clothing Industry

The study by Macedo (2000) considers a model of the supply chain, in which requirements of “make-to-order” production are combined with the need of urgent changes in production. This supply chain is called Fast Innovation Reinforcement

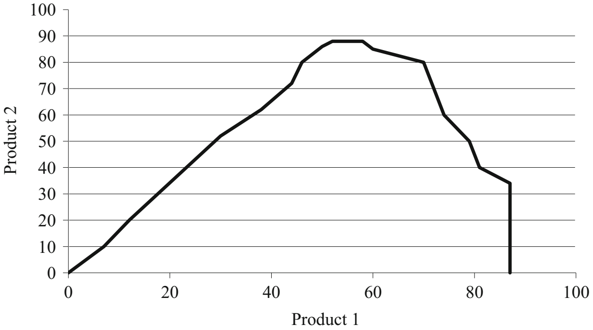

Fig. 7.6 Relationship of output values of core and by-products [based on Li and Ierapetritou (2009)]

(FIR). The reasons for the need of urgent changes can be urgent orders, as well as promotion of new product models, the demand for which cropped up in the market.

FIR chains differ in the fact that they involve supplying of not fully finished products but individual components (modules), the properties of which are regularly updated according to the needs of the market. At the same time, the FIR chain allows bringing the selling price of the product in accordance with the properties required by an individual customer (mass customization) to a level that is sufficiently close to the usual wholesale request supplies. The FIR chain's operation is provided by information technologies application to all stages of production.

Usually, each product module in the FIR chains is a family of several species of single-purpose components that are produced in the same cell manufacturing site. If the production technology is so complex that it cannot be fully implemented on a single physical site, then for each such technological process the so-called virtual object site is organized in the information space. This approach greatly facilitates the ability to constantly monitor the sites workload and adjust the operational plan of production.

Figure 7.7 shows the basic operations for the FIR supply chain in the manufacture of clothes. The material flows are indicated by solid lines with arrows and information flows by dashed lines with arrows. Dash–dotted lines designate the information flows describing the demand seasonality.

At the order collection stations, to determine individual sizes, the electronic scanning of the customer's body is performed according to the type of garment to be sold. At the same time, the client is able to vary the different parameters of clothing, thus establishing its most suitable properties.

Simultaneously, the information system collects the statistical data about the parameters of sales in order to determine the most popular designs. The company of clothes model based on the statistical data develops new models and performs their pilot production. Using new components clothing designs are tailored. The data on them are recorded in the database after their finalization. These data on new materials are transferred to the textile factory, and the data on the components of clothes are transferred to the tailoring company.

Textile production is characterized by stable processes. Therefore, the seasonal increase in the production scale is only possible at the expense of reserve capacity.

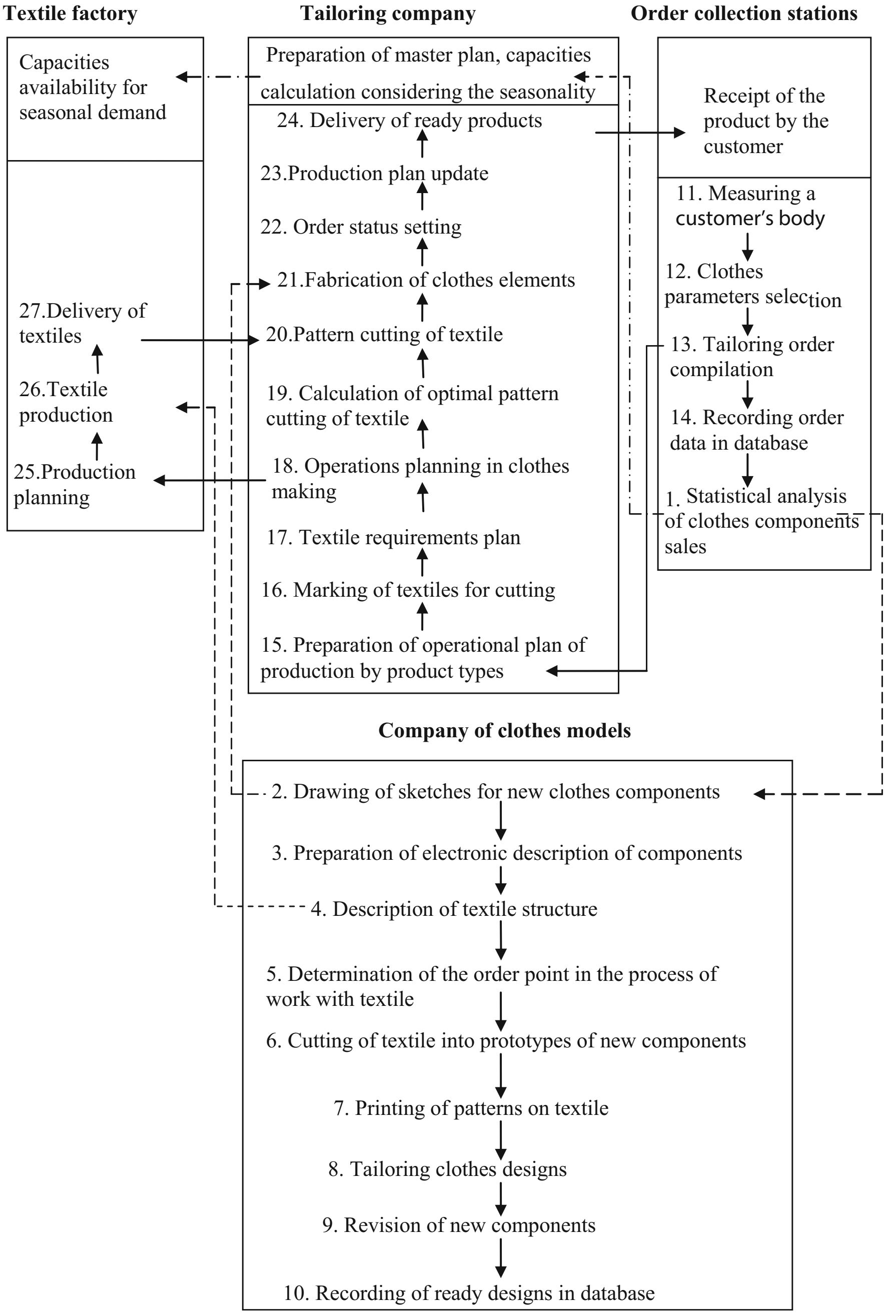
Textile factory
Tailoring company
Order collection stations
Capacities availability for seasonal demand
Preparation of master plan, capacities calculation considering the seasonality
Receipt of the product by the customer
24. Delivery of ready products
23.Production plan update
22. Order status setting
21.Fabrication of clothes elements
20.Pattern cutting of textile
19. Calculation of optimal pattern cutting of textile
18. Operations planning in clothes making
17. Textile requirements plan
16. Marking of textiles for cutting
15. Preparation of operational plan of production by product types
27.Delivery of textiles
26.Textile production
25.Production planning
11. Measuring a customer's body
12. Clothes parameters selection
13. Tailoring order compilation
14. Recording order data in database
1. Statistical analysis of clothes components sales
Company of clothes models
2. Drawing of sketches for new clothes components
3. Preparation of electronic description of components
4. Description of textile structure
5. Determination of the order point in the process of work with textile
6. Cutting of textile into prototypes of new components
7. Printing of patterns on textile
8. Tailoring clothes designs
9. Revision of new components
10. Recording of ready designs in database

Fig. 7.7 Operations of quick supply chain in the manufacture of clothes [based on Macedo (2000)]

Increased volume of tailoring can be possible in various ways including increasing the number of personnel, their workload, etc.

The main difference and at the same time advantage of the described reference model is that for the majority of its operations informational support is provided in the form of special computer systems. Systems Lectra and Lawson Fashion are mainly used for this purpose. The first one provides designing of clothes, measurement of customer's sizes and refinement of the designed by customer's requirements, as well as the optimal making-up and cutting of fabrics. The second one is used for scheduling and managing the production of clothing on the shop level. Both of these systems also provide data transfer between companies via Internet.

Logistics methods of the quick supply chains are very popular now and being intensively developed. A number of studies of this kind were published in the book by Kestner et al. (2008).

7.5 Schedule Model for a Machine Shop

One of the most important and frequently discussed problems of planning is the problem of optimal scheduling for the job shop (type 5a in Sect. 1.3.1). As a rule, this category includes machine shops of machine-building plants, furniture factories and other enterprises producing serial and small-serial complex products. To manufacture parts of such products, it is often necessary to have the process consisting of a number of successive machining operations, and the order of these operations is arbitrary. The sequence of operation of this production is convenient to display in the form of so-called mixed graph (Fig. 7.8).

The circles in Fig. 7.8 denote the operations performed on workpiece *i* on machine *j*. For the convenience of graph construction the fictive general starting point of schedule S and finishing point E are introduced. The processing time of each operation are indicated as p_{ij}. The solid lines with arrows indicate the movement of the lot of parts from one machine to another according to the procedure specified by the process.

The dashed lines with arrows designate the possible procedure of processing different lots of parts on a certain machine. For example, since each of three kinds

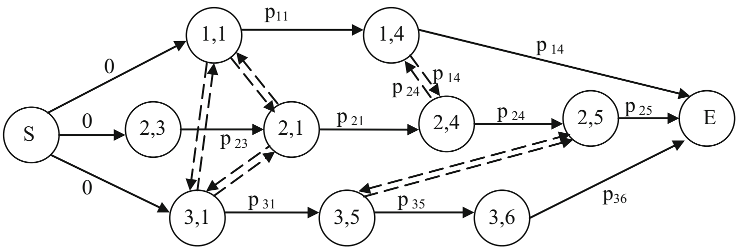

Fig. 7.8 Oriented mixed graph of job shop production

of parts requires processing on machine 1, then six variants of sequences of three kinds of machining on this machine are possible:{1, 2, 3}, {1, 3, 2}, {2, 1, 3}, $\{2, 3, 1\}$, $\{3, 1, 2\}$ and $\{3, 2, 1\}$. It is easy to see that each of these variants is shown in Fig. 7.8 in the form of sequence of two dashed arrows. Each of these arrows corresponds to the processing duration, such as shown for arrows relating to machine 4.

The problem of the optimal scheduling, in terms of getting the best option of mixed graph 7.8, is to make optimal choice of dashed arrows describing the sequence of operations on each machine. Practically, this means that of all the possible permutations of this sequence only one should be selected, which ensures the best value of an objective function. Obviously, the choice of the optimal sequence obtained this way depends primarily on the type of the objective function.

7.5.1 Schedule Model with Specified Processing Stages

In the model (Prilutsky and Vlasov 2007) it is assumed that at the beginning of the planning period we know the list of orders subject to processing. Each order is a set of interdependent jobs that are performed on the machines. Equipment units are arranged in groups of interchangeable machines, which fulfil various stages of processing, herewith the machines in the group can have different performance. In general, to perform each job i, it should be processed sequentially on the machines of each stage l, and for each job order the procedure of passing through the stages may differ.

The latter means that each job i should match the collection (vector)

$$\vec{r}^i = \left(r_1^i, r_2^i, \ldots r_k^i\right) \text{ for } r_l^i \in K; \quad l = 1, 2, \ldots k_i; \quad i \in I. \tag{7.14}$$

In Eq. (7.14) r_l^i is the stage number, where the l-th operation of job i shall be performed, K is a set of stages (equipment groups), and k_i is the quantity of operations for job i. For example, if there are four groups of equipment—(1) turning, (2) drilling, (3) milling, and (4) grinding, and the process for part 1 provides for sequential operations of milling, drilling, turning, and grinding then vector $\vec{r}^1 = (3, 2, 1, 4)$. It is assumed that at the next stage the job can be performed on any machine of this stage.

It is necessary to schedule the jobs performance on the machines, for which the aggregate cost is minimal. This schedule is represented by a set of values S_{ijl} of start moments of operation l of job i on machine j. The time period, during which such operation is performed, is determined by binary function value y_{ijl}. If the operation is performed, then $y_{ijl} = 1$, if not—$y_{ijl} = 0$. The sequence of the entire set of operations is determined during scheduling and described by a set of serial numbers z_{ijl}. The quantity of these numbers is equal to the total amount of all transactions for all jobs.

7.5.2 Optimality Criteria and Constraints

In the paper by Prilutsky and Vlasov (2007), the schedule quality assessment is determined by three main components: the cost of jobs on the machines, the cost of setup, and penalties imposed on the system for violation of specified due dates. The cost of all operations on the machines:

$$\sum_{j\in J}\sum_{l=1}^{k_i} c_{ijl}^{p} p_{ijl} y_{ijl}, \tag{7.15}$$

where c_{ijl}^{p} is the cost of manufacture time unit for an operation on particular equipment and p_{ijl} is the operation duration.

Similarly, the costs for setup excluding the dependence of setup duration on its sequence

$$\sum_{j\in J}\sum_{l=1}^{k_i} c_{ijl}^{s} s_{ijl} y_{ijl}, \tag{7.16}$$

where c_{ijl}^{s} is the cost of setup time unit for an operation on particular equipment and s_{ijl} is the setup duration.

In this paper, it is assumed that non-observance of timely (directive given) performance of each job leads to penalties, which for various jobs may depend on the delay to various extents, that is, for each job they have the form:

$$g_i \max\left(0, S_{ijk} + p_{ijk} - d_i\right), \tag{7.17}$$

where g_i is the penalty amount per 1 day of tardiness of job i, S_{ijk} is the start moment (date) of the last (k-th) operation performed on machine j for job i, p_{ijk} is the duration of this operation in days, and d_i is the due date of job i.

The first of the constraints of the described model is that each operation of any job must be performed on a certain machine, that is

$$\sum_{j\in J} y_{ijl} = 1 \text{ for all } l = 1, 2, \ldots k_i; \;\; i\in I. \tag{7.18}$$

Besides, any operation of job i can be started only after completion of all operations that precede it according to the process:

$$S_{ijl} \geq S_{ij(l-1)} + p_{ij(l-1)} \text{ for all } l = 1, 2, \ldots k_i; \;\; i\in I, j\in J. \tag{7.19}$$

Similarly, any new operation l of job i on machine j can be started only after completion of operation $l^{'}$ of previous job $i^{'}$ on this machine, and after the corresponding setup, i.e.

$$S_{ijl} \geq S_{i'jl'} + p_{i'jl'} + s_{ijl} \text{ for all } l$$
$$= 1, 2, \ldots k_i; \quad l' \in 1, 2 \ldots k_{i'}; \quad i \in I, \ i' \in I, j \in J. \tag{7.20}$$

By analysing the schedules quality criteria (7.15) and (7.16), we see that both of these values are cost input. In this case, their aggregate value describes the criterion of direct costs K1 (Table 2.3). At the same time, indicator (7.17) characterizes the delay of job performance and meets criterion C1 in Table 2.2.

The simultaneous use of two such criteria to assess the quality of the schedule makes it necessary to solve a multi-objective problem. Considering the formulation of this problem in terms of usefulness (Sect. 4.1.3), we see that it corresponds to the points of small-serial or serial production on the curve of the UV-diagram for operational plans, for which it was just recommended to use multi-criteria approach in Sect. 4.1.3.

Let us draw up a classification model according to Sect. 2.2.2, the same way as it was done in Sect. 7.3.2:

$$J \big| prec, d_i \big| F_l(c, T). \tag{7.21}$$

In expression (7.21) according to the designations in Appendix C: J is the job shop designation, *prec* is the presence of constraints for process sequence of operations, d_i is the directly set due date for jobs performance, and $F_l(c, T)$ is the achievement of the minimum of the objective function, which can be presented as the liner combination of two criteria—cost c and tardiness T.

In the paper by Prilutsky and Vlasov (2007), in order to solve the problem stated above, the examples of using several different algorithms are given. For example, in the simplest of them some sequence of jobs is input. This sequence defines the priority of tasks and operations that make up this job. The algorithm builds a feasible solution to the original problem by distributing the jobs to machines and ordering them according to the following principle: the next job, determined by the original permutation is fixed to the machine of the next stage, for which the cost of the job (taking into account setups and possible violations of the deadlines) are minimal.

The scheduling results on all the proposed algorithms are local. This means that the calculated best value of the plan criteria is optimal in small neighbourhood of the problem parameters. Therefore, the study proposes to use several solution algorithms consistently and to compare the results.

It should be noted that, as the schedule quality criterion is a minimum of sum of two values calculated from (7.15–7.16), then in this approach the stated multi-objective problem is actually replaced by the single-objective problem during solving, in which a priori values g_i of penalties for tardiness must be set. This example is, therefore, substantially different from the example of multi-objective scheduling problems in Sect. 4.3, for the solution of which the algorithm for constructing compromise curves was used. In this case, obviously, we can build these curves as well, if we set several different values g_i successively.

Various specific algorithms for scheduling with multi-step processing are listed below in Chap. 14.

7.6 Multi-stage Logistics Chain Model

Here, we discuss the example of supply chain optimization providing the operation of complex manufacturing enterprise.

7.6.1 Some Notions in Logistics Chain Modelling

By multi-stage enterprise, we understand (Pleshchinsky et al. 2008) a stable set of production capacities processing some products at different stages of the process, as well as a number of auxiliary facilities serving the core production. A multi-stage enterprise is characterized by presence of two or more independent productions, diverse range of products, presence of several technological conversions. Such enterprise may be geographically dispersed and have different geographical markets. Products, being intermediate in the process, can either be manufactured at the enterprise or supplied from the third party, and here competition is even possible.

Logistics chain is (Rodnikov 1995) a linearly ordered set of natural and legal entities (manufacturers, distributors, etc.) engaged in logistics operations to bring the material flow (products) from one entity or individual to another. Here, a logistics operation means a series of actions aimed at the transformation of the material and information flow. Examples of logistics operations are loading, unloading, transportation, warehousing, etc.

The objects of the logistics chain of a multi-stage enterprise are their own production capacities, storage facilities and transport. All products at the multi-stage enterprise can be divided into three groups: original raw material, intermediate products (semi-finished), and finished products. The first group includes also resources such as water, electricity, natural gas, etc.

A mathematical model of the logistics chain is based on the database containing information about suppliers, potential sales volumes, prices, etc. For each type of products supplied the so-called delivery basis must be known. The delivery basis is the point at which the supplier's obligations are over and the consumer's obligations begin. The delivery price is fixed on its basis.

7.6.2 Dynamic Logistics Chain Optimization Model in Multi-stage Production

Here is the presentation of this problem in application for one enterprise according to Pleshchinsky (2004). In this case, the transport factor of the chain is considered to be of little importance and may be included directly in the prices for finished products and semi-finished products. From the point of view of the production classification in Sect. 1.3.1, the multi-stage enterprise belongs to type 5c.

In this statement, the planning horizon has T periods of time, i.e. $t = 1, 2, \ldots T$. It is assumed in the model that the enterprise produces N types of products $i = 1, 2, \ldots N$, and the length of one production stage of each type of product is less than the duration of the one planning and accounting period. In accordance with the master plan, with each period t the amount of product $x_i(t) \geq 0$ is produced; the quantity the product for sale $q_i(t)$.

Some intermediate products, manufactured by the enterprise, can be purchased from third parties in amounts $V_i(t)$. Let us denote the stock of products of i-th type by $z_i(t)$ at the end of period t with $t = 1, 2, \ldots T$. In this case, $z_i(0)$ represent stock at the beginning of the first period. Also we denote the minimum necessary reserve stock of product of i-th type at the end of period T, which ensures operation of the enterprise in future periods, as $\underline{z}_i$.

Suppose that a_{ij} is the quantity of product of i-th type necessary for manufacturing a unit of semi-finished or finished product of j-th type on the next stage of production. Many types of semi-finished and finished products, the manufacture of which requires just product of i-th type, we denote as P_i.

Let us write the equation of products receipt and expenditure balance in period t, defining the residual stock at the end of the period:

$$\begin{aligned} z_i(t) \quad &= z_i(t-1) + x_i(t) + V_i(t) - \sum_{j \in P_i} a_{ij} x_j - q_i(t) \text{ for all } t \\ &= 1, 2, \ldots T; \quad i = 1, 2, \ldots N; \text{and with condition } z_i(t) \geq \underline{z}_i \text{ as well.} \end{aligned} \tag{7.22}$$

Some part of ρ_i product of i-th type, manufactured within period t can be spent within the same period for manufacturing of other products j-th type out of set P_i. The condition of sufficiency of intermediate products for manufacturing within period T has the form:

$$\begin{aligned} &z_i(t-1) + \rho_i x_i(t) + V_i(t) - \sum_{j \in P_i} a_{ij} x_j - q_i(t) \geq 0 \text{ for all } t \\ &= 1, 2, \ldots T; \quad i = 1, 2, \ldots N. \end{aligned} \tag{7.23}$$

Besides the restriction on material resources, the model considers the restrictions on labour resources:

$$\begin{aligned} &\sum_{i=1}^{N} p_{ik} x_i(t) + s_{kt}^- - s_{kt}^+ = S_{kt} \text{ for all } t = 1, 2, \ldots T; \ k \in K \text{ and with } s_{kt}^+ \\ &\leq S_k. \end{aligned} \tag{7.24}$$

In expression (7.24) p_{ik} is the run time of labour resource k to fulfil the job (product) i, s_{kt}^{-} is the aggregate time of idleness of resource k for period t, s_{kt}^{+} is the total time of overtime jobs for the same period, S_{kt} is the time fund of labour resource k for period t, S_k is the maximal time of overtime jobs for resource k for period t, and K is the set of various kinds of labour resource.

Restrictions on the time resources of equipment are written similarly:

$$\sum_{i=1}^{N} p_{ij} x_i(t) = S_{jt} \text{ for all } t = 1, 2, \ldots T; j \in J, \tag{7.25}$$

where p_{ij} is the run time of equipment j resource to fulfil the job (product) i, S_{jt} is the time fund of equipment j resource for period t,and J is the set of various kinds of equipment.

To manufacture finished products the quantity of source raw material is required

$$y_l(t) = \sum_{i=1}^{N} y_{il} x_i(t) \text{ for all } t = 1, 2, \ldots T; \quad l \in L, \tag{7.26}$$

where y_{il} is the source raw material consumption norm for the unit of product i and L is the set of various kinds of raw materials.

Another condition of the described model is correspondence of the quantity of marketed product $q_i(t)$ and market demand $D_i(t)$. In this case this condition is used in the form:

$$q_i(t) \leq D_i(t) \text{ for all } t = 1, 2, \ldots T; \quad i = 1, 2, \ldots N. \tag{7.27}$$

The objective function of the problem is the value of the highest possible profit for period T in question. The profit margin depends primarily on the values of four groups of variables: volume of finished product $x_i(t)$,quantities of raw materials used $y_l(t)$, stock of purchased intermediates $V_i(t)$, and sales volumes $q_i(t)$. In general, the value of optimality (profit) criterion

$$\begin{aligned} F(x, y, V, q) = \sum_{t=1}^{T} \Bigg[& \sum_{i \in M_{out}} c_{it} q_{it} - \sum_{i \in M_{in}} c_{it} V_i(t) - \sum_{l \in L} c_{lt} y_l(t) - \\ & -(1 + \delta) \sum_{k \in K} \left(c_k \sum_{i=1}^{N} p_{ik} x_i(t) + c_k^{-} s_{kt}^{-} - c_k^{+} s_{kt}^{+} \right) - c_t - \beta \sum_{i=1}^{N} c_i z_i(t) \Bigg] \end{aligned} \tag{7.28}$$

should tend to maximum.

In expression (7.28) M_{out} is a set of kinds of semi- and finished products, manufactured by the enterprise and marketed; M_{in} is a set of similar kinds of intermediate product, which are manufactured by other enterprises and purchased on the market for manufacturing finished products.

Other designations in expression (7.28): c_{it}—product price, c_{lt}—raw material price, δ—unified social tax fee, c_k—rate per time unit of labour resource job, c_k^- and c_k^+—payment per time unit of idleness and extra payment for overtime job, c_t—constant expenditures including depreciation, β—alternative return on assets for one period t, and c_i—i-type product cost.

To achieve the maximum of function (7.28) it is necessary to vary the values of variables $x_i(t)$, $y_l(t)$, $V_i(t)$, and $q_i(t)$, including the constraints (Eqs. 7.22–7.27). In fact, the set objective is the same as the problem of sales and operations scheduling, which is discussed below in Chap. 10.

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Part II

Planning Processes

Single-Echelon Inventory Planning 8

8.1 Inventory Types and Parameters

This chapter discusses the stocks, their characteristics, and methods of stock planning. Stocks are generally classified (Grigoriev et al. 2007) by their intended purpose and economic and logistical functions. From the planner's point of view, the main interest is in the division of all stocks into parts by functions in the logistics process as follows:

- Current;
- Safety;
- Preparatory;
- Seasonal.

Preparatory stocks are for special preparation or incoming inspection prior to the production process—for example, wood drying, quality assurance laboratory tests, etc. The basis for the stocks calculation is demand values defined by the methods discussed above in Chap. 6.

For stocks the so-called ABC classification into three categories is widely used. Category A includes the most valuable goods comprising 20 % of stocks and providing 80 % of sales. Category B includes 30 % of stocks, which give 15 % of sales; category C is the least valuable goods comprising 50 % of stocks, giving 5 % of sales.

Stock changes over time are characterized by a number of parameters influencing the quantity of the individual parts of the stock. Let us consider the stock graph in the simplest case (Fig. 8.1), when the demand is constant, the stock is replenished instantaneously, and delivery is performed within a specified time interval.

In this case, the stock decreases from the highest value $\dot{S}$ to the lowest permissible Z_c at a rate determined by demand value D. At time T of reaching the

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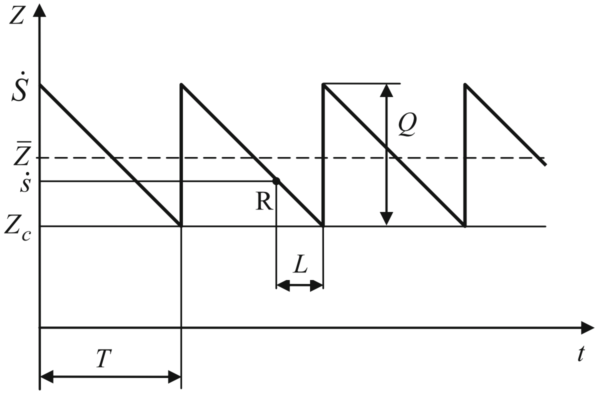


Fig. 8.1 The simplest stock graph

smallest value, the new product lot of size Q is received, which increases the stock to the initial value. Time interval T is called the cycle time.

Average value of stock $\overline{Z}$ for this period

$$\overline{Z} = \frac{\dot{S} + Z_c}{2}. \tag{8.1}$$

Point R in Fig. 8.1 is called the reorder point. The supplying order for new product lot at this point ensures new receipt of product precisely at the moment when the stock reaches the minimum allowable value. Herewith the stock value at point R

$$\dot{s} = Z_c + DL, \tag{8.2}$$

where L is the interval length of new product lot supply.

Minimal allowable stock Z_c is reserve stock and serves to satisfy the customers' requests during possible fluctuations in demand, delivery time, etc.

8.2 Inventory Management Models

When controlling the stock of any product there are two questions: (a) what the order value should be and (b) when the order is to be made? Existing management models provide three possible answers to these questions:

- Fixed reorder quantity;
- Fixed reorder cycle;
- Simultaneous change of the reorder quantity and the reorder cycle.

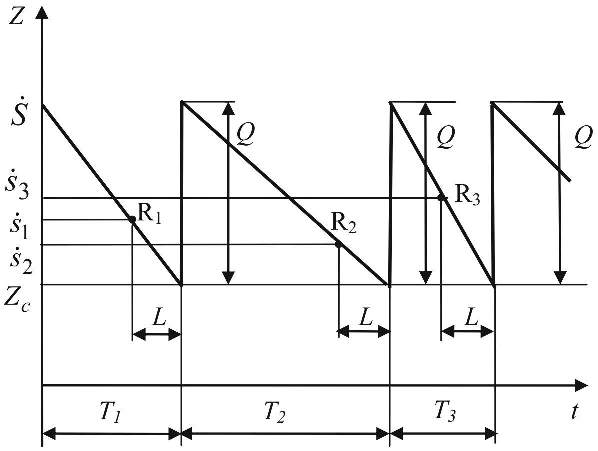


Fig. 8.2 Stock graph with fixed reorder quantity

8.2.1 Model with Fixed Quantity of Order

Figure 8.2 shows the stock graph at a constant quantity of the order. This quantity can be represented by economic order quantity defined by formula (2.4). Let us refer to the example considered in Sect. 2.1.1, in which the quantity of monthly demand D is equal to 300 pcs., the ordering cost of one order c_o amounts to 2000, and the cost of monthly storage c_h is 200. In this case

$$Q^* = \sqrt{\frac{2 \times 300 \times 2000}{200}} = 77.5 \ \text{pcs.}$$

By approximating we take fixed order quantity $Q = 80$ pcs. Since it was decided that the monthly demand is 300 pcs., the average daily demand is 10 pcs. Let us assume that the delivery time $L = 2$ days, and the safety stock equals the average 3-day consumption, i.e. $Z_c = 30$ pcs. The largest value of the stock in this case $\dot{S} = Z_c + Q = 110$ pcs.

Consider changing of the cycle parameters in the event of possible deviations of demand from the average value. Given that

$$T = Q/D, \tag{8.3}$$

and formulas (8.1)–(8.3), we obtain the values shown in Table 8.1.

Time values in Table 8.1 are approximated up to half a day. Note that the average value of the stock in the model with fixed reorder quantity is the same in all cycles. Note that the described version of the model with fixed reorder quantity

Table 8.1 Cycle parameters with fixed order quantity

	Cycle number		
Parameter	1	2	3
Demand D pcs per day	10	9	11
Cycle time T	8	9	7.5
Stock $\dot{s}$ at reorder point R	50	48	52
Average stock $\overline{Z}$	70	70	70

is often called (S, Q) model in the literature as the stock is replenished with fixed lots to maximal value $\dot{S}$.

In addition to this model, a model with fixed value Q and a constant stock quantity $\dot{s}$ at reorder point R is often used. It is often called (R, Q) model. Obviously, in this model, the maximum value of the stock does not remain constant.

8.2.2 Model with Fixed Reorder Cycle

The stock graph with fixed reorder cycle is shown in Fig. 8.3. All three reorder cycles, shown in Fig. 8.3, have the same cycle period T.

Due to different rates of consumption the angle of the graph lines in different periods is different. Therefore, with constant cycle time the order quantity changes. The optimal order period value in this model is set by the value of optimal lot defined by formula (2.4), namely,

$$T^* = Q^*/\overline{D}, \tag{8.4}$$

where $\overline{D}$ is the average daily consumption.

The stock value at the end of period:

$$Z_{\min} = \dot{S} - TD, \tag{8.5}$$

order quantity,

$$Q = \dot{S} - Z_{\min} = TD, \tag{8.6}$$

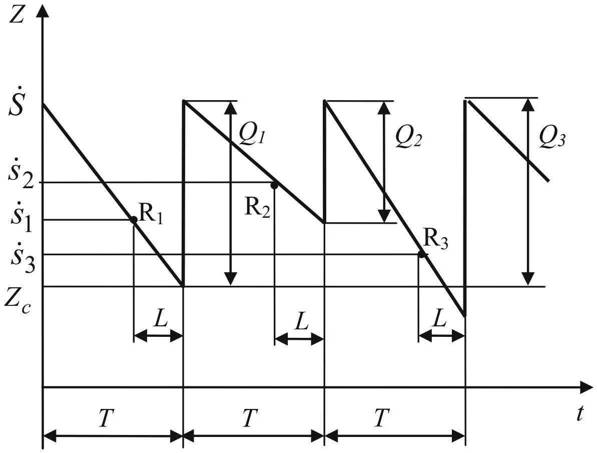
and reorder point,

$$\dot{s} = Z_{\min} + DL = \dot{S} - D(T - L). \tag{8.7}$$

The quantity of average order

$$\overline{Z} = \frac{\dot{S} + Z_{\min}}{2} = \dot{S} - \frac{TD}{2}. \tag{8.8}$$

Fig. 8.3 Stock graph with fixed reorder cycle



For the example, considered in the previous paragraph, optimal period value $T = 80/10 = 8$ days. Assume that the highest proffered quantity of order is as previously $\dot{S} = 110$ pcs.

The changes in the cycle parameters for possible deviations in demand from the average value in this case are presented in Table 8.2.

As can be seen from Table 8.2, in this model the value of average stock increases with the decrease of demand. The model with fixed reorder cycle and fixed maximal stock value is called (S, T) model.

8.2.3 Two-Tier Inventory Management Model

This model establishes two levels of stock: maximal required stock $\dot{S}$ and the level defining the reorder point $\dot{s}$. The order is made as soon as the actual value of the stock reaches level $\dot{s}$ in order to replenish the stock up to level $\dot{S}$.

Figure 8.4 shows the stock graph in this model. In this case, if the demand varies, both the reorder cycle and the order quantities vary. The reorder cycle as shown in Fig. 8.4:

$$T = \left(\dot{S} - \dot{s}\right)/D + L, \tag{8.9}$$

order quantity

$$Q = \dot{S} - \dot{s} + DL, \tag{8.10}$$

stock at the end of period

Table 8.2 Cycle parameters with fixed reorder cycle

Parameter	Cycle number		
	1	2	3
Demand D pcs per day	10	9	11
Cycle time T	80	72	88
Stock $\dot{s}$ in reorder point R	50	56	44
Average stock $\overline{Z}$	70	74	66

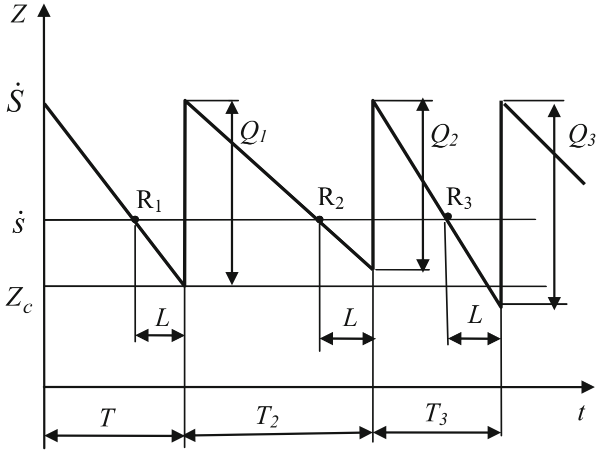


Fig. 8.4 Stock graph with two-tier model

$$Z_{\min} = \dot{S} - Q,$$

and average value of stock for a cycle

$$\overline{Z} = \frac{\dot{S} + Z_{\min}}{2} = \frac{\dot{S} + \dot{s} - DL}{2}. \tag{8.11}$$

Returning to the above discussed example, where safety stock $Z_c = 30$ pcs., the maximal allowable stock $\dot{S} = 110$ pcs., delivery time $L = 2$ days, and average value of consumption $D = 10$ pcs./day, let us define reorder point $\dot{s}$ from expression (8.2)

$$\dot{s} = Z_c + DL = 30 + 10 \times 2 = 50 \text{ pcs.}$$

The cycle parameters for possible deviations in demand from the average value in this case are presented in Table 8.3.

In literature the two-tier model is often referred as (s, S) model or a threshold model, because value $\dot{s}$ is threshold (trigger) point of reorder, and the highest value of stock $\dot{S}$ is fixed.

Table 8.3 Cycle parameters with two-tier model

	Cycle number		
Parameter	1	2	3
Demand D pcs per day	10	9	11
Order quantity Q	80	78	82
Cycle time T	8	8.5	7.5
Average stock $\overline{Z}$	70	71	69

8.2.4 Benchmarking of Inventory Management Models

The simplest model is a fixed reorder cycle model. This model is common among small businesses (shops), the procurement of which is done quickly and regularly from sustainable suppliers. In this case, the order quantity is usually determined manually considering the current consumption and the forecast for the next period. Comparing the data in Tables 8.1–8.3, we see that with fixed reorder cycle (Table 8.2), even with small fluctuations in demand, there are significant changes in both the order size and the average stock value.

The fixed reorder quantity model is used when the possible value changes with a big step—by the amount of product per pallet, container, tank, etc. In these cases, at small variation in demand the changes in order quantities are undesirable. Furthermore, this model is usually used if the product which is consumed in small amounts has the so-called transit rate—minimum quantity shipped to one consignee. Where there is the transit rate of the order quantity is forced to coincide with the value of this rate.

From Table 8.1, it is clear that even a slight reduction in demand in this model leads to a significant increase in the cycle time that is fraught with the possibility of obsolescence of a long stored product. The moment of order in the fixed reorder quantity model is usually determined manually.

Unlike the first two models, the two-tier model is well suited for automatic reorder and therefore widely used in automated stock management systems. The analysis of Table 8.3 shows that here small variations in demand lead to slight deviation of the order quantity, cycle time, and the average stock value.

It should be noted that the two-tier model allows two possibilities of reorder launch: (a) when the stock value becomes equal to value $\dot{s}\cdot$ and (b) when the stock value is less than $\dot{s}\cdot$ by one unit. The difference between these variants is important in those cases where the value of demand is low and the stock changes slowly, especially at small quantity of the safety stock.

Since the product is normally consumed by portions, the amount of stock rarely coincides exactly with threshold value $\dot{s}$. Therefore, the most common rule of reorder launch is condition

$$Z \leq \dot{s}, \tag{8.12}$$

i.e. to perform this launch it is necessary that the current value becomes equal or less than threshold $\dot{s}$.

When determining the current value of the stock, it is necessary to take into account the stock in transit. The value of this stock depends on delivery duration L. If $L \leq T$, i.e. the delivery duration is less than or equal to the cycle of stock fluctuations, then one product lot should be in transit; otherwise the number of lots in transit can be several. The above stock models are for the first case as a rule.

In the two-tier model, the order is often approximated to a quantity of a number of allowable values. With small fluctuations in demand in such cases the quantity of the order is simply equal to the optimal fixed value. However, with such approximation the stock after another receipt should not exceed the volume (area) of its storage container.

8.2.5 Kanban Inventory Management Model

Currently system Kanban has become widely known. It is usually defined as a system of "pulling" products from the previous echelon of production. In terms of models outlined above, system Kanban is some kind of two-tier model, which uses fixed reorder quantity (Axseter 2006).

This system's information medium is the turnover mechanism of cards (kanbans) of two kinds used for production management. The information in the cards of the first type—withdrawal kanbans—describes the number of objects that must be delivered to the place of consumption. In the second type of cards—production-ordering kanbans—the number of products, which must be made at the production site, are recorded. Figure 8.5 shows the diagram of kanban circulation between two echelons of the production process—machines 1 and 2.

Besides machines 1 and 2, the diagram shows two buffers (storages) B1 and B2 of cards exchange. In buffer B1, the selection card that has come with empty container is replaced by the order card, which then together with this container is sent to machine 1. The withdrawal kanban from buffer B1 is transferred to B2. Container 1 with the product of machine 1, accompanied by the production-ordering kanban, goes into B2 too. In B2 on this container the production-ordering kanban is replaced by a withdrawal kanban and then the container is transferred for processing on machine 2.

Replacement of the withdrawal kanban by ordering kanban happens every time after another container that is fed to machine 2 becomes empty. Since the capacity of all containers is the same, the order quantity Q is equal to the capacity. The stock value at reorder point $\dot{s}$ is also constant and equal to the product quantity in all containers in transit from machine 1 to machine 2. This amount is determined by demand value D and cycle time L for delivery of product produced on machine 1 to machine 2. In addition, the need for the safety stock should be taken into account. Thus, the stock at the reorder point

$$\dot{s} = DL + Z_c. \tag{8.13}$$

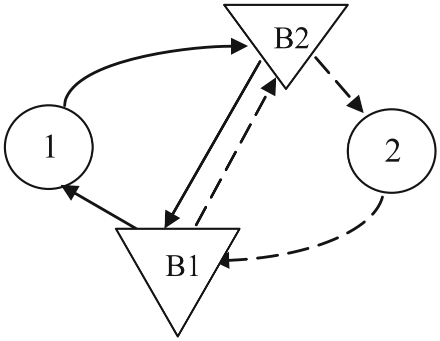


Fig. 8.5 Kanban turnover diagram. *Solid arrows*—production-ordering kanbans; *Dashed arrows*—withdrawal kanbans

To calculate the duration of delivery cycle of the product we take into account, which as can be seen from Fig. 8.5, it is made of duration of the following operations (Productivity Press Development Team 2002):

- Delivery of a withdrawal kanban from machine 2 to buffer B1;
- Replacement of a withdrawal kanban by a production-ordering kanban and its delivery to machine 1;
- Production of a product lot on machine 1;
- Delivery of the container with product from machine 1 to buffer B2;
- Replacement of a production-ordering kanban by a withdrawal kanban and delivery to machine 2.

The number of containers themselves, as well as cards (kanbans), is equal to the safety stock at order point s, divided by the container volume Q. Note that in this case the cycle of fluctuations in the stock at the consumption point is usually less than the duration of delivery L.

The model under consideration is some combination of (S, Q) and (s, S) models described in Sects. 8.2.1 and 8.2.3 above. In this case, there is a management variant (s, Q) with fixed reorder point $\dot{s}$ and fixed order quantity Q, which is sometimes referred to as (R, Q). In this management model, the same as in (S, Q) model with fixed reorder quantity, the average value of stock $\overline{Z}$ in the delivery cycle does not change with small variations in demand.

If we compare the management models discussed in this paragraph with two ways of production organization—“push” and “pull,” referred to in Sect. 1.2.2, it is possible to conclude that the fixed reorder cycle model (S, T) is close enough to the first one, and the second one corresponds to kanban model (R, Q).

As it was shown in Table 8.2, when using model (S, T) with decreasing demand the value of average stock increases. Therefore, model (R, Q), in which the average stock quantity is not dependent on the demand and, accordingly, the pull method provides significantly smaller need for stocks compared to the push method.

The stock graph for model (R, Q) is much more complicated than the graphs in Figs. 8.2–8.4, because the stock change in this case is determined not just by consumption in one echelon but by performance of two coupled machines, as well as the number of containers (and kanbans) on the line. The less kanbans are in the chain, the lower stock is in the system, but the more rigid connection between the machines in this chain. Ideally, only two containers are used and model (R, Q) becomes the so-called two-tier system, in which there is no excess stock, but in this case the performance of both coupled machines must be exactly the same and this management model is transformed into a model of automatic line of type 3a (Sect. 1.3.1).

Under circumstances where the performances of the coupled machines are markedly different, the need for more containers on the line becomes mandatory. The book (Piasecki 2009) states that since formula (8.13) implies that the stock of these containers depends on demand value D, this demand must be predicted on a time horizon. The need for such forecast actually brings the pull method close to the push method, which is entirely based on demand forecasting. Therefore, in all probability, with a decrease in serial production the capabilities of both methods as to the stock quantity are gradually equalized.

8.3 Inventory Management Model Under Uncertainty

Above in Sect. 1.6.1, it was remarked that the most important quality of the supply chain in the market of any type is the high level of customer service. At the same time, from Sect. 6.1 it followed that demand value D is an independent random variable. Therefore, it is clear that the stock model under uncertainty must ensure the amount of current reserves that the service level would be high enough at forecasted demand. However, since too much stock has negative impact on economic performance, the objective of the model under such circumstances is to optimize stock, taking into account the requirements described above.

8.3.1 Customer Service Level

In practice, a convenient definition of the service level is the understanding of this value as the level of fulfilment of external orders, i.e. as a part of the immediate satisfied demand for a certain period. This value can be calculated as a percentage of the complete order fulfilment, the percentage of fulfilment of order items, percentage of sale of specific physical units of the product as well as percentage of orders fulfilment in terms of value. The order item is understood here as the line (entry) indicating the quantity of a particular product. Table 8.4 shows the example of execution of some orders and Table 8.5 shows service level values calculated in a different way.

As seen from Table 8.5, there is a significant difference between the different calculation methods of satisfied demand. Obviously, the most difficult is the complete fulfilment of orders, but it is unlikely this indicator should be used. In

Table 8.4 Execution of orders

Order number	Product type	Ordered quantity	Dispatched quantity	Deficiency	Order cost	Supply amounting to
1	A	20	20		100	100
	B	40	40		400	400
	C	12	12		180	180
2	B	30	20	10	300	200
	D	25	25		300	300
	E	15	15		300	300
	F	20	20		400	400
3	A	40	40		200	200
	C	15	15		225	225
	E	20	0	20	400	0
4	D	20	20		240	240
	F	10	10		200	200

Table 8.5 Demand satisfaction indicators

Indicators	Percentage	Remark
Complete order fulfilment	50	Out of 4 orders 2 are fulfilled completely
Fulfilment of order items	83.3	Out of 12 items 10 are fulfilled completely
Fulfilment of supply scope	88.7	Out of 267 units of products 237 are supplied
Fulfilment by cost	84.5	Instead of amount 3245 the supply amounts to 2745

all probability the rate of execution of order items is the most impartial, which we will use in future.

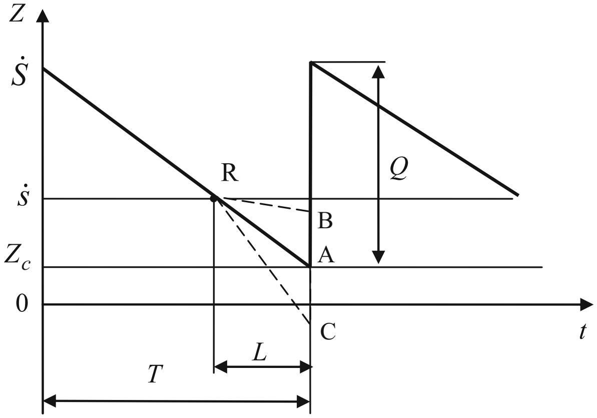
Each order item can be executed if there is sufficient available stock of the corresponding product. In this case, the level of service can be understood as the probability that during the execution of the order its quantity will not exceed the amount of current stocks. For example, the service level of 97 % means that the available stock is sufficient for deficit-free operation during the order processing with a probability of 97 %. We designate this definition of service level as S_L.

8.3.2 Shortages Permitted Inventory Management Model

Figure 8.6 shows the graph of current stock changing within one supply cycle in the case allowing shortage.

Figure 8.6 uses the same designations as in Figs. 8.1–8.4. On the stock graph, the reorder point is shown by straight line $\dot{S}R$ describing its uniform decrease corresponding to consumption D. As mentioned above, the position of reorder point $\dot{s}$ is determined by relationship (8.2), where the consumption rate during delivery time L of the order is assumed to be the same as on the section $\dot{S}R$.

Fig. 8.6 The stock graph in case of possible shortage



If this assumption is correct, then by the time of receipt of a new product lot (the end of the cycle) stock value Z_A will be equal to the quantity of safety stock Z_c. However, if the consumption rate after the order placing is different from the rate on section $\dot{S}R$, then by the end of the cycle the stock quantity may be in the range from Z_B to Z_C. In the latter case, it may appear that the safety stock is not enough, and there is deficiency.

Straight line $\dot{S}A$ corresponds to the mean value of the demand during the cycle, while at different moments in time the demand is a random variable fluctuating near the average. Figure 8.7 shows a typical histogram of daily demand within the cycle period. On the abscissa the values of demand divided by 10 are plotted and on the vertical axis the demand values' frequency is plotted. For example, histogram column 4 for demand 40 shows that the demand values ranging from 30 to 40 account for 20 % of all events of demand for the period.

Obviously, in this case the most probable value of the daily demand D is 50 pcs./ day. Assume that the order delivery duration L is 1 day, and the quantity of safety stock is 20 units. In this case, the stock value at the reorder point $\dot{s} = 70$ pcs.

At the same time, there is a possibility that the consumption within the delivery time with 4 % chance will be in the range of 70–80 pcs., and with 1 % chance will be in around 80–90 pcs. Hence, the probability of shortage under the given conditions can be 5 % and, accordingly, service level $S_L = 95$ %.

8.3.3 Demand Distribution Functions

The distribution function of random value ξ is the probability P that ξ takes the value less than the value of some variable x, i.e.

$$\Phi(x) = (\xi < x). \tag{8.14}$$

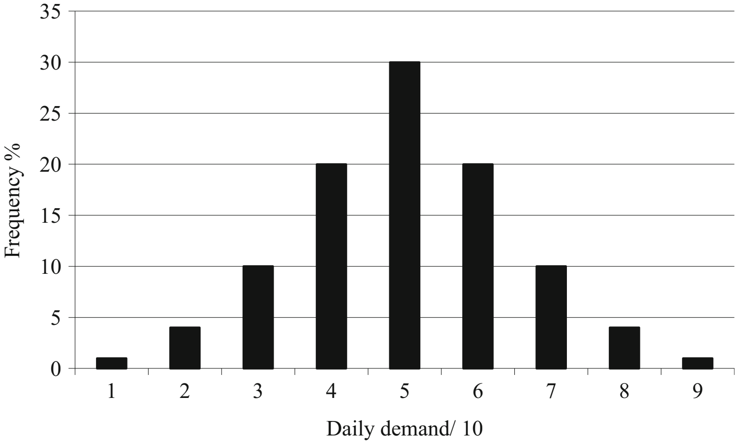


Fig. 8.7 Daily demand histogram

The histogram in Fig. 8.7 corresponds to the so-called Poisson distribution (Sect. 3.2.2) of a random value—demand in this case. This distribution is distinctive in that the probability of a zero value of the random variable is definitely equal to 0, while any large value of the random variable has small but finite probability.

Instead of the Poisson distribution the so-called normal distribution is considered, which is symmetrical to a mean value of the variable. Normal distribution, corresponding to the histogram in Fig. 8.7, is shown in Fig. 8.8. On the ordinate the density of normal distribution of demand D is plotted

$$\varphi\left(\frac{D-\mu}{\sigma}\right)=\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{D-\mu}{\sigma}\right)^2},\tag{8.15}$$

where μ is the mean value of demand D and σ is the standard deviation.

The mean value is equal to the mathematical expectation of the random variable

$$\mu=E(D)=\int\limits_{-\infty}^{\infty}\frac{D}{\sigma}\varphi\left(\frac{D-\mu}{\sigma}\right)dD.\tag{8.16}$$

The function of normal distribution depends on the demand value and is represented by the area under the distribution density curve from $-\infty$ to the demand value

$$\Phi\left(\frac{D-\mu}{\sigma}\right)=\frac{1}{\sigma\sqrt{2\pi}}\int\limits_{-\infty}^{D}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}dx.\tag{8.17}$$

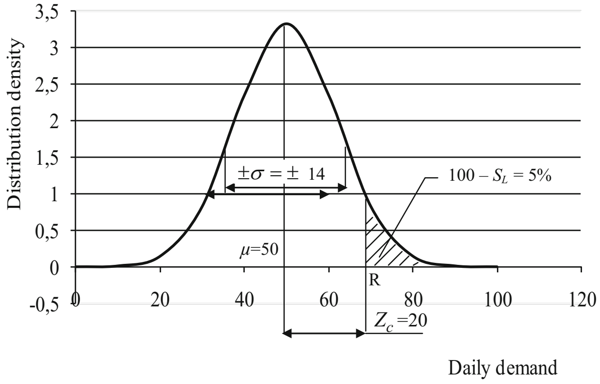


Fig. 8.8 Normal distribution of daily demand

Standard deviation σ is related to forecast mean absolute error $\overline{M}_a$, determined by formula (6.10), by relation

$$\sigma = \sqrt{\pi/2}\overline{M}_a \approx 1.25\overline{M}_a. \tag{8.18}$$

The shaded area of behind point R in Fig. 8.8 is equal to shortage probability 100 % $-S_L$ at safety stock $Z_c = 20$ pcs. Recall that the normal distribution function shown in Fig. 8.8 is equal to service level S_L only if order lead time L is equal to 1 day.

The value of demand deficiency can be described by the so-called loss function (Axseter 2006)

$$G(D) = \int\limits_{D}^{\infty} (\nu - D)\varphi(\nu)d\nu, \tag{8.19}$$

the derivative of which according to the rule of definite integral differentiation by parameter

$$G^{'}(D) = \Phi(D) - 1. \tag{8.20}$$

8.3.4 Newsvendor Problem

This problem is a classic example of the use of the distribution function in the inventory theory. Let us consider one cycle (day) of an order and sale of some quantity of newspapers Q. If the quantity of newspapers Q ordered for sale is too large, then a part of the order is not sold, and in this case the seller suffers a loss from each excess newspaper equal to its ordering price c_o. If the number of purchased newspapers for subsequent sale is less than the actual need, then there is loss of potential profit c_u from each unsold newspapers.

Considering that demand D is a random variable with mean value μ and standard deviation σ, we define value Q, at which vendor's loss $c(D)$ is minimal. To do this, we note that

$$\begin{aligned} &c(D) = (Q - D)c_o \ \text{ at } \ D < Q \\ &\text{and} \\ &c(D) = (D - Q)c_u \ \text{ at } \ D > Q. \end{aligned} \tag{8.21}$$

Considering that demand is a random function with distribution density $\varphi\left(\frac{D-\mu}{\sigma}\right)$, the total losses of the vendor can be defined as the mathematical expectation of loss amount (formula 8.21), i.e. $c = E[c(D)]$ or

$$c = c_o \int\limits_{-\infty}^{Q} \frac{Q - D}{\sigma} \varphi\left(\frac{D - \mu}{\sigma}\right) dD + c_u \int\limits_{Q}^{\infty} \frac{(D - Q)}{\sigma} \varphi\left(\frac{D - \mu}{\sigma}\right) dD. \tag{8.22}$$

Using expression (8.16), we have

$$c = c_o(Q - \mu) + (c_o + c_u) \int\limits_{Q}^{\infty} \frac{(D - Q)}{\sigma} \varphi\left(\frac{D - \mu}{\sigma}\right) dD \tag{8.23}$$

and taking into account (formula 8.19), we obtain

$$c = c_o(Q - \mu) + (c_o + c_u)\sigma G\left(\frac{D - \mu}{\sigma}\right). \tag{8.24}$$

To define the minimal value of losses, we take the derivative and using formula (8.20) we obtain the following equation:

$$\frac{dc}{dQ} = c_o + (c_o + c_u)\left[\Phi\left(\frac{Q - \mu}{\sigma}\right) - 1\right] = 0, \tag{8.25}$$

from which we find the solution of the problem in the form

$$\Phi\left(\frac{Q - \mu}{\sigma}\right) = \frac{c_u}{c_o + c_u}. \tag{8.26}$$

As an example, consider the case, in which the purchase price $c_o = 10$, the sales up-lift $c_u = 15$, and daily demand distribution correspond to the graph in Fig. 8.8. We have

$$\Phi\left(\frac{Q-50}{14}\right)=\frac{15}{10+15}=0.625.$$

To define the optimal value of order quantity Q, we use integral function NORMDIST from MS Excel, selecting integral values Q so that the value of function NORMDIST $\left(\frac{Q-50}{14}\right)$ is as close as possible to 0.625. This value (0.639) is $Q=55$. As you can see, the optimal quantity of the order in this case is somewhat higher than the mean value of demand.

The problems similar to those described are often found when ordering various perishable or rapidly obsolescent products.

8.4 Inventory Management Using Logistic Operating Curves

Storage Operating Curves method can be used to define the optimal stock level as well as to account the supply deviations' influence on the capability to provide service.

8.4.1 Storage Curves and Their Applications

A model of logistic chain of the procurement process can be represented by a diagram of stock movement at the storage similar to the diagram of processing time flow in manufacturing above in Sect. 3.2.1 in Fig. 3.6. In this case, the steps in the diagram are determined by the values of receipt and withdrawal from the storage. The current stock is equal to the vertical segment between the curves of receipt and withdrawal, and the horizontal line between these curves represents time spent in the storage.

The dilemma of operational planning for the case of storage management described above in Sect. 3.2 is to find a balance between high level of customer service and low value of stocks. A measure of the service level may be a compliance degree of the plan of requests fulfilment to receive from the storage and actual values of goods or materials withdrawal, wherein the mean delivery delay for a certain period may be determined as the sum of productions of the shortage quantity per number of days of waiting. The dependence of mean delivery delay $\overline{T}$ on the average value of stock $\overline{Z}$ for the period can be represented graphically in the form of so-called storage operating curve.

In a similar way to the above logistics theory for production, the theory of storage operating curve is based on a comparison of this curve with some ideal curve. It is assumed that the withdrawal from the storage is uniform and in the same quantities of some product. For such conditions (Nyhuis and Wiendahl 2009), it is found that the relationship between $\overline{T}$ and $\overline{Z}$ is described by formula

$$\frac{\overline{T} = \overline{Q}/2 - \sqrt{2\overline{Q}\,\overline{Z}} + \overline{Z}}{\overline{D},} \tag{8.27}$$

where $\overline{Q}$ is the average size of goods or material lot, received at the storage, and $\overline{D}$ is the average rate of withdrawal of goods or material from the storage.

Function (8.27) equals 0 at

$$\overline{Z}_0 = \frac{\overline{Q}}{2}. \tag{8.28}$$

The second typical point of dependence (formula 8.27) is determined at $\overline{Z} = 0$

$$\overline{T}_0 = \frac{\overline{Q}}{2\overline{D}}. \tag{8.29}$$

The family of curves, which includes both an ideal storage curve and the actual curves, by analogy with the family of logistic operating curves can be described by parametric dependencies

$$\overline{Z} = \overline{Z}_1 \times t; \quad \overline{T} = \overline{T}_1 \times \sqrt[c]{1 - t^c} \text{ at } 0 \le t \le 1, \tag{8.30}$$

where value $\overline{Z}_1$ corresponds to the stock with zero delay with regard to necessary safety stock and value $\overline{T}_1$ corresponds to the delay value at zero stocks and possible deviations from the ideal conditions of supplies. For the case of the ideal curve, degree indicator c equals 0.5; $\overline{Z}_1 = \overline{Z}_0$; $\overline{T}_1 = \overline{T}_0$.

The farther the actual storage curve is separated from the ideal, the greater, at the set delay value, stocks must exist, which are necessary for customer service. Modern production typically uses a very large range of materials and components and, as a rule, it is impossible to hold stocks, immediately providing user requests, for all items at the same time. Therefore, it is important to establish such quantities of stocks for certain items that would ensure acceptable level of delays in actual practice.

Since the entire set of storage items is usually divided into consumption groups in accordance with the widely used ABC classification, then for each group it is advisable to carry out logistical positioning, which is to determine the rational operating point on the relevant storage curve. At the same time, the form of the storage curve may vary for the same product depending on the nature of the supplier's activity.

Suppliers can also be divided into groups by the so-called UVW classification based on the delays of shipments: group U includes suppliers with small delay, group V—average delays values, and group W—with the values significantly higher than the average ones. Assessment of the reliability of suppliers is probably the most relevant for suppliers of W group. If some item is supplied by various suppliers in parallel, then to evaluate their relative reliability it is possible to build

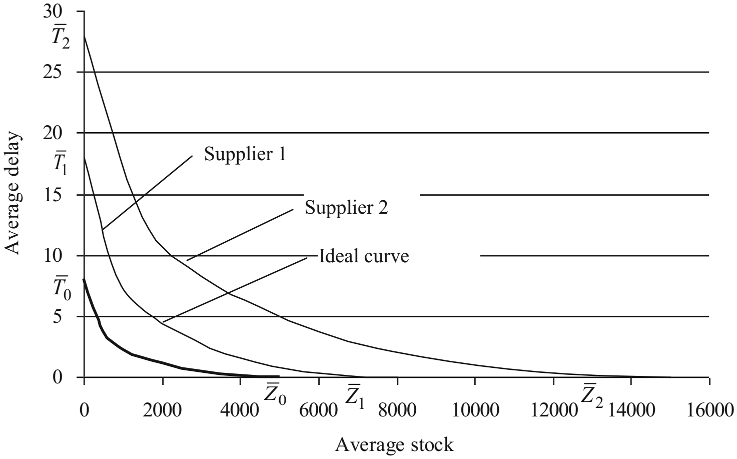


Fig. 8.9 Comparison of suppliers using storage curves [based on Nyhuis and Wiendahl (2009)]

logistics storage curves, using expressions (8.28) and (8.29), and compare them with the ideal curve (Fig. 8.9). Table 8.6 gives comparative data for two suppliers and relevant ideal storage curve.

Advantages of supplier 1 over supplier 2 are obvious.

8.4.2 Finished Product Inventory Sizing to Optimize the Overall Production Performance

Let us consider the case when all manufactured products can be sold without delays. Then the overall production efficiency E is defined by formula (3.26)

$$E = \frac{(\overline{z} + \overline{w})\left[a\overline{P}/\left(b_1 + b_2\overline{P} + b_3\overline{P}\overline{z}\right) - 1\right]\overline{P}}{P_{\max}\overline{F}},$$

where $\overline{F}$ is the mean duration of production cycle, $\overline{z}$ is the mean stock of finished products in days of production, $\overline{w}$ is the mean waiting period, for which the customer agrees, from the moment of order placement to its fulfilment, $\overline{P}$ is the mean output rate, and $P_{\max}$ is the maximal output rate.

To find the optimal stock $\overline{z}$, providing the most production efficiency, we take a derivative and set it equal to zero

$$\frac{dE}{d\overline{z}} = \frac{\overline{P}}{P_{\max}\overline{F}}\frac{d}{d\overline{z}}\left[(\overline{z} + \overline{w})\left(\frac{\overline{P}}{\alpha + \beta\overline{P} + \gamma\overline{P}\overline{z}} - 1\right)\right] = 0, \qquad (8.31)$$

where $\alpha = b_1/a$, $\beta = b_2/a$, and $\gamma = b_3/a$.

Table 8.6 Comparative data of suppliers

Curve	Minimal stock level $\overline{Z}_1$	The maximum delay of supply $\overline{T}_1$
Ideal	5000 pcs.	8 days
Supplier 1	8000 pcs.	18 days
Supplier 2	15,000 pcs.	28 days

By calculating the derivative in formula (8.31), using a number of algebraic transformations we obtain for the optimal value of stock

$$\overline{Z}^* = \frac{1}{\gamma}\left(\sqrt{\frac{\alpha}{\overline{P}} + \beta - \gamma\overline{w}} - \frac{\alpha}{\overline{P}} - \beta\right). \tag{8.32}$$

Figure 8.10 shows the graphs of optimum stock variation depending on the allowable waiting period of customers at different output rates $\overline{P}$, h/day. In this example, we used the values of the coefficients from Sect. 3.3.3: $a = 2000$ unit/h, $b_1 = 350$ thous. of units/day, $b_2 = 500$ unit/h, and $b_3 = 50$ unit/h/day.

As can be seen from Fig. 8.10, the optimal stock decreases with an increase in the allowable waiting period and at some point becomes equal to 0. For it is assumed that when output rate $\overline{P}$ is actually determined by the volume of sales, the graphs indicate that the optimal quantity of the stock with an increase in sales volume increases first and then stabilizes.

The optimal quantity of the stock depends on the values of input coefficients b_1, b_2, and b_3 and profit margin a, and the decrease of the input coefficients or the increase of profit margin leads to increase in the stock.

8.5 Safety Stock Sizing

Upper limit $\dot{S}^{\cdot}$ of the storage stock, as it can be seen from Fig. 8.6 for example, is determined as the sum of order quantity Q and the quantity of safety stock Z_c. The optimal quantity of the order is calculated by the above formula (2.4), and the calculation is discussed here below in Chap. 11.

In the inventory theory, mainly two approaches are used to determine the level of safety stocks. The first of them (normative), using different rules, calculates the level of stock, which should guarantee fully balanced operation. Several of these rules are described in detail in Lukinsky (2007). The comparative analysis of calculation by these rules, which was carried out in this paper, shows that the results obtained in various ways are markedly different and, of course, hardly actually ensure no shortage.

The more modern approach (stochastic) is that not the full absence of shortage should be guaranteed but the set level of customer service, as described in Sect. 8.3.

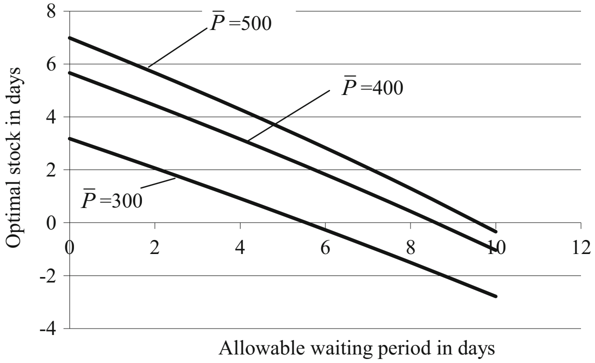


Fig. 8.10 Dependence of optimal stock on the allowable waiting period

8.5.1 Calculation of Safety Stock with Random Demand

Consider first the case when demand is a random variable, and other parameters of the stock management process, lead time, interval between requests, etc., are determined. It is easy to establish a link between level of service S_L and the quantity of safety stock Z_c, if you plot a graph of demand distribution for lead time L in days. This graph is completely similar to the graph in Fig. 8.8, and its parameters $\mu^{'}$ and $\sigma^{'}$ are related to the parameters of the daily demand through dependencies

$$\mu^{'} = L\mu; \quad \sigma^{'} = \sigma\sqrt{L}. \tag{8.33}$$

As the area under the curve of demand density function (Fig. 8.8) to the point determined by the stock value is equal to the service level, then

$$S_L = \Phi\left(\frac{R-\mu^{'}}{\sigma^{'}}\right) \tag{8.34}$$

and since $R - \mu^{'} = Z_c$, we obtain

$$S_L = \Phi\left(\frac{Z_c}{\sigma^{'}}\right). \tag{8.35}$$

Dependency (formula 8.35) is usually written in the form

$$Z_c = \kappa\sigma^{'}, \tag{8.36}$$

where safety factor κ is fully determined by service level S_L and can be found using function NORMSINV MS Excel. For example, NORMSINV (0.99) = 2.326.

Standard deviation $\sigma^{'}$ of demand during delivery can be calculated not only according to the daily demand, but, in general, according to available data of distribution function of demand for any forecasting interval I in days as

$$\sigma^{'} = \sigma\sqrt{\frac{L}{I}}. \tag{8.37}$$

Let the lead time be $L = 2$ days. To distribute the daily demand in Sect. 8.3.3 with $\sigma = 14$ and service level $S_L = 0.99$, we obtain

$$Z_c = 2.326 \times 14\sqrt{2} \approx 46 \quad \text{pcs.}$$

8.5.2 Sizing of Safety Stock with Two Random Variables

In fact, the process of stock management takes place under the conditions, where not only the demand but also other process parameters are also random variables. First, a random value is the order lead time. To calculate the quantity of safety stock in this case it is natural to try to use formula (8.36), where we can try to determine somehow the value of standard deviation $\sigma^{'}$ regarding the influence of random variable L.

The simplest option of such integration is described, for example, in Vollmann et al. (2005) as the function

$$\sigma^{'} = \sqrt{\overline{D}\sigma_D^2 + \overline{L}\sigma_L^2}, \tag{8.38}$$

where $\overline{D}$ is the mean demand, units/days; $\overline{L}$ is the mean lead time in days; σ_D is standard deviation of demand in units; and σ_L is standard deviation of lead time in days. It is obvious that components under the radical sign in expression (8.38) have different dimensions, but, oddly enough, recognizing this fact, the authors of the book (Vollmann et al. 2005) continue using this expression.

More famous is the option of the joint account for deviations in demand and lead time in the form

$$\sigma^{'} = \sqrt{\overline{L}\sigma_D^2 + \overline{D}^2\sigma_L^2}. \tag{8.39}$$

In this expression, as in the previous one, the components under the radical sign also have different dimensions. This fact clearly indicates the impossibility of its use. In the book (Grigoriev et al. 2007), the following relationship is proposed in order to close the dimensions gap:

$$\sigma^{'} = \sqrt{\overline{L}^2\sigma_D^2 + \overline{D}^2\sigma_L^2}; \tag{8.40}$$

however, as shown in (Lukinsky 2007), this expression is valid only if the demand-time dependence is linear.

In our opinion, it seems unacceptable to use the dependence like (formulas 8.38–8.40), at least as long as the experts in the field of probability theory do not find out about the origin of the paradox. In the current situation, we can suggest to use semi-empirical approach based on a number of theoretical and experimental data on the use of dependencies (formulas 8.36 and 8.37).

First of all, in Axseter (2006) it is indicated that dependence (formula 8.37) is satisfied exactly if the values of demand standard deviation σ in different time periods are independent. Since this is often not the case, then theoretically, instead of formula (8.37), we should use a more general dependency

$$\sigma' = \sigma\left(\frac{L}{I}\right)^c, \tag{8.41}$$

where index $0.5 \le c \le 1$. Value $c = 0.5$ corresponds to the independence of demand values in different time periods, value $c = 1$—the case of full correlation, and actual value c should be between 0.5 and 1.

In the paper of Rosenfield (1994), extensive experimental data were obtained to determine the dependence of the demand standard deviation on the mean lead time $\overline{L}$ and the mean value of demand $\overline{D}$ for consumer goods. It turned out that this dependence is well described by expression

$$\sigma_{T,D} = a\overline{D}^{0.7}\overline{L}^{0.7}, \tag{8.42}$$

where a is the coefficient determined for each specific item of goods. To calculate value a, it is necessary to obtain the value of the standard deviation by experiment with known and constant values of $\overline{D}$ and $\overline{L}$.

To reconcile the result (formula 8.42) with dependence (formula 8.41), we introduce the concept of the basic forecasting interval. We take the most convenient for observation time interval for this basic interval for assessing the demand distribution. For example, as in construction of histogram (formula 8.7), it is most convenient to define the parameters of daily demand; the basic forecasting interval here is obviously 1 day.

We denote the value of mean demand $\overline{D}$ on the basic interval through μ and the value of the standard deviation through σ. Then, by analogy with expressions (8.41) and (8.42) we can write

$$\sigma' = \sigma\left(\frac{\overline{D}}{\mu}\right)^{0.7}\left(\frac{L}{I}\right)^{0.7} \tag{8.43}$$

and

$$Z_c = \kappa\sigma'. \tag{8.44}$$

From formula (8.43), it follows that with $\overline{D} = \mu$ and $\overline{L} = I$, deviation $\sigma^{'}$ automatically becomes equal to σ. If $\overline{D} = \mu$, and $\overline{L} \neq I$, then (formula 8.43) turns into (formula 8.41) with indicator $c = 0.7$.

If we use expression (8.43) for the example given in the previous Sect. 8.5.1, then as in this example $\overline{D} = \mu$

$$\sigma^{'} = \sigma\left(\frac{L}{I}\right)^{0.7} = 14\left(\frac{2}{1}\right)^{0.7} \approx 22.7$$

and the safety stock with service level $S_L = 0.99$

$$Z_c = \kappa\sigma^{'} = 2.326 \times 22.7 \approx 53 \quad \text{pcs.}$$

8.5.3 Sizing of Safety Stock with Three Random Variables

In addition to demand and time of delivery, the value of reserve stock can be significantly influenced by the order cycle time. If duration of the cycle is short (Piasecki 2009), i.e. with frequent and small orders, probability of shortage is significantly higher than with long duration.

The reason for this situation is the fact that with increasing quantity of order Q, at the same time the value of the maximal stock $\dot{S}^{\cdot}$ increases and according to formula (8.1) the value of mean stock $\overline{Z}$ as well. Since fluctuations in demand D within the order cycle are random, the probability of shortage occurrence with increasing mean stock in the cycle decreases.

In Piasecki (2009), it is assumed that the random variable of time T of the order cycle is independent of the random variable of demand within lead time $L_{,}$ and therefore, the probabilities of shortage for these reasons may be multiplied. Let us name the level of service, defined only by fluctuations in demand as the estimate level of service, and denote it as $S_L^{'}$. Probability of shortage due to fluctuations in demand within the lead time equals $1 - S_L^{'}$. Probability of shortage against random changes in the cycle duration with $L < T$ is the ratio of the mean lead time to the mean cycle time. Hence, the actual level of service S_L based on the cycle time is

$$\frac{S_L = 1 - \left(1 - S_L^{'}\right)\overline{L}}{\overline{T} \quad \text{with} \ \ L < T.} \tag{8.45}$$

Assuming that the desired level of service to be equal to the actual, and given that the cycle average duration $\overline{T} = Q/\overline{D}$, we obtain for the calculated level of service

$$S_L^{'} = 1 - (1 - S_L)\frac{Q}{\overline{LD}}, \tag{8.46}$$

where, as above, $\overline{D}$ is the mean demand in the pc/day, $\overline{L}$ is the mean lead time in days, and S_L is the desired level of service. Now to consider the influence of the cycle time, safety factor κ in expression (8.36) with $L < T$ should be determined not from the value of the desired level of service but from the value of estimate level of service $S_L^{'}$. If $L \geq T$, safety factor κ is determined by value S_L.

Let us continue discussing the example in Sect. 8.5.2 taking into account the cycle time. To do this, first of all, it is necessary to determine value Q, for which we use formula (2.4) of economic order quantity. Suppose, as in Sect. 8.2.1, cost of a single order c_o is 2000 units, cost of storage per product unit c_h equals 200 units/month. The value of mean demand in this example (Sect. 8.3.2) $\overline{D} = 50$ pcs/day, and the demand for the month is $\overline{D} \times 30 = 1500$ pcs/month.

In this case

$$Q^{*} = \sqrt{\frac{2 \times 1500 \times 2000}{200}} \approx 174 \text{ pcs.}$$

Considering that $\overline{L} = 2$ days, $S_L = 0.99$ (Sect. 8.5.2), we obtain from formula (8.46):

$$S_L^{'} = 1 - \frac{1 - 0.99}{2} \times \frac{174}{50} = 0.982.$$

Safety factor κ will be determined using function MS Excel NORMSINV (0.982) = 2.096.

Using formulas (8.43 and 8.44) and data of Sect. 8.5.2, we obtain

$$Z_c = \kappa\sigma^{'} = 2.096 \times 22.7 \approx 48 \text{ pcs.}$$

By comparing this result with the result of Sect. 8.5.2, we see that in this example, consideration of the influence of the cycle time reduced the quantity of safety stock by about 10 %. Generally speaking, with regard to large ration $Q/\left(\overline{LD}\right)$, i.e. when the order quantity is much more than the demand within the lead time, the estimate level of service can dramatically decrease. With value $S_L^{'} = 0.5$ function NORMSINV (0.5) = 0, which means that there is no need for safety stock. Therefore, there is no need for calculation of safety stock for $S_L^{'} \leq 0.5$.

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Supply Chain Inventory Dynamics 9

9.1 Stock Distribution Planning in the Chain

Supply Chain Management (SCM) mentioned in Sect. 1.4.1 has many different functions: rational networking for storage and distribution of goods, selection of suitable vehicles and their loading, determining the required number of items in the various echelons of the supply chain, timely transfer of information between the echelons, optimization of the expanses for stocks promotion in the network, etc.

This chapter covers only a small part of these issues relating to distribution resource planning (DRP) in the associated echelons of the supply chain. Expediency of presentation here of this problem is due to the fact that information on stocks in the chains can be directly used in the sales and operations planning.

On one hand, DRP ensures information transfer on demand from points of sale to sources of supply and, on the other hand, transmits data on supplies from sources of supply to the points of sale. All the main echelons of the supply chain must use DRP for selection of transport, its loading, and storage provision. DRP allows you to choose rational packaging of products, enabling to reduce the amount of product in one package consistently as the supply approaches the ultimate consumer.

In the tables, shown below, the quantity of products is measured in some physical units.

9.1.1 DRP Technique

To perform planning for stock-keeping unit (SKU) usually the following input data are used:

- Demand forecast for the planning period (week, month);
- Current stocks at storage sites;
- Safety stocks at storage sites;
- Recommended sizes of delivery lots;

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Table 9.1 Stock movement of one product at the local storage

Product A_1; initial balance $Z_0 = 20$; safety stock $Z_c = 10$; lot $Q = 50$; lead time $L = 1$ period; previously ordered (in transit) 0 units.							
Parameter	**Planning periods**						
	1	2	3	4	5	6	7
Forecast	15	15	18	18	20	23	25
To be received		50		50			50
Estimated balance	5	40	22	54	34	11	36
New order	50		50			50	

- Lead time to different echelons of the supply chain.

DRP is performed by preparing a number of DRP tables, where the current balance is calculated consistently for a particular product and the required needs for the future are determined. Table 9.1 shows an example of such a table for one product for multiple future periods (weeks) with the horizon of seven periods constructed similarly to Vollmann et al. (2005).

In the header of Table 9.1 the main parameters of planning and data about the initial state of the stock are shown. In this case, there is management version (R, Q) with fixed quantity of order Q and reorder point R determined by sum $\dot{s}$ of safety stock Z_c and forecasted consumption D. To determine the quantity of the order lot, formula (2.4) of economic lot can be used; the safety stock is calculated based on the procedure described in Sect. 8.5.

The value of the estimated current balance is determined at the end of corresponding period n by the following dependence:

$$Z_n = Z_{n-1} + Q_n - D_n; \tag{9.1}$$

herewith the order in period n is placed if

$$Z_n \leq \dot{s} = Z_c + D_{n+1}. \tag{9.2}$$

For example, at the end of period 1 estimated balance Z_1 is equal to the difference of initial balance Z_0 of 20 units and forecast D_1 of 15 units, i.e. 5 units. Since this value is less than sum $\dot{s}$ of safety stock of 10 units and the forecast for the second week equalling 15 units, i.e. 25, the next order should be made for the first week. The value of this order is reflected as expected arrival in period 2 in line “to be received”.

By the end of period 2 the balance is equal to

$$Z_2 = Z_1 + Q_2 - D_2 = 5 + 50 - 15 = 40$$

Table 9.2 Stock movement of the product at the central storage

Product A_1; initial balance $Z_0 = 100$; safety stock $Z_c = 50$; lot $Q = 80$; lead time $L = 1$; previously ordered (in transit) 80 units.

Parameter	**Planning periods**						
	1	2	3	4	5	6	7
Local storage B_1	50		50			50	
Local storage B_2		70				70	
Orders from places	50	70	50			120	
To be received	80		80		80	80	
Estimated balance	130	60	90	90	170	130	130
New order		80		80	80		

This quantity provides the demand of the next, the third period of 18 units, and the remaining value of 22 units is more than the required safety stock of 10 units. Therefore, there is no need for the order in the second period.

In the third period, the existing stock meets the needs of the fourth period, but the remaining quantity is less than the safety stock and therefore it is necessary to place another order in the third period. The planned stock of the fourth and fifth periods meets the needs of the fifth and sixth periods accordingly, and in the sixth period, there is a need for a new order. This method of stocks management in DRP reminds of "Kanban" management model described in Sect. 8.2.5.

Orders, defined by the tables such as Table 9.1 in local storages, enter the central distribution storage and provide the basis for production orders (Table 9.2).

Table 9.2 has not the forecasts but the orders based on these forecasts from each of the local storages. Suppose, for example, product A_1 is distributed from the central storage to two local storages; herewith the orders from the first storage match Table 9.1, and from the second one are similar to the table for this stock.

The values of the safety stock and lot quantity at the central storage, of course, are much higher than at each of the local storages. As can be seen from Table 9.2, the total order from places can be very uneven, which in general leads to significant fluctuations in stock at the central storage. Therefore, after determining the estimated orders on the total order of local storages, actual orders for production are made on their basis. For period 1, the estimated balance is

$$Z_1 = Z_0 + Q_1 - D_1 = 100 + 8 - 50 = 130.$$

In period 1 there is no need for a new order, since the current balance of 130 units is more than the sum of safety stock of 50 units and consumption in the second period of 70 units. On the contrary, in the second period a new order is clearly necessary because the rest in the third period equal to 10 units is less than the safety stock. Generally speaking, the need for a new order does not occur until period 5 and in the fifth period the stock of 90 units will be kept.

When ordering in the fifth period in the amount of one lot of 80 units, the balance of the sixth period will be equal to 50 units, that is exactly equal to the safety stock. If we stick to the position corresponding to condition (9.2), in which the current reorder point is the stock value less than or equal to the safety stock, then in the fifth period it is necessary to order the quantity of two lots at once, i.e. 160 units.

For more even flow of orders into production, in this case it is advisable to make two orders of 80 units and spread them in the periods. It should be borne in mind, however, that increase of the uniformity of orders leads generally to increase in the mean value of the stock. For example, in the above case, the current balance of the fifth period has increased to 170 units, which is much higher than the mean stock.

9.1.2 Regular Maintenance of DRP Tables

If the sale coincides with the forecast, and the scope and time of delivery corresponds to the order in the DRP table, then going to the planning in the new period in the DRP table, simply add a new column. However, such a successful situation is rare; in most cases some of the scheduled operations are performed partially or untimely. Table 9.3 shows four DRP tables for a single product, made in sequential order.

In each of these tables three lines are added, which record the actual state at the end of current period *t*—actual consumption $D^{'}$, and actual receipt $Q^{'}$ and actual balance $Z^{'}$ equalling

$$Z_t' = Z_{t-1}' + Q_t' - D_t'. \tag{9.3}$$

The value of the planned balance in the period immediately following the current one is calculated by formula

$$Z_{t+1} = Z_t' + Q_{t+1} - D_{t+1}, \tag{9.4}$$

and in the rest periods it is calculated according to dependence (9.1).

If the current period is the first period (first part of Table 9.3), then at the end of it

$$Z_1' = Z_0' + Q_1' - D_1' = 20 + 0 - 12 = 8.$$

The stock in the second period is equal to $Z_2 = Z_1' + Q_2 - D_2 = 8 + 50 - 15 = 43$ and in the third period $Z_3 = Z_2 + Q_3 - D_3 = 25$. In the third period (Table 9.1) a new order is necessary. Thus, in the fourth period $Z_4 = Z_3 + Q_4 - D_4 = 57$, etc.

In the second part of Table 9.3 the current period is period 2. Let the actual consumption in this period is 20 units and the receipt is 50 units. Then the actual balance in this case $Z_2' = Z_1' + Q_2' - D_2' = 8 + 50 - 20 = 38$. The estimated balance of the third period will make $Z_3 = Z_2' + Q_3 - D_3 = 38 + 0 - 18 = 20$, and here a new order is needed. The balance of the fourth period is $Z_4 = Z_3 + Q_4 - D_4 = 52$. As the planning horizon was set for seven periods then now we consider new period 8, for which the demand of 20 units is forecasted.

The balance of the fifth period is calculated by formula (9.1) and is equal to 32 units, which is less than the sum of the forecast for the sixth period of 23 units

Table 9.3 Consistent planning of the product at the local storage

Product A_1; initial actual balance $Z_0^{'} = 20$; safety stock $Z_c = 10$; lot $Q = 50$; lead time $L = 1$ period; previously ordered 0 units.							
Parameter	**Planning periods**						
	1	2	3	4	5	6	7
Forecast	15	15	18	18	20	23	25
To be received		50		50			50
Estimated balance	5	43	25	57	37	14	39
New order	50		50			50	
Actual demand	12						
Actual receipt							
Actual balance	8						
Parameter	**Planning periods**						
	2	3	4	5	6	7	8
Forecast	15	18	18	20	23	25	20
To be received			50		50		
Estimated balance	43	20	52	32	59	34	14
New order		50		50			
Actual demand	20						
Actual receipt	50						
Actual balance	38						
Parameter	**Planning periods**						
	3	4	5	6	7	8	9
Forecast	18	20	20	23	25	20	20
To be received		50		50		50	
Estimated balance	20	46	26	53	28	58	38
New order	50		50		50		
Actual demand	22						
Actual receipt							
Actual balance	16						
Parameter	**Planning periods**						
	4	5	6	7	8	9	10
Forecast	20	22	23	25	20	20	20
To be received		50	50		50		
Estimated balance	46	21	48	23	53	33	13
New order		50		50			
Actual demand	23						
Actual receipt							
Actual balance	−7						

and the safety stock of 10 units. Therefore, the new order will have to be placed not in the sixth period, as it was planned earlier, but in the fifth already. Further balance calculations for periods 6–9 are made according to formula (9.1).

In the third part of Table 9.3 period 3 is current with the actual consumption of 22 units. Comparing the actual consumption with the forecast for the last two periods, we see that in both cases there is an increase in actual demand compared with the forecast, i.e. we can see underestimation of the demand. Therefore, in further planning we increase the forecast for the fourth period from 18 to 20 units. The actual balance in the third period $Z'_3 = Z'_2 - D'_2 = 38 - 22 = 16$, the estimated balance of the fourth period $Z_4 = 16 + 50 - 20 = 46$, etc. Decrease in the estimated balance in the seventh period from 34 to 28 units necessitates a new order in this period. We add period 9 and calculate the balance in periods 5–9.

The fourth part of Table 9.3 refers to the time, when period 4 becomes the current period. During this period, according to the previously made plan, a new lot was to be supplied, but for some reason, there is a delay, and a new supply did not occur. The existing demand was not satisfied fully, and the balance became negative, i.e. the storage backlog of seven units was formed. Just as in the previous planning period, because of the apparent increase in demand, it is advisable here to adjust the forecast for the fifth period from 20 to 22 units.

Assuming that the receipt will occur in period 5, we recalculate the estimated balance and the need for new orders for periods 5–10. For example, balance of the fifth period $Z_5 = -7 + 50 - 22 = 21$.

9.1.3 Parallel Multi-product Planning

Since a local storage usually receives more than one type of product from a single supply source, then it is advisable to combine orders for different types of products in one period, which increases loading of vehicles and significantly reduces shipping costs. For such joint orders the combined DRP table can be used (Table 9.4).

When combining DRP tables for different products, it is necessary first to reduce calculations to the same units—kilograms, containers, pallets, etc. Assume that in Table 9.4 this reduction takes place. We also assume that, for example on the ground of load capacity, the largest possible number of co-delivered products shall not exceed 200 units. At the bottom of Table 9.4 the values ordered and received products of all three considered types are summarized.

When drawing up Table 9.4, in accordance with dependencies (9.1) and (9.2), it appears that in period 3 it is necessary to order, and in period 4 to receive simultaneously products A_1, A_2, and A_3 in amounts of 50, 80, and 100 units, respectively, which exceeds the maximum allowable amount. To make the receipts within the limit of 200 units it is possible, for example, move the order of product A_2 from the third to the second period. In Table 9.4 the calculated values, which were then adjusted, are in brackets.

As a result of the order movement to the second period, the product deliveries have become much more uniform—the total order in the second period has increased from 0 to 80, and in the third period has decreased from 230 to 150.

Table 9.4 Joint planning of several products

Product A_1; initial balance $Z_0 = 20$; safety stock $Z_c = 10$; lot $Q = 50$; lead time $L = 1$ period; previously ordered (in transit) 0 units.							
Parameter	**Planning periods**						
	1	2	3	4	5	6	7
Forecast	15	15	18	18	20	23	25
To be received		50		50			50
Estimated balance	5	40	22	54	34	11	36
New order	50		50			50	
Product A_2; initial balance $Z_0 = 50$; safety stock $Z_c = 20$; lot $Q = 80$; lead time $L = 1$ period; previously ordered (in transit) 80 units.							
Parameter	**Planning periods**						
	1	2	3	4	5	6	7
Forecast	30	28	28	25	25	22	22
To be received	80		80 (0)	(80)			
Estimated balance	100	72	(44) 124	99	74	52	30
New order		80 (0)	(80)				
Product A_3; initial balance $Z_0 = 100$; safety stock $Z_c = 40$; lot $Q = 100$; lead time $L = 1$ period; previously ordered (in transit) 0 units.							
Parameter	**Planning periods**						
	1	2	3	4	5	6	7
Forecast	50	54	54	60	60	62	65
To be received		100		100	100		100
Estimated balance	50	96	42	82	122	60	95
New order	100		100	100		100	
Total number of products in general measuring units							
Parameter	**Planning periods**						
	1	2	3	4	5	6	7
To be received	80	150	80 (0)	50 (130)	100		150
Total order	150	80 (0)	150 (230)	100		150	

However, it should be borne in mind that this kind of order shift leads to premature receipt of product A_2 in stock and its excessive amount in the third period.

9.1.4 Inventory Dynamics at Long Lead Cycles

In the previous examples it was assumed that the duration of delivery does not exceed one planning period and accordingly only one lot of the product can be in transit. If lead time L is several periods, these lots may be several (Table 9.5).

With long delivery, the planning horizon should be increased in order to cover several periods in which the delivery is scheduled. In this case, the horizon is assumed to be eight periods, which allows us to consider two cases of placing a new order.

Table 9.5 Stock movement with several lots in transit

Product A_4; initial balance $Z_0 = 80$; safety stock $Z_c = 50$; lot $Q = 100$; lead time $L = 4$ periods; previously ordered (in transit) 200 units.

Parameter	**Planning periods**							
	1	2	3	4	5	6	7	8
Forecast	45	50	50	55	60	60	60	60
To be received	100		100		100		120 (100)	
Estimated balance	135	85	135	80	120	60	120	60
New order	100		120 (100)					

The value of the estimated balance in Table 9.5 is defined as before according to formula (9.1). At the same time, the determination of the need for a new order is complicated because for this purpose we have to take into account the demand and receipt forecasts for the period equal to the lead time. The order in period n is placed, if

$$Z_n \le Z_c + \sum_{j=n+1}^{n+L} \left(D_j - Q_j\right). \tag{9.5}$$

In this case, when planning in the first period it is known that in periods 1 and 3 one lot of product for each should arrive in quantity of 100 units. Stock value $Z_1 = 135$ units, calculated by formula (9.1), available at the end of the first period is less than

$$Z_c + \sum_{j=n+1}^{n+L} \left(D_j - Q_j\right) = 50 + \sum_{j=2}^{5} \left(D_j - Q_j\right) = 50 + [50 + (50 - 100) + 55 + 66] = 165,$$

which necessitates a new order. The new order in quantity of 100 units as well shall arrive in the fifth period according to the schedule.

Verification of the need for an order in the second period leads to the inequation

$$85 > Z_c + \sum_{j=3}^{6} \left(D_j - Q_j\right) = 50 + [(50 - 100) + 55 + (60 - 100) + 60] = 75$$

and there is no need for a new order.

Similarly to the first period, the order is required in the third period; however, its quantity is likely to be greater than in the previous case.

In fact, with the order of 100 units in the third period (Table 9.5, this value is given in brackets), the planned balance of the fourth period may be insufficient to ensure the consumption in future periods. In this case, in the fourth period, condition (9.5) gives

$$80 < Z_c + \sum_{j=5}^{8} \left(D_j - Q_j\right) = 50 + [(60 - 100) + 60 + (60 - 100) + 60] = 90,$$

which causes the need for a new order. If the quantity of the ordered lot is not determined by any additional terms and conditions, in this case it is advisable to increase the size of the lot, and so satisfy a tendency to increase in demand. Suppose, for example, the size of the lot ordered in the third period is 120 units. In this case, the need for additional orders in the fourth period will disappear.

The estimated balance in this case can be set for all periods up to the horizon under consideration; at the same time to determine the need for a new order in the fifth period, the forecast horizon of eight periods is insufficient and therefore this need has to be defined in the next planning cycle.

9.2 Supply Chain Fluctuations

Changes in demand, as discussed above in Sect. 6.1, are defined by the trend, on which random fluctuations are superimposed. Besides, the demand can make regular oscillatory motion about the main direction (trend) due to seasonal changes in demand, promotions, etc. The direction of the trend can also vary due to changes in fashion, competition, general economic situation, etc.

Changes in the demand at the point of final sales of the product cause changes in orders for the product itself, and then to its components, materials, and subsequently to the equipment for manufacturing the product and all its components. Passing through the supply chain, the changes in the order quantity gradually deviate from the initial changes in demand, and such deviations are often growing in nature.

The reasons for this phenomenon are rooted in the use of forecasting for order placement. As the forecasts are based on statistical data, they always contain errors, for which the possibility of correcting is provided by safety stocks in orders. The accumulation of errors by moving up the supply chain leads to an increase in safety stocks. When the trend direction in demand changes, the wave of changes in safety stocks propagate throughout the chain. The amplitude of the wave increases with distance from the end user. This phenomenon is called Forrester effect or bullwhip effect. The latter name is due to the analogy with the shape of oscillations with increasing amplitude, which a shepherd's whip takes.

9.2.1 Bullwhip Effect

To demonstrate the mechanism of the bullwhip effect, we shall consider the process of change in sales and orders for the chain consisting of three echelons (Table 9.6). In Table 9.6, 13 periods of supply chain operation are considered in form of a relevant DRP table, and it is believed that up to and including the first period sales

D_i were stable and amounted to 100 units. In the echelons of the supply chain the model with a fixed reorder cycle is used, as described in Sect. 8.2.2.

In Sect. 9.1 it was assumed that safety stock Z_c does not change in the planning process up to the planned horizon. Generally speaking, this is not the case, and the safety stock should depend on the demand value. For example, when considering the bullwhip effect they often believe that the safety stock should be equal to the expected demand for the lead time. When drawing up Table 9.6 we assume that lead time L is equal to one period, the initial stocks of the product and its components are equal to 100 units, and the same number are in transit.

At the end of each i-th period an order is placed for the supplier, which is equal to

$$Q_i = D_i + Z_{ci} - Z_i, \tag{9.6}$$

where Z_i is the balance by the end of the period. For example for period 1, we have

$$Q_1 = D_1 + Z_{c1} - Z_1 = 100 + 100 - 100 = 100.$$

The similar situation occurs with ordering components as well and

$$Q_1' = Q_1 + Z_{c1}' - Z_1' = 100 + 100 - 100 = 100.$$

Starting with period 2, the sales are increased with increments in each period by 10 % from the previous value up to period 6, and after this period they are reduced at the same rate until period 11, after which they remain constant. In period 2 the increase in sales from 100 to 110 units causes the decrease in stocks by the end of the period down to 90 units. In this case the order for the supplier

Table 9.6 Generation of bullwhip effect in the supply chain

Period	Sales	To be received	Balance	Order for supplier	To be received	Balance	Order for components
1	100	100	100	100	100	100	100
2	110	100	90	130	100	70	190
3	121	130	99	143	190	117	169
4	133	143	109	157	169	129	185
5	146	157	120	172	185	142	202
6	160	172	132	188	202	156	220
7	146	188	174	118	220	258	0 (−22)
8	133	118	159	107	0 (−22)	151	63
9	121	107	145	97	63	117	77
10	110	97	132	88	77	106	70
11	100	88	120	80	70	96	64
12	100	80	100	100	64	60	140
13	100	100	100	100	140	100	100

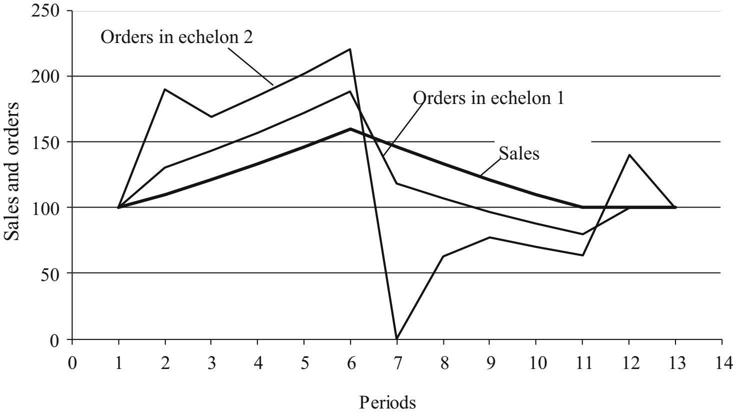

Fig. 9.1 Graphs of sales and orders in the chain echelons

$$Q_2 = D_2 + Z_{c2} - Z_2 = 110 + 110 - 90 = 130,$$

and for components order we will obtain

$$Q_2' = Q_2 + Z_{c2}' - Z_2' = 130 + 130 - 70 = 190.$$

Determining the quantities of orders in the same way up to period 13, we obtain the values given in Table 9.6. Exact calculations by formula (9.6) for period 7 lead to a negative amount of −22 in the order for components. In view of impossibility of negative order, in Table 9.6 this order is considered to be zero.

The graphs in Fig. 9.1 indicate that in each successive echelon in the supply chain the amplitude of fluctuations increases. Besides, the graphs show that there are oscillations with increasing amplitude in the chain during orders transfer up the chain. As a result, even after sales return to its original value, in the chain there are still damped oscillations, duration of which depends on the chain echelon distance from its beginning.

We assume that the demand changes occur in the echelon located at 0. We will define the notion of fluctuations transfer factor from demand fluctuations to the first echelon (level) of the supply chain

$$k_{01} = \frac{\Delta Q}{\Delta t} / \frac{\Delta D}{\Delta t}, \tag{9.7}$$

where ΔQ is the increment of the quantity of the order for the supplier on interval Δt and ΔD is the increment of demand on this interval. This factor indicated the ratio of the slope angles of the order curve in echelon 1 to the sales curve for each interval in Fig. 9.1. Similarly, the fluctuations transfer factor from the first echelon to the second one is equal to

$$k_{12} = \frac{\Delta Q'}{\Delta t} \Big/ \frac{\Delta Q}{\Delta t}. \tag{9.8}$$

The analysis of the graphs in Fig. 9.1 shows that the value of fluctuations transfer factor in echelon 1 becomes large at the points of sudden changes of sales trend and the factor in echelon 2—at break points on order curve in echelon 1. Thus, the transfer factor is a function of changing in trend direction. If graphs in Fig. 9.1 were smooth, it could be assumed that the transfer factor depends on the second derivative of the fluctuations curve in the previous echelon. However, the curves in Fig. 9.1 are not differentiable, and therefore the fluctuations transfer factor $k_{01}(n)$ in period n depends on the difference

$$\frac{\Delta D_n}{\Delta t} - \frac{\Delta D_{n-1}}{\Delta t}, \tag{9.9}$$

and accordingly factor $k_{12}(n)$ depends on the difference

$$\frac{\Delta Q_n}{\Delta t} - \frac{\Delta Q_{n-1}}{\Delta t}. \tag{9.10}$$

Fluctuations of the discussed type can occur not only in the cooperation of several enterprises, but also among departments of one enterprise engaged in sequential processing and assembly of products (Sachko 2008).

9.2.2 Bullwhip Effect Factors

A detailed analysis of the factors contributing to the occurrence and scope of the bullwhip effect is shown in the papers of Lee et al. (1997a, b). The article (Lee et al. 1997a) lists the main causes of the bullwhip effect:

- Regular updating of the forecasted demand values;
- Demand aggregation for several periods into one lot;
- Fluctuations in the price of the product;
- Restriction of sales and fears of shortages.

Of all these factors, according to the authors (Lee et al. 1997a) the need for a regular accounting of changes in demand is the most important. Indeed, with demand changes it is necessary to replenish the changed current stock and to provide necessary quantity of safety stock. As a result, as shown above in Sect. 9.2.1, in the supply chain there is the oscillation process, which increases as the distance from the supply echelon of the end consumer.

In the paper (Lee et al. 1997b), this process is studied for the case of so-called autoregressive demand of the first order. In auto-regression, each value of the time series is a linear function of the previous one or several previous values of this series. In the first-order auto-regression only the previous value is used, in the second order auto-regression—two, etc. Thus, the demand in period n is described by the dependence

$$D_n = a + \rho D_{n-1} + \varepsilon, \tag{9.11}$$

where a is a certain constant, ε is the random deviation of demand, and factor ρ is within—$1 \leq \rho \leq 1$. For this demand the optimal size of a lot ordered at the end of observation period n is defined by the dependence

$$Q_n = \frac{\rho(1 - \rho^{L+1})}{1 - \rho}(D_n - D_{n-1}) + D_n, \tag{9.12}$$

where L is the lead time in period of observation.

The most interesting case of this dependence is when $\rho \to 1$. Expanding the resulting uncertainty by the L'Hospital's rule, with $\rho = 1$ we have

$$Q_n = (L + 1)(D_n - D_{n-1}) + D_n = D_n + (L + 1)\Delta D_n. \tag{9.13}$$

From Eq. (9.13), it follows that the quantity of the ordered lot directly depends on the lead time, which naturally leads to an increase in the amplitude of fluctuations in the supply chain with the increase of lead time.

By starting consideration of the influence of demand aggregation, we note that the order for a new receipt is created, for the most part, not after each consumption, but only after its reduction to a certain value determined by some stock management model (Sect. 8.2). Here all the demand for stock replenishment cycle is aggregated into a single lot that can significantly reduce transportation costs. Figure 9.2 shows the graphs of sales and orders in the first echelon of the supply chain for the data in Table 9.6 with the order lot of 300 units. The orders schedule in echelon 2 in this case coincides with the graph for echelon 1.

From Fig. 9.2, it follows that the application of the model with fixed order quantity leads to significant fluctuations in demand at all levels of the supply chain. At the same time, the change of the parameters of such fluctuations due to changes in demand is small.

Temporary, for instance during the holidays, discount prices of the product may cause a significant increase in demand and therefore sales revenue. However, such measures often entail a drop in demand in subsequent periods, resulting in the observed significant fluctuations in orders, destabilizing the production process. Therefore, such pricing policy is justified mostly only when there are excessively large stocks complicating their storage.

If demand for the product significantly exceeds the capacity of its production, restrictions are often imposed on the product deliveries to consumers. Usually the manufacturer distributes such products among consumers proportionally according to the orders placed. Naturally, the consumers knowing the possible limitation exaggerate their needs. This leads to the fact that, in reality, when the production

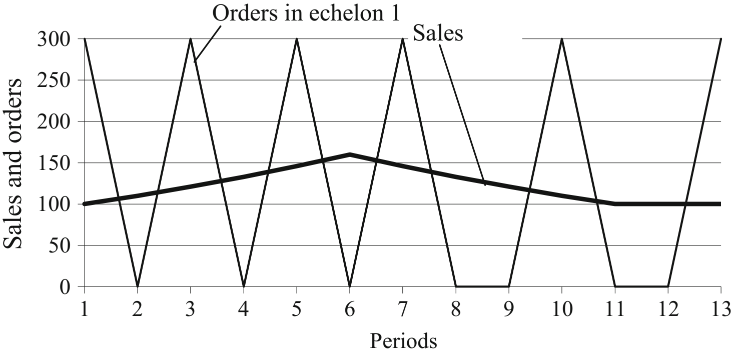

Fig. 9.2 Graphs of sales and orders with fixed size of lots

is aligned with the demand, there are multiple cancellations of previously ordered product. This practice leads to serious losses in production.

9.2.3 Methods of Reducing Supply Chain Fluctuations

The fluctuations transfer factor in period n from the level of demand (zero level) to m-th echelon is equal to the product of all intermediate transfer factors, i.e.

$$k_{0m} = \prod_{i=0}^{m-1} k_{i,i+1}, \tag{9.14}$$

for example, $k_{02} = k_{01} \times k_{12}$.

A number of studies of the bullwhip effects in the supply chain have shown that in the situation similar to that described above in Sect. 9.2.1, the average fluctuations transfer factor for the trend change period in one echelon is approximately 1.7 (Vollmann et al. 2005). In this case, the average $k_{0m} = 1.7^m$. Obviously, the decrease in the number of m echelons in the chain significantly reduces potential fluctuations scale, but the removal of unnecessary echelons from chain is rarely possible.

The main method to fight the bullwhip effect, as indicated in Lee et al. (1997a), is to coordinate the estimation and execution of orders at all echelons of the supply chain including information sharing, channel alignment, and operational efficiency.

Thanks to the Internet, the rapid exchange of information about the real state of demand is absolutely real, though not always corresponds to the desires and goals of the information sources. To facilitate the transfer of the product to the consumer, of course, if it is a major consumer, on the territory of the consumer a special warehouse is often organized, which is, however, at the vendor's disposal (Vendor-managed Inventory, VMI). Naturally, in such circumstances, the response to changing requirements of demand becomes very fast. Operational efficiency of supply is determined by two factors: lead time and the ability to identify and account the total amount of product over the entire supply chain.

Fluctuations in supply chains can be somewhat reduced due to rejection from various temporary price changes, but in general, these measures do not influence the order quantities very much, as they usually last only until reduction of the stock to the standard value.

To have a correct idea of the actual needs of customers in the event of stock-out, the manufacturer may be guided not by the ratio of current orders of different consumers, but by similar ratios before stock-outs. The point of this approach is explained by the fact that before stock-outs, consumers had no intention to exaggerate their needs. In addition, in such cases penalties for cancellation of previously made orders can be of great importance.

Fluctuations in the supply chain can be significantly smoothed if changes of the safety stock will not be directly proportional to changes in the values of the demand. First of all, we note that according to expressions (8.44 and 8.45) in Sect. 8.5.2, calculated safety stock Z_c is proportional to demand D with index 0.7. In addition, it is possible to reduce the dependence of the safety stock on the demand at the moment of sudden changes in demand and orders, i.e. at break points of sales and orders curves in Fig. 9.1.

We assume that the reserve stock in echelon 1 depends on the difference of increments of demand in neighbouring periods (Eq. 9.9), and in echelon 2—on similar difference (Eq. 9.10), namely

$$\begin{aligned} Z_{c1}(n) &= a\left[1-\psi\left(\frac{\Delta D_n}{\Delta t}-\frac{\Delta D_{n-1}}{\Delta t}\right]D_n^{0.7},\right. \\ Z_{c2}(n) &= a\left[1-\psi\left(\frac{\Delta Q_n}{\Delta t}-\frac{\Delta Q_{n-1}}{\Delta t}\right]Q_n^{0.7},\right. \quad \text{etc.,} \end{aligned} \tag{9.15}$$

where factor ψ defines the dependence of the safety stock on sudden changes in demand, and factor a can be defined by the set value of the safety stock for the established value of the demand. For example in Fig. 9.6 in its initial state demand $D(0) = 100$ units, safety stock $Z_{c1}(0) = 100$ units, and $a = Z_{c1}(0)/D(0)^{0.7} = 3.98$.

Since the length of time interval Δt equals 1 in all above examples, then

$$\begin{aligned} &\frac{\Delta D_n}{\Delta t}-\frac{\Delta D_{n-1}}{\Delta t}=\Delta D_n-\Delta D_{n-1}=(D_n-D_{n-1})-(D_{n-1}-D_{n-2}) \quad \text{or} \\ &\frac{\Delta D_n}{\Delta t}-\frac{\Delta D_{n-1}}{\Delta t}=D_n-2D_{n-1}+D_{n-2}. \end{aligned} \tag{9.16}$$

While setting the value of factor ψ and using dependence (9.15), (9.16), we can obtain the values of safety stock, responding to sudden changes of demand in different ways (Table 9.7).

Above when drawing up Table 9.6, we believed that the safety stock is exactly equal to the changing demand. From Table 9.7 it is clear that the use of dependencies (9.15) leads to reduction in safety stock, and with the growth of factor ψ the changes

Table 9.7 Changes in safety stock depending on the demand

	Periods											
Parameters	0	1	2	3	4	5	6	7	8	9	10	11
Demand	100	100	110	121	133	146	160	146	133	121	110	100
Stock at $\psi = 0$	100	100	107	115	122	130	139	130	122	114	107	100
Stock at $\psi = 0.002$	100	100	105	114	121	130	138	137	122	114	107	100
Stock at $\psi = 0.003$	100	100	104	114	122	130	138	141	121	113	106	100

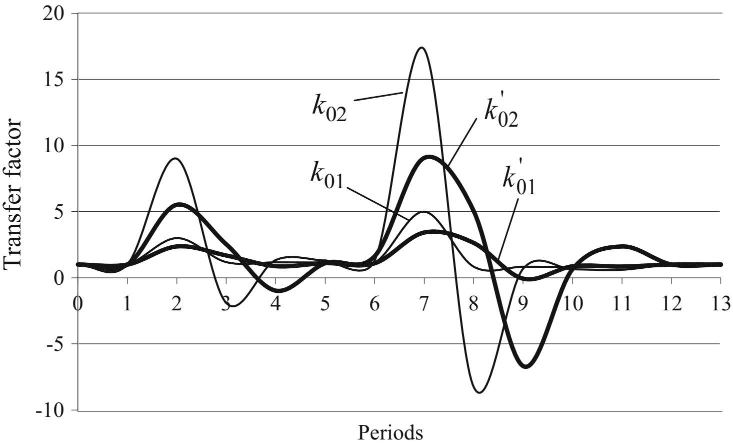

Fig. 9.3 Fluctuations transfer factors with various safety stocks

in safety stock at the points of sudden change in demand (i.e. in periods 2 and 7) smoothen (Fig. 9.1).

The efficiency of such safety stock reduction is clearly noticeable when comparing the corresponding graphs of fluctuations transfer factors (Fig. 9.3). The use of dependencies (9.15) for smoothing sudden changes in demand leads to significant decrease in occurring orders peaks and more rapid damping of the fluctuations.

k_{01}—transfer factor from level 0 (demand) to echelon 1 with the safety stock equalling to the demand; k_{02}—transfer factor from level 0 (demand) to echelon 2 with the safety stock equalling to the demand; k'_{01}—transfer factor from level 0 (demand) to echelon 1 with the safety stock calculated by formulas (9.15); k'_{02}—transfer factor from level 0 (demand) to echelon 2 with the safety stock calculated by formulas (9.15).

9.3 Application of Logistics Operating Curves in Supply Chains

Since any supply chain is a complex of production sites and warehouses, so logistic operating curves, the theory of which was described above in Sects. 3.2 and 8.4, can be applied for the chain analysis. Operation parameters of individual echelons of a supply chain are interrelated as a rule. For example, the level of orders fulfilment from the finished goods storage is significantly associated with the receipt of materials to the relevant warehouse. Deviation of the actual receipts from the supply plan causes disruption of production deadlines, delay of commercial products, and inability to fulfil customers' orders timely.

Figure 9.4 shows the connection of logistic parameters in the supply chain (Nyhuis and Wiendahl 2009). The influence of each of the parameters in Fig. 9.4 extends down the supply chain, with one exception—the number of manufactured

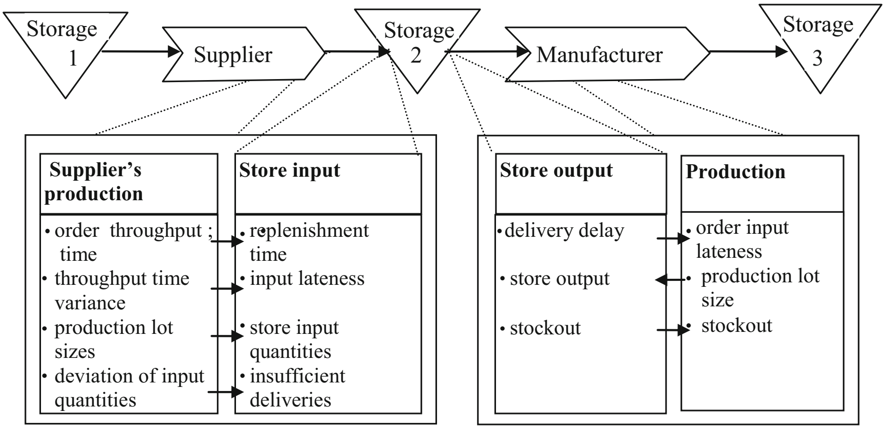

Fig. 9.4 Interrelation among the logistic parameters of the supply chain [based on Nyhuis and Wiendahl (2009)]

products in the order or the lot. The latter value is determined by demand and actually sets the values of all the rest parameters of the delivery process, extending up the chain.

Such parameters of the process, as deviations in lead time, deviations in quantity of supplied materials along with fluctuations in demand, necessitate for safety stock. Material stock-outs and delays in stock removal complicate putting orders into production and can cause its complete halt. The production efficiency improvement is determined by the ability to enhance the quality of logistics parameters—reduce lead times, reduce the lead time spread, etc. For this purpose, it is necessary to conduct systematic analysis of all echelons of the supply chain starting from the last echelon—finished products storage, and then of other echelons up the chain.

In Sect. 3.3.1 it was shown that the achievement of high-quality production process is determined by the correct logistics positioning, which involves selecting of Work-in-process (WIP), optimally corresponding to the current operational state. Section 8.4 described the method of quality assessing of storage facilities as the balance between high level of customer service and low stocks. When using the method of logistic operating curves, these curves are consistently built for each echelon of the chain, and on each of these curves, the optimal positioning of the process is established.

The book (Nyhuis and Wiendahl 2009) presents an example of the logistic analysis of the supply chain relating to production of tools. This chain matches the chain in Fig. 9.4, where at storage 1, all required raw materials are stored, at storage 2—various semi-finished products, and at storage 3—ready tools.

The analysis begins with examination of the situation at the finished products storage (storage 3 in Fig. 9.4). For this purpose, logistic curves similar to the curves in Fig. 8.9 are built for each tool item in stock. Following this study it was found that for the most part the tool items the actual average stock level for a period $\overline{Z}$ was greater than stock level $\overline{Z}_1$ providing zero delay of delivery with regard to the required safety stock, i.e. clearly excessive stocks of finished products were observed. For some items, on the contrary, the stock was insufficient and did not provide tools supply with the required level of customer service.

The results of this operation led to the fact that the initial number of tools of 3.5 million pcs. valued at 5.9 million euros has been reduced to the quantity of 2.2 million pcs. valued at 3.3 million euros. Thus, only due to the performance analysis of the finished products storage it has appeared to be possible to reduce the stock by approx. 40 %.

Logistic analysis has also helped to establish the structure of stocks typical for the production under study. It turned out that the variable component of stocks caused by the receipt and withdrawal of the product lots amounts to only 43.1 % of the stock, and the bigger part (56.9 %) is the safety stock. All safety stock in the study was divided into three parts: the safety stock for fluctuations in demand, which is 36.7 % of the total stock; stock for delays in orders delivery amounting to 19.6 %; and the stock insuring an insufficient number of goods in the supply is only 0.7 % of the total value of the stock. Determining the stock structure helped to define the measures to further reduce stocks—marketing efforts to stabilize demand, reducing the production lot size for individual items, delivery regulation, etc.

At the next stage the tool production as such was studied. For production bottlenecks logistics operating curves similar to the curves in Figs. 3.7 and 3.10 were built and the solutions were developed to improve the performance of bottlenecks using the methods described in Sect. 3.3.2. Following these activities the average duration of production cycle $\overline{F}$ could be reduced from 32.5 working days to 15.5 working days. This result was achieved due to reducing average workload $\overline{W}$ by about 50 %.

Analysis of the performance of the manufacturer's storage for incoming materials and semi-finished products (storage 2 in Fig. 9.4) showed that the supplying enterprise fulfils the orders of the manufacturing enterprise with a standard deviation of lead time of 6.4 working days. As the planned deviation of the finished product output is only 1.2 working days, the lead time spread of 6.4 days makes the manufacturer keep high safety stocks.

The second reason for large stocks at the incoming materials storage is that the size of the delivery lots source materials and semi-finished products are usually much higher than the size of the finished product lots. The use of ABC classification allowed grouping the various items of materials into groups A, B, and C and matching each group with the required service level. For example, for group A, as known, being 20 % of all the supply items, level of service S_L is required to approach 100 %. Accordingly, by setting a lower level of service for groups B and C, the required amount of safety stock was reduced.

At the last stage of the described study, the production activity of the suppliers was examined. For this purpose, the above-described technique of identification of bottlenecks and logistics positioning was applied.

As a result of all taken measures the volume of stocks at the storage was reduced by 72 %—up to 1.4 million pcs. The average total amount of material objects in the entire supply chain during the study period (approximately 2 years) decreased from 9.1 million units to 4.1 million units, i.e. approximately by 60 %. The level of customer service for group A of manufactured tool was 98 %; groups B and C were close to 95 %; herewith the average lead time was 1.2 working day.

9.4 Inventory Echelon Accounting

The methods of determination of reorder points and calculation of safety stock, described in Chap. 8, require some adjustments when used in supply chains. The reason for this situation is the fact that stocks in the connected echelons of the chain cannot be considered as independent. The main problem for the supply chain is the need to find the right balance across the individual echelons of the system.

It should be noted that stocks in one of the echelons can maintain normal operation of other echelon. For example, if large stock of finished products is available in the final echelon, there is no need for large stocks of components in the echelons that are higher up the chain; in the chain consisting of local storages and a distribution center, large stocks at central storage can significantly reduce stocks at the storages intended for end users, etc. However, it is very difficult to arrange mathematically optimal distribution of stocks among the echelons in the chain. Therefore, for this purpose, various models based on reasonable assumptions and heuristic methods are applied.

9.4.1 Inventory Echeloning

The total stock of a certain product, which is simultaneously available at all levels of the supply chain, is called echelon stock of this product. Let us consider the chart of product A_1 shown in Fig. 9.5.

According to the chart in Fig. 9.5, the product arriving at incoming storage 4 is distributed then to two directions—to assembly 1 and to the chain of subassemblies 3 and 2. Each of the subassemblies 1, 3, 2 can contain several units of the source product and this product can be included into the assembly unit either directly (echelons 1 and 3) or as part of other assembly unit (echelon 2).

Table 9.8 shows the chain structure composed of four objects (echelons) corresponding to Fig. 9.5. According to the chart in Fig. 9.5, objects 1 and 2 are the end products in the chain and are not included in any other objects. Object (assembly unit) 3 is included in each object 2 in amount of two units, object 4 (product A_1) is included in object 3 in amount of one unit, and in object 1 in amount of three units.

Column “Current availability of object” contains the number of the relevant objects that are currently in stock. Echelon stock of object 3 is made up of the number of this object at the storage and the number of objects 3 included in objects

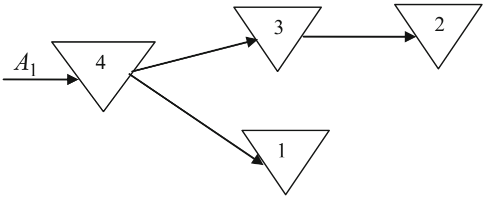

Fig. 9.5 Product chart

Table 9.8 Chain structure and stock of objects in the chain

Object in the chain	In which object included directly	Quantity in one upper object	Current availability of object	Echelon stock of object
1	–	–	5	5
2	–	–	3	3
3	2	2	4	10
4	3	1	8	33
4	1	3		

2, i.e. $4 + 2 \times 3 = 10$. Similarly, for object 4, we have $8 + 3 \times 5 + 1 \times 10 = 33$. Note that the current availability of an object in the storage changes at transferring the objects from the storage to the assembly, but the echelon stock of this object in the chain does not change at such transferring.

Let us consider two variants of the two-stage supply chain (Fig. 9.6). In the simplest case (Fig. 9.6a), a two-stage chain consists of two sequential echelons. The existence of this chain can be justified if the storing cost at intermediate storage 0 is significantly lower than at storage 1, from which the product is issued. For example, for a product consumed by only one department of the enterprise, it may be appropriate to store not only in this department but also at the central storage. In general, as shown in Fig. 9.6b, the product stored at the central storage 0 can then be shared among local storages 1, 2, 3, etc., for issuing to end users.

In two-stage supply chains there is an issue of the correct distribution of stocks among all echelons. This distribution depends on a number of factors—level and random fluctuations of demand, the cost of storing, losses arising from the inability to meet risen demand, and the delivery time to each echelon and some others.

9.4.2 Sequential Supply Chain

For the case of the sequential chain (Fig. 9.6a) by accounting of stock echeloning it appears to be possible to find the exact optimal solution with known values of the listed factors (Clark and Scarf 1960). In this case, the optimal value of the safety stock is determined based on the minimum cost arising due to storage and losses due to backlog demand.

For each *i*-th echelon of the chain we introduce the following values:

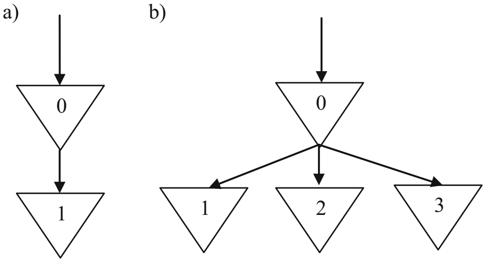

Fig. 9.6 Two-stage supply chains. (**a**) Sequential chain; (**b**) chain with distribution

T_i—time of fixed period of delivery;
L_i—time of product delivery to the i-th echelon from the previous echelon;
Z_i—current stock;
Z_i^e—current echelon stock of product starting with the i-th echelon;
Z_{ci}—safety stock;
Z_{ci}^e—echelon safety stock;
$\dot{S}_i$—max. quantity of stock;
S_i^e—max. quantity of echelon stock;
μ_i—the mean value of the random variable of demand for the reference period of demand determination with duration of I days;
σ_i—standard deviation of demand in the i-th echelon during the reference period;
$D_i(n)$—demand for n periods is equal to the reference one;
c_{hi}—cost of storage per product unit in the reference period;
c_{bi}—value of losses from the backlog demand unit in the reference period.

We will consider the model with a fixed reorder cycle, as described above in Sect. 8.2.2. We will consider the nature of the change in stocks under such conditions in each of the echelons of the sequential supply chain, as well as changes in the echelon stock during the delivery cycle (Fig. 9.7).

Echelon stock in echelon 1 is not different from the regular stock of this echelon, as echelon 1 is the last in the chain. At the same time, the echelon stock in echelon 0 is the sum of the regular stocks in echelons 0 and 1. The order for new supply of echelon stock is placed at time R_0, and the order to replenish echelon 1—at time R_1.

Let us assume that at the initial moment of the chain operation, a product lot is received by echelon 0 from an external source and at the same moment a lot of the same product, but smaller, came from echelon 0 to echelon 1. At this point the stocks in both echelons have the highest values $\dot{S}_0$ and $\dot{S}_1$ and the echelon stock takes the highest possible value S_0^e.

As the consumption of the stock in echelon 1 falls and the echelon stock lowers at the same rate in echelon 0. At the time of the order placement in echelon 0 its stock reduces by the amount of the order, and thereafter remains constant until the next order, or until the arrival of the lot from the external source. According to the model with fixed reorder cycle, at the end of the period a new delivery should be

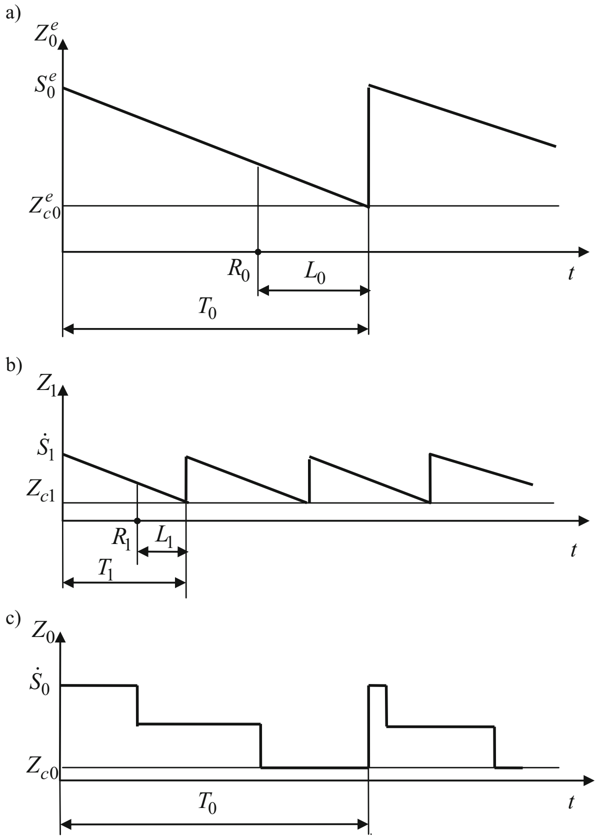

Fig. 9.7 Graphs of stock changing in the sequential supply chain: (**a**) echelon stock in echelon 0; (**b**) stock in echelon 1; (**c**) stock in echelon 0

made, and the echelon stock should be equal to the safety stock of the chain as a whole.

The method for determining optimal values $\dot{S}_0$ and $\dot{S}_1$ proposed in Clark and Scarf (1960) makes it possible to determine stocks in the echelons sequentially, starting from the last echelon of the chain. Since the total cost of storage and penalties in echelon 1 depends only on the parameter of this echelon, it is possible to find the minimum value of this cost as a function of value $\dot{S}_1$. This approach (Axseter 2006) leads to the expression of optimal value S_1^*

$$\Phi\left(\frac{S_1^* - \mu_1'}{\sigma_1'}\right) = \frac{c_{h0} + c_{b1}}{c_{h1} + c_{b1}}, \tag{9.17}$$

where the function of normal distribution $\Phi\left(\frac{S_1^* - \mu_1'}{\sigma_1'}\right)$ is determined as an integral of the demand distribution density according to Eq. (8.17).

In the model (S, T) with fixed reorder cycle unlike with models (S, Q), and (s, S), described in Sect. 8.2, when calculating the order at the moment of its placing R_1 it is assumed that the demand will remain equal to the current demand and constant from this moment to the planned moment of new delivery, i.e. until the end of the next cycle. That's why the calculation of the mean value of random demand μ_1' and standard deviation of demand σ_1' in echelon 1 includes not only lead time L_1, as in models (S, Q), and (s, S), but also cycle duration T_1. Hence, similarly with dependencies (8.34) and (8.38) we have

$$\mu_1' = \frac{L_1 + T_1}{I}\mu_1, \quad \sigma_1' = \sqrt{\frac{L_1 + T_1}{I}}\sigma_1. \tag{9.18}$$

In the simplest case $T_1 = I = 1$ day and expressions (9.18) take the form

$$\mu_1' = \mu_1(L_1 + 1); \quad \sigma_1' = \sigma_1\sqrt{L_1 + 1}. \tag{9.19}$$

Let us consider the example with data from Sect. 8.3.3, in which $\mu_1 = 50, \sigma_1 = 14$, $L_1 = 1$ and expressions (9.19) are valid, and $c_{h0} = 1, C_{h1} = 2\,и\,C_{b1} = 10$. We have

$$\Phi\left(\frac{S_1^* - 50 \times (1+1)}{14 \times \sqrt{1+1}}\right) = \frac{1+10}{2+10} = 0.916.$$

Using function NORMSINV MS Excel, we obtain NORMSINV (0.916) = 1.378 and

$$S_1^* = 1.378 \times 14 \times 1.41 + 50 \times 2 = 127.$$

After determining S_1^*, according to the described method, it is necessary to make an expression for the value cost of echelon stock and find optimal value S_0^e, which gives minimum of this cost. The corresponding expression is given in Axseter (2006), but the related calculations cannot be reduced to elementary functions and therefore are quite complex.

However, value S_0^e can be calculated in different way by using dependence (Fig. 9.7a)

$$S_0^e = Z_{c0}^e + \mu T_0. \tag{9.20}$$

The standard deviation value of the echelon stock in echelon 0 can be defined similarly (Eq. 9.19). For the case $I = 1$, we obtain

$$\sigma_0' = \sigma_1\sqrt{L_0 + T_0}, \tag{9.21}$$

and using the expression (8.37), we will have

$$Z_{c0}^e = \kappa\sigma_0' = \kappa\sigma_1\sqrt{L_0 + T_0}, \tag{9.22}$$

where, as in Sect. 8.5.1, safety factor κ is defined by level of service S_L and can be found using function NORMSINV MS Excel. Let for example $S_L = 0.99$ and accordingly $\kappa = \mathrm{NORMSINV}(0.99) = 2.326$. With $L_0 = 5, T_0 = 10$ in the considered example we get $Z^e_{c0} = 2.326 \times 14 \times \sqrt{5+10} = 126$ and from Eq. (9.20) we have $S^e_0 = 126 + 50 \times 10 = 626$.

By safety stocks directly in each echelon, we mean the minimum value of the stock in the process of the cycle. It is obvious that such stock in echelon 1

$$Z_{c1} = S^*_1 - \mu T_1 \tag{9.23}$$

and for the given example we have $Z_{c1} = 127 - 50 \times 1 = 77$.

The minimal value of stock in echelon 0 occurs at the end of cycle T_0 and is equal to

$$Z_{c0} = Z^e_{c0} - S^*_1, \tag{9.24}$$

which gives $Z_{c0} = 126 - 127 = -1$.

In this example it appears that by the end of cycle T_0 stock Z_0, being directly in echelon 0 (Fig. 9.7c), will be consumed completely and in echelon 1 there will be stock Z_1, approximately equal to the highest value S^*_1.

9.4.3 Supply Chain with Distribution

The problem of optimal distribution of stocks in a two-stage chain with distribution on several echelons (Fig. 9.6b) is extremely complex and has no exact theoretical solution. Currently, to solve this problem the method, which is based on the so-called balance assumption, is widely used. According to this assumption, the deliveries of the central echelon to the local echelons may be both positive and negative, which allows you to "balance" the stocks in local echelons during fluctuations in demand in them.

The use of this assumption allows (Axseter 2006) obtaining the following expression for the optimal value of the stock in the k-th local echelon S^*_k

$$\Phi\left(\frac{S^*_i - \mu'_i}{\sigma'_i}\right) = \frac{c_{h0} + c_{bi} - \lambda}{c_{hi} + c_{bi}} \text{ with } 0 \le \lambda < c_{h0} + c_{bi}, \tag{9.25}$$

where λ is the so-called Lagrange multiplier being the same for each of the local echelons. The Lagrange multiplier in expression (9.25) reflects the need to take into account an important constraint arising in this case during the search of the optimal solution. This restriction is that the sum of stocks in all local echelons N should not exceed the echelon stock throughout the supply chain, i.e. must be

$$\sum_{i=1}^{N} S_i^* \leq S_0^e. \tag{9.26}$$

In most of the numerous papers devoted to the study of this problem, attempts are made to find the limits, within which the stocks at all echelons of the two-stage chain can be present. Thus, in the same way as above in Sect. 9.4.2, they are based on the minimum cost arising due to storage costs and losses due to backlog demand. Since, however, the optimal distribution of the chain can be strongly affected by various other factors such as transport costs, ability to transfer stocks from one storage to another, etc., the practical application of the results of the above studies is difficult.

In this situation, it seems appropriate to use expression (9.25) as the basis for selection of the relations between stocks in echelons of the real-life chain. To do this, by varying value λ, it is possible to determine the relation of the minimum value of the stock in echelon 0 and echelon safety stock in this echelon and then evaluate its validity on the basis of available experience.

The value of echelon safety stock with reference demand period $I = 1$ for the chain with distribution is equal to (Cao and Silver 2005)

$$Z_{c0}^e = \kappa \sqrt{(L_0 + T_0) \sum_{i=1}^{N} \sigma_i^2}. \tag{9.27}$$

As stock changes in local echelons are random, the minimum value of the stock in echelon 0, similarly to Eq. (9.23), we have

$$Z_{c0} = Z_{c0}^e - \sqrt{\sum_{i=1}^{N} \left(S_i^*\right)^2}. \tag{9.28}$$

Let us consider an example, in which a two-stage chain has a central echelon and three local echelons (Fig. 9.6b) with initial data given in Table 9.9.

For the data in Table 9.9 according to Eq. (9.27) the value of the echelon safety stock with safety factor $k = 2.326$, providing level of service $S_L = 0.99$

$$Z_{c0}^e = 2.326 \times \sqrt{(5 + 10)\left(14^2 + 14^2 + 20^2\right)} = 254.$$

Table 9.10 presents the calculation results of optimal stocks in echelons 1, 2, and 3 and safety stock in echelon 0 for different values of λ within the range meeting the condition (9.25), i.e. at $\lambda < 11$.

Value of the optimal stock in echelon 1 at $\lambda = 0$, of course, coincides with the value of the stock received in Sect. 9.4.2 above, since the corresponding original data match. The values of safety stock in echelon 0 are calculated from expression (9.28); coefficient of safety stock in echelon 0 is defined as the ratio of the safety

Table 9.9 Parameter of the chain with distribution

Echelon	T	L	μ	σ	c_h	c_b
0	10	5	–	–	1	–
1	1	1	50	14	2	10
2	1	2	50	14	2	10
3	1	1	100	20	3	10

Table 9.10 Calculation results of stocks in the chain

	Parameter λ						
Stock characteristics	0	6	8	9	10	10.5	10.8
Optimal stock in echelon 1	127	95	86	80	72	65	58
Optimal stock in echelon 2	183	144	133	126	116	108	98
Optimal stock in echelon 3	228	191	179	171	159	149	138
Maximal stock in echelon 0	1934	1995	2014	2026	2043	2057	2074
Safety stock in echelon 0	−66	−5	14	26	43	57	74
Coefficient of safety stock in echelon 0	−0.25	−0.01	0.05	0.1	0.17	0.23	0.29

stock in this echelon to the echelon safety stock. For example, if $\lambda = 9$, according to expression (9.28) the safety stock $Z_{c0} = 254 - \sqrt{80^2 + 126^2 + 171^2} = 26$ and safety factor $K_0 = 26/254 \approx 0.1$. Note that the difference of the largest stock in echelon 0 and the corresponding safety stock is exactly equal to the total demand for the product for T_0 cycle time of 2000 units.

The results, shown in Table 9.10, are a good illustration of the application of the above-mentioned method of stock balance (relations changing) in central echelon and local echelons of the chain. Indeed, with increasing of λ the stocks of local echelons gradually decrease, and the stock in the central echelon increases. At that even the sign of safety (minimum) stock can change. For example, if $\lambda \leq 6$ the calculated safety stock of the central echelon is less than 0, and then becomes positive.

From the results in Table 9.10 it also follows that with the initial data of the present example, the real safety stock in echelon 0 can be obtained at values of λ greater than 8. Then with increase of λ the safety stock grows rapidly and at $\lambda = 10.8$ it reaches approximately 30 % of echelon stock. Table 9.10 makes it possible to determine the corresponding parameter and optimal values of stocks in the echelons of the chain through value K_0 known from the experience.

9.4.4 Dependency Between Echelon Stock and Number of Links of One Level in the Supply Chain

Each supply chain exists to provide a certain set of products for a specified number of consumers. For this, different number of end echelons in the chain can be used. Similarly, the number of distribution centres providing operation of local echelons can also be various. It is obvious that in organization of the supply chain it is necessary to achieve cost reductions for the delivery and maintenance of required stocks.

The issues of optimal arrangement of chains echelons are beyond the scope of this book. However, since the problem of calculation of safety stocks is very important here, it makes sense to focus on the nature of the changes in echelon safety stock when the number of echelons of the same level in the chain changes.

It turns out (Maister 1976), with such changes, the so-called Square Root Law is valid, according to which the quantity of the total stock in the chain echelons of the same level depends on their quantity to power of ½. For example, for local chain echelons in Fig. 9.6b, the following dependence is valid:

$$\sum_{i=1}^{N(k_2)} Z_i = \sum_{i=1}^{N(k_1)} Z_i \times \sqrt{N(k_2)/N(k_1)}, \tag{9.29}$$

where k is the number of chain structure variant and $N(k)$ is the quantity of local echelons of variant k. If the number of echelons change, for example, from three in the first variant to four in the second variant, $N(k_1) = 3$, $N(k_2) = 4$, and formula (9.29) has the form $\sum_{i=1}^{4} Z_i = \sum_{i=1}^{3} Z_i \times \sqrt{4/3}$.

For the data given in Table 9.10 at $\lambda = 0$, $\sum_{i=1}^{3} Z_i = 127 + 183 + 228 = 538$. With increasing number of local echelons up to four $\sum_{i=1}^{4} Z_i = 538 \times \sqrt{4/3} = 717$ and accordingly echelon stock Z_0^e increase by value $717 - 538 = 179$.

Since in case of increase in the number of local echelons it was assumed that the total demand of end users did not change, but only was redistributed to the new number of local echelons, the increase in the echelon stock leads to the same increase in echelon safety stock. Thus, we see that the increase in the number of local echelon serving the non-changing number of consumers leads to significant increase in the required safety stock. Of course, reduced number of echelons yields the reverse result.

9.5 Inventory Planning in Spare Parts Supply Chains

In the supply of spare parts two-level model (s, S), described above in Sect. 8.2.3, is usually used. Therefore, application of the method of stock echeloning given in the preceding paragraph and based on the model with a fixed reorder cycle is not justified here.

The reason for feasibility of (s, S) model in this case is that, firstly, the intensity of demand for spare parts is low, and secondly, it is irregular. Recall that in the (s, S)

model, the largest value of stock $\dot{S}$ is set as the sum of the safety stock and the expected demand for some average supply cycle. Since the duration of this cycle is very uncertain, stock $\dot{S}$ is established either on the basis of existing experience or by a directive. In the latter case, it becomes necessary to maintain stock at the constant level, so that after each issue of at least one spare part, there is the need for another purchase order for a new part of the relevant type. As a result, the threshold of order $\dot{s} = \dot{S} - 1$ and (s, S) model is transformed into the model $(S - 1, S)$.

With a relatively stable demand, which is true for the known range of customers, model (R, Q) described in Sect. 8.2.1 can be used. In this model, lot with fixed quantity Q is ordered from the manufacturer at fixed reorder point R, i.e. when the product stock becomes equal to the value of threshold $\dot{s}$.

9.5.1 METRIC Method in Spare Parts Supplies

For the supply chain using model $(S - 1, S)$ in Sherbrook (1968) a method for planning stocks was developed, which is called METRIC. In this method, to describe the demand the Poisson distribution (3.13) is used in Sect. 3.2.2, and for storage operation FIFO priority rule (Sect. 2.3.1) is used, i.e. "First in – first out".

Let us consider a two-stage supply chain of the type shown above in Fig. 9.6b and having N of local echelons. For each i-th echelon of the chain we use the following designations:

L_i—time of product delivery to the i-th echelon from the previous echelon;
Z_i—current stock;
B_i—current backlog;
λ_i—mean demand intensity;
$\dot{S}_i$—max. quantity of stock;
w_0—random value of delay when issuing from the central storage (echelon 0).

In METRIC method, the chain analysis, unlike the method of echelon stock, begins not with local echelons, but the central storage. Suppose that at time t in echelon 0, current stock Z_0 is equal to $\dot{S}_0$. During the time of lot delivery from an external supplier the stock reduces by random variable $D_0(L_0)$

$$Z_0(t + L_0) = \dot{S}_0 - D_0(L_0); \tag{9.30}$$

herewith the average value of spending during this time will make λL_0. Following this consumption, stock Z_0 can take the value equal to integer j within from 1 to $\dot{S}_0$.

According to Poisson distribution (3.13) the probability of equation $Z_0 = j$ is defined by dependence

$$\mathrm{P}(Z_0 = j) = \frac{(\lambda t)^n}{n!} e^{-\lambda t}. \tag{9.31}$$

From Eq. (9.30) it is obvious that $Z_0 = j$ in that case, if the quantity of the spend product $D_o(L_0) = \dot{S}_0 - j$. As in model $(S - 1,\ S)$ it is considered that if the consumption is performed by a piece at a time then the number of orders n for period L_0 equals $\dot{S}_0 - j$. Regarding that duration of period t in formula (9.31) equals L_0, we have

$$\mathrm{P}(Z_0 = j) = \frac{(\lambda L_0)^{\dot{S}_0 - j}}{(\dot{S}_0 - j)!} e^{-\lambda L_0} \ at\, j \leq \dot{S}_0. \tag{9.32}$$

In this case the mean value of the stock in echelon 0

$$\overline{Z}_0 = \sum_{j=1}^{\dot{S}_0} j \times \ \mathrm{P}(Z_0 = j), \tag{9.33}$$

and the mean value of backlog of the central echelon 0 for external customers (Axseter 2006)

$$\overline{B}_0 = \overline{Z}_0 - \left(\dot{S}_0 - \lambda L_0\right). \tag{9.34}$$

Going to determination of the stock and backlog of local echelons we shall note that the mean lead time in the central echelon according to Little's law (formula 3.15 in Sect. 3.2.2)

$$\overline{w}_0 = \overline{B}_0 / \lambda_0. \tag{9.35}$$

Waiting time for each of the local echelons is the same, since FIFO rule is used. Therefore, the average duration of order delivery to the i-th local echelon is

$$\overline{L}_i = L_i + \overline{w}_0. \tag{9.36}$$

Now for the probability of equation $Z_i = j$ in the i-th local echelon, similar to Eq. (9.32), we obtain

$$\mathrm{P}(Z_i = j) = \frac{(\lambda \overline{L}_i)^{\dot{S}_i - j}}{(\dot{S}_i - j)!} e^{-\lambda \overline{L}_i} \ with\, j \leq \dot{S}_i. \tag{9.37}$$

Accordingly, the mean values of the stock and backlog in the local echelon are defined by the dependencies similar to Eqs. (9.33) and (9.34)

Table 9.11 Parameters of the supply chain, mean stocks and backlogs

Echelon	λ_i pcs/day	$\dot{S}_i$ pcs	L_i days	$\overline{L}_i$ days	$\overline{Z}_i$ pcs	$\overline{B}_i$ pcs	$\overline{w}_i$ days
0	1.2	4	4	–	0.49	1.29	1.08
1	0.1	1	1	2.08	0.81	0.02	0.20
2	0.7	3	1	2.08	1.62	0.08	0.17
3	0.4	2	1	2.08	1.23	0.06	0.16

$$\overline{Z}_i = \sum_{j=1}^{\dot{S}_i} j \times \mathrm{P}(Z_i = j) \text{ } and \text{ } \overline{B}_i = \overline{Z}_i - (\dot{S}_i - \lambda \overline{L}_i). \tag{9.38}$$

The mean lead time in the local echelon, similarly as in Eq. (9.35)

$$\overline{w}_i = \overline{B}_i / \lambda_i. \tag{9.39}$$

Table 9.11 shows the input parameters of the supply chain in Fig. 9.6b and the corresponding mean values of stocks and backlogs as determined by the above dependencies. Obviously, the intensity of orders in echelon 0 is the sum of the intensities in echelons 1, 2, 3, and value $\lambda_0 L_0 = 4.8$.

The mean value of the stock in echelon 0 according to expressions (9.32) and (9.33)

$$\overline{Z}_0 = \sum_{j=1}^{\dot{S}_0} j \times \mathrm{P}(Z_0 = j) = \left(1 \times \frac{4.8^3}{3!} + 2 \times \frac{4.8^2}{2!} + 3 \times \frac{4.8^1}{1!} + 4 \times \frac{4.8^0}{0!}\right) e^{-4.8}$$
$$= 0.493.$$

The mean value of backlog in echelon 0 from Eq. (9.34) $\overline{B}_0 = 0.49 - 4 + 4.8 = 1.29$, and the mean waiting time for order in echelon 0, according to Eq. (9.35) $\overline{w}_0 = 1.29/1.2 = 1.08$ day. As a result, the lead time of the order from the central echelon increases from 1 day to 2.08 days. Results for stocks and backlogs in the local echelons in Table 9.11 are calculated by formula (9.38). In the cases, where mean lead time $\overline{w}_i$ of the order is set regulatory, Table 9.11 can be used to determine the relevant regulatory stocks $\dot{S}_i$ in each echelon of the chain.

The described method can be used if the order quantity of the central echelon for an external supplier is greater than unity, i.e. for the order in the form of a lot with quantity Q. In this case, stock management model (S, Q) (Sect. 8.2.1) is used, in which the reorder threshold in echelon 0 is equal to

$$\dot{s}_0 = \dot{S}_0 - Q_0. \tag{9.40}$$

The probability of equation $Z_0 = j$, similarly to Eq. (9.32), has the form (Clark and Scarf 1960)

$$\mathrm{P}(Z_0 = j) = \frac{1}{Q_0} \sum_{k=\max(j,\dot{s}_0+1)}^{\dot{S}_0} \frac{(\lambda L_0)^{k-j}}{(k-j)!} e^{-\lambda L_0} \quad \text{with} \quad j \leq \dot{S}_0. \tag{9.41}$$

Assume that with the data in Table 9.11 lot quantity $Q_0 = 2$ and accordingly $\dot{s}_0 = 4 - 2 = 2$. In this case, for example, for $j = 1$ $k = \max(1, 2+1) = 3$ and

$$\mathrm{P}(Z_0 = 1) = \frac{1}{2} \sum_{k=3}^{4} \frac{(1.2 \times 4)^{k-1}}{(k-1)!} e^{-1.2 \times 4} \quad \text{or}$$

$$\mathrm{P}(Z_0 = 1) = \frac{1}{2} \left[\frac{(1.2 \times 4)^{3-1}}{(3-1)!} + \frac{(1.2 \times 4)^{4-1}}{(4-1)!} \right] e^{-4.8} = 0.123.$$

By calculating in the same way the probabilities of stocks at $j = 2, 3, 4$, we will obtain the mean stock in echelon 0 according to Eq. (9.33):

$$\begin{aligned} \overline{Z}_0 &= \sum_{j=1}^{\dot{S}_0} j \times \mathrm{P}(Z_0 = j) = 1 \times 0.123 + 2 \times 0.067 + 3 \times 0.023 + 4 \times 0.004 \\ &= 0.35. \end{aligned}$$

Comparing the obtained value with the stock calculated above when the quantity of the order is equal to unity, we see that the value of the mean stock with increase in the quantity of the order reduces. In contrast to ($S - 1$, S) model used in case of rare and irregular demand, ordering by lots in (S, Q) model is applied at relatively predictable demand. An example of irregular demand is operation of military equipment; an example of stable demand for spare parts can be maintenance of the vehicle fleet. In the latter case, to plan spare parts supply by lots it makes sense to use DRP-tables described in Sect. 9.1.

In those cases where for maintenance it is required to have n parts of different types, the mean waiting time for the required set of parts in the i-th local echelon can be determined using a variation of the above-described method, which is called MOD-METRIC (Muckstadt and Thomas 1980). The corresponding expression has the form

$$\overline{w}_i = \sum_{j=1}^{n} \lambda_{i,j} \overline{w}_{i,j} \Big/ \sum_{j=1}^{n} \lambda_{i,j}. \tag{9.42}$$

For example, assume that in echelon 2 (Table 9.11), besides the spare part with number 1 and intensive demand $\lambda_{2,1} = 0.7$, for the maintenance it is required to have also spare part 2 with demand intensity $\lambda_{2,1} = 0.5$. The mean waiting time for part 1, according to Table 9.11, is 0.17. Assume that the waiting time for part 2 is 0.25. Then we obtain

$$\overline{w}_i = \sum_{j=1}^{2} \lambda_{i,j}\overline{w}_{i,j} / \sum_{j=1}^{2} \lambda_{i,j} = \frac{0.7 \times 0.17 + 0.5 \times 0.25}{0.7 + 0.5} = 0.2.$$

9.5.2 Inventory Planning for Central Spare Parts Storage Using (*R*, *Q*) Model

Usually, the demand for spare parts is random, and at the same time it requires fast fulfilment of orders. High level of orders service, of course, necessitates maintaining large stocks in the distribution storage. At the same time, stocks availability is associated with significant costs for their keeping.

The article (Hopp et al. 1997) presents the development of a method for optimal control of the storage, which has a wide range of spare parts, and as a computational model the so-called (R, Q) model (Sect. 8.2.1) is used. The goal of optimal control in this case is to minimize the total cost of stocks when there are two constraints: on mean frequency of orders for replenishment and on mean level of customer service at the storage.

The method suggested in Hopp et al. (1997) was further developed in Zhang et al. (2001), and here mainly the results obtained in this paper are given. In accordance with Zhang et al. (2001) for the i-th type of parts we denote the annual demand as D_i, the cost per nomenclature unit as c_i, the lead time as l_i in years, the consumption of spare parts during the lead time as $\theta_i = D_i l_i$, and the number of items in the nomenclature of parts as n. We also denote the total demand for all nomenclature items as $D = \sum_{i=1}^{n} D_i$ and total cost as $c = \sum_{i=1}^{n} c_i$.

The probability density of demand distribution is denoted by P_i, and standard deviation of demand during delivery is denoted as σ_i. If we assume that the demand for any type of i-th parts within the shipping time has Poisson distribution (Sect. 3.2.2), then $\sigma_i^2 = \theta_i$ (Zhang et al. 2001).

For each i-th product, generally speaking, the directive constraint can be set for the highest frequency of orders Ω_i, as well as for own level of customer service S_{Li}. However, in the presence of a large range of spare parts it is possible to set some average values of these parameters Ω and S_L.

The value of the mean stock $\overline{Z}_i$ of the i-th type parts depends on order quantity Q_i and reorder threshold $\dot{s}_i$

$$\overline{Z}_i(\dot{s}_i, Q_i) = \dot{s}_i - \theta_i + Q_i/2. \tag{9.43}$$

In the strict sense, expression (9.43) is valid if $\dot{s} - \theta \geq 0$, but in Hopp et al. (1997) it is said that there will be no big error when it is used, if $\dot{s} - \theta < 0$. In this case the optimization problem is finding the minimum of value

$$\sum_{i=1}^{n} c_i(\dot{s}_i - \theta_i + Q_i/2) \tag{9.44}$$

with constraints

$$\frac{1}{n}\sum_{i=1}^{n}\frac{D_i}{Q_i} \leq \Omega \tag{9.45}$$

and

$$\sum_{i=1}^{n}\frac{D_i}{D}[1 - A_i(\dot{s}_i, Q_i)] \geq S_L. \tag{9.46}$$

Value $A_i(\dot{s}_i, Q_i)$ in expression (9.46) is the probability of refusal from order fulfilment and is defined as (Zhang et al. 2001):

$$A_i(\dot{s}_i, Q_i) = \frac{1}{Q_i}[\alpha_i(\dot{s}_i) - \alpha_i(\dot{s}_i + Q_i)], \tag{9.47}$$

where

$$\alpha_i(v) = \sum_{u=v+1}^{\infty}(u - v)\mathrm{P}_i(u). \tag{9.48}$$

Problem (Eqs. 9.44–9.46) is solved (Hopp et al. 1997) using Lagrange method, wherein constraints (9.45) and (9.46) are accounted by special parameters (Lagrange multipliers), similarly to Sect. 9.4.3. As a result, for the optimal quantity of the order of the *i*-th product we have obtained the expression

$$Q_i = \sqrt{\frac{2c\lambda_1 D_i}{nc_i}}, \tag{9.49}$$

where λ_1 is Lagrange multiplier for order quantities.

Comparing expression (9.49) with the value determined by formula EOQ (2.4) we see that the role of the costs for ordering c_o here is played by product average cost c/n, and storage costs—by product cost c_i. Using expression (4.10), we can assume that Lagrange multiplier serves as correction factor χ, and this factor is the same for products of all kinds.

Reorder threshold value $\dot{s}_i$ is defined as the sum of the parts consumption during delivery time θ_i and safety stock Z_{ci} required to provide the required level of service S_{Li}, and according to Eq. (8.37), we have

$$\dot{s}_i = \theta_i + Z_{ci} = \theta_i + \kappa_i \sigma_i', \tag{9.50}$$

where σ' is the mean square deviation of demand during the delivery and safety factor κ_i is completely determined by the level of service S_{Li} and can be found using function NORMSINV MS Excel.

When solving optimization problem (Eqs. 9.44–9.46) in Hopp et al. (1997) the expression for κ_i was found in the form

$$\kappa_i = \sqrt{-2\ln\left(\sqrt{2\pi}\frac{\sqrt{l_i}c_iD}{\sqrt{D_i}\lambda_2}\right.}, \tag{9.51}$$

in which λ_2 is Lagrange multiplier of reorder point.

Direct use of dependences (9.49) and (9.51) to determine the parameters of the order is difficult because of the uncertainty in determining the values of Lagrange multipliers. Therefore, in the paper of Zhang et al. (2001) it is proposed to use the formula following from dependence (9.49), to determine the order quantity taking into account the constraints (9.45), namely:

$$Q_i = \frac{\sum_{i=1}^{n}\sqrt{D_i c_i}}{n\Omega}\sqrt{\frac{D_i}{c_i}}. \tag{9.52}$$

The situation with determination of safety factor κ_i is more difficult, as for this purpose it is necessary to have required value of service level for each i-th type of spare parts S_{Li}, while in the source problem the average values of this parameter S_L are defined. At the same time it can be noted that from expression (9.51) it follows that value κ_i depends on parameter $\varsigma_i = \frac{D_i}{l_i C_i^2}$, and with increase of this parameter κ_i should increase too.

In the paper (Zhang et al. 2001) it is suggested instead of one mid-level service S_L to introduce three groups of spare parts with different levels of service similar to the widely used ABC classification of goods (Sect. 8.1). Herewith it is proposed to sort the range of spare parts by the increasing value of parameter ς_i.

Accordingly, category A amounting to 20 % of all items of nomenclature should include spare parts with small values ς, for which the lowest level of service must be set. Category B with mid-level of service is suggested to include 30 % with average values ς, and for category C, covering 50 % of the nomenclature it is suggested to set the highest level of service.

In this case, to determine the reorder threshold, you can use formula (9.50). The example of calculation is shown in Table 9.12, where it is accepted that the highest frequency of orders per year $\Omega = 6$, service levels $S_{LA} = 0.65$, $S_{LB} = 0.92$, $S_{LC} = 0.97$. Various products are sorted in Table 9.12 by ascending order of parameter ς_i. As in this example it is assumed that the number of spare part types is ten, the first two types belong to category A, the next three to category B, and the rest to category C.

Table 9.12 Optimal quantities of orders and stocks at reorder points

Type of product	c_i	l_i	D_i	θ_i	Q_i	Ω_i	ς_i	$\dot{S}_{Li}$	$\dot{s}_i$	$\overline{Z}_i$
1	250	0.2	20	4	2	10	0.00	0.65	5	2
2	150	0.2	20	4	3	7	0.07	0.65	5	2.5
3	40	0.11	100	11	12	9	0.57	0.92	16	10
4	25	0.11	130	14.3	17	8	1.89	0.92	20	14
5	13	0.06	120	7.2	22	5	11.8	0.92	11	15
6	6	0.06	200	12	42	5	92.6	0.97	19	28
7	5	0.01	200	2	46	4	800	0.97	5	26
8	4	0.01	300	3	64	5	1875	0.97	6	35
9	3	0.01	300	3	73	4	3333	0.97	6	40
10	2	0.01	300	3	90	3	7500	0.97	6	48

Let us consider, for example, the calculation for product 2. From formula (9.52), we obtain

$$Q_2 = \frac{\sum_{i=1}^{n}\sqrt{D_i c_i}}{n\Omega}\sqrt{\frac{D_2}{c_2} = \frac{\sum_{i=1}^{10}\sqrt{D_i c_i}}{10\times 6}\sqrt{\frac{20}{150}} = \frac{440}{60}\times 0.36 \approx 3.}$$

In this case the frequency of orders $\Omega_2 = 20/3 \approx 7$. Parameter $\varsigma_2 = \frac{D_2}{l_2 c_2^2} = \frac{20}{0.2\times 150^2} = 0.07$ and product 2 belongs to category A with level of service $S_{LA} = 0.65$.

To define the quantity of reorder $\dot{s}_2$ we use formula (9.50). In this case safety factor $\kappa_2 = \mathrm{NORMSINV}(0.65) = 0.385$, mean square deviation $\sigma_2' = \sqrt{\theta_2} = \sqrt{4} = 2$. Hence we obtain $\dot{s}_2 = \theta_2 + \kappa_2\sigma_2' = 4 + 0.385\times 2 \approx 5$. The value of mean stock by formula (9.43) $\overline{Z}_2 = \dot{s}_2 - \theta_2 + Q_2/2 = 5 - 4 + 3/2 = 2.5$.

According to the data from Zhang et al. (2001), the service level values calculated by formulas (9.47 and 9.48) are usually more than value $\dot{S}_{Li}$, accepted in Table 9.12. This means that service levels in Table 9.12 are minimal possible and their actual values have somewhat higher values.

9.6 Coordinated Planning Between Two Supply Chain Members

As an example of optimal joint action of the two members in the chain, we consider the Newsvendor Problem described above in Sect. 8.3.4. In this case, the chain includes newspapers supplier (the publisher) and the vendor. Each of them, of course, is committed to the highest own profit, but the profit of each depends on the total profits from the sales of newspapers to consumers. Therefore, they have a common interest in the most quantity of sold product.

The newspapers vendor, as followed from Sect. 8.3.4, usually prefers to order from the supplier the amount of product that maximizes its profits in accordance with expression (8.26). At the same time, it turns out that this number is not optimal in terms of the total profit that can be obtained in the process of production and sales of the product. Similar to Sect. 8.3.4, we introduce the following designations:

Q—quantity of newspapers ordered from the supplier for sale within 1 day;
c_o—price per newspaper, for which the vendor purchases newspapers from the supplier;
c_u—vendor's profit received from sale of one newspaper;
c_m—market price, $c_m = c_o + c_u$;
c_p—production cost of one copy;
c_b—compensation to the vendor per one unsold newspaper;
μ—mean demand;
σ—mean square deviation of demand;
q—statistically expected quantity of sales;
Π_s—expected profit of supplier;
Π_r—expected profit of vendor;
Π—total expected profit.

The expected profit of the vendor is the difference of expected profit from sales and expenses for purchasing the newspapers from the publisher

$$\Pi_r = c_m q - Q c_o. \tag{9.53}$$

Statistically expected quantity of sales according to Axseter (2006) is defined by dependence

$$q = \mu - \sigma G\left(\frac{Q-\mu}{\sigma}\right),$$

where $G\left(\frac{Q-\mu}{\sigma}\right)$ is the so-called loss function defined above in Sect. 8.3.3 by dependence (8.19).

Since the expected profit of the supplier

$$\Pi_s = Q(c_o - c_P), \tag{9.54}$$

then the total profit

$$\Pi = \Pi_r + \Pi_s = c_m\left[\mu - \sigma G\left(\frac{Q-\mu}{\sigma}\right)\right] - Qc_P. \tag{9.55}$$

To determine quantity Q that provides the greatest value of the total profit it is necessary to differentiate expression (9.55) by Q and equate the value obtained to zero, which is the same operation as in Sect. 8.3.4 above. As a result, we obtain the expression for optimal $Q*$

$$\Phi\left(\frac{Q^{*}-\mu}{\sigma}\right)=\frac{c_o-c_p+c_u}{c_m}\ with\ c_o>c_P. \tag{9.56}$$

However, the vendor orders the quantity of newspapers less than $Q*$ from the supplier, since for him optimal order quantity Q_r^* is defined by dependence (8.26)

$$\Phi\left(\frac{Q_r^{*}-\mu}{\sigma}\right)=\frac{c_u}{c_m}, \tag{9.57}$$

the right side of which is obviously less than the right side of expression (9.56).

In order to encourage the vendor to do the order equal to $Q*$, the supplier must share a part of its profit. For example, perhaps the supplier can take back the newspapers unsold during the day with predetermined compensation c_b. Let the size of the compensation depend on the market price

$$c_b=(1-g)c_m, \tag{9.58}$$

and the selling price is set in the amount of

$$c_0=(1-g)c_m+gc_P\,\text{with}\,0<g<1. \tag{9.59}$$

In this case the vendor's profit

$$\Pi_r=c_mq-Qc_o+c_b(Q-q)=gc_mq-gQc_o=g\Pi, \tag{9.60}$$

and the supplier's profit

$$\Pi_s=Q(c_o-c_P)-c_b(Q-q)=(1-g)c_mq-(1-g)Qc_o=(1-g)\Pi. \tag{9.61}$$

Thus, with coordinated action of the supplier and vendor the optimal order quantity should be determined from expression (9.56), and the members of the supply chain must agree on the sharing of common profits, establishing acceptable rate g.

Let us consider an example of such calculation using the data from the example in Sect. 8.3.4, in which $\mu=50$; $\sigma=14$ and purchasing price $c_o=10$, sales liftup $c_u=15$, and $c_P=5$. From Eq. (9.56) given that $c_m=c_o+c_u=25$, we obtain

$$\Phi\left(\frac{Q^{*}-50}{14}\right)=\frac{10-5+15}{25}=0.8\,\text{and NORMSINV}\,(0.8)=0.841.$$

The obtained value $Q^{*}\approx 62$ is slightly higher than the optimal quantity of order of 55 pcs., obtained in Sect. 8.3.4 above by formula (9.57). Suppose, for example, number of newspapers actually sold q equals mean value $\mu=50$. In this case the vendor's profit at his usual not coordinated order is equal to $\Pi_r = 15 \times 50 - (55 - 50) \times 10 = 700$, and the total profit of the entire chain amounts to $\Pi = q(c_m - c_P) = 50 \times (25 - 5) = 1000$. Thus, without coordination the vendor receives 70 % of the profits on average.

It is obvious that coordination is possible only if for each of the members the coordinated result will not be worse than the result without coordination. Therefore, offering the vendor to place the order of 62 units, the supplier must agree that the vendor's share of the total profit g will be not less than 70 %.

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Planning of Supplies to Consumers 10

10.1 Sales and Operation Planning

The sales and operations plan is elaborated according to the business plan or similar document describing the enterprise development strategy and is the basis for its operations. This plan connects the main goals of the business plan with planning of production, finance, sales, and other enterprise services.

10.1.1 Interrelation Between Various Planning Directions with Sales and Operations Plan

Figure 10.1 shows the basic planning directions related to the sales and operations plan. As can be seen from Fig. 10.1, the indices of this plan are generated based on a range of figures determined during strategic planning, demand planning, and finance planning as well as the constraints set during planning of production, procurement, and sales.

The input data for sales and operations planning are key figures of output of the main aggregated types of products, defined in strategic planning, and demand forecasting results for these products. During plan elaboration, these data are analysed and compared with the available resources.

The quality of the sales and operations plan under development is primarily determined by the balance of demand for products and resources used. If the demand exceeds the capabilities of the enterprise significantly during its normal operation, this leads to an increase in cost due to overtime payments and a reduction in the product quality. On the contrary, when the production plan is in excess of actual demand, it causes surplus products and financial and economic indicators decline.

Another very important aspect of the sales and operations plan consists in the right ratio of the aggregate products planning and its detailing. In developing the sales and operations plan, first it is necessary to define the consolidated production

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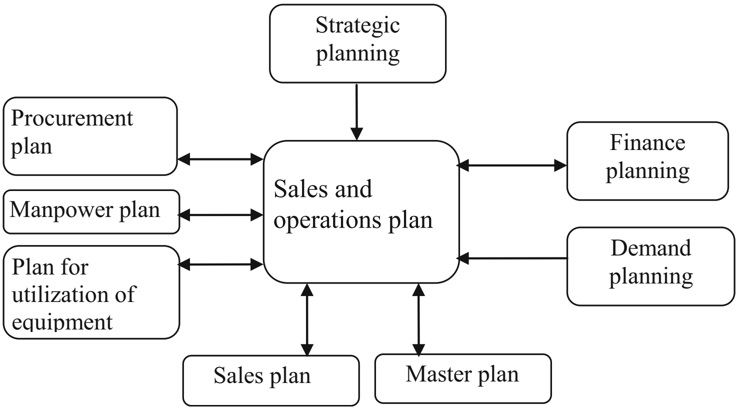


Fig. 10.1 Interactions of various planning directions

output for aggregated groups of products and only then to specify such groups by specific standard sizes of products.

The need for mandatory planning of aggregated groups is due to the fact that the adequate balancing of demand and production for an extended period can only be at the aggregate level. The plan drawn up at this level is the basis of medium-term planning of material and manpower resources.

The plan by the aggregated product groups can be calculated in different units. The most common indicator is, of course, the production output in terms of value. This figure, however, says little to production managers and employees of procurement services. For these employees, for example, values of output in physical units, tons, or even in labour hours are more convenient.

The amount of aggregated groups are generally (Vollmann et al. 2005) is in the range from 6 to 12. This grouping allows covering the main types of products and duly analysing their contribution to the earned income, production, and resources use. Aggregation of production into groups can be carried out by various parameters: type of product, size or properties, market segments, and even individual consumers, etc. The most frequent and successful aggregation is by commodity groups in the market.

The production plan by a group in a certain period does not necessarily equal the forecasted demand in this period. The output in some period may provide the demand of future periods and can less be for some reason than the current demand. The practice has proven that absence of aggregate sales and operations plan or poor elaboration of it causes a number of unpleasant consequences—excessive commodity and material stocks, low level of customer service, equipment downtime, long duration of the production cycle, nervousness occurring when dispatching, overspending of payroll, and finally great difficulties in introducing new products.

10.1.2 Sales and Operation Planning Methods

The most common way of sales and operations planning is to draw up tables containing the values of the corresponding parameters at different time intervals—usually monthly (Table 10.1).

In the example shown in Table 10.1, the sales and operations plan is made on a quarterly basis with a horizon of 6 months. Here you can see a version of the plan for the second and third quarters for some aggregated group of products, which are measured in pieces. According to the last quarter actual sales trends and their deviation from the original plan can be judged. Similarly, we can analyse the results of the production for the last quarter. In the above example, the increase in production and the tendency to fulfil the plan are clearly observed.

The main indicator of the production plan is the value of the planned monthly operating load. Table 10.1 assumes that manufacturing of a product unit of this group requires 12 h. Considering that the production plan uses the chase strategy, in which the production should accurately track the demand, the production load varies quite widely—from 3360 to 4320 h. Accordingly, the average daily load varies as well—from 156 to 206 h.

The targets stock at the end of each period is usually set by the standard stock in the days of consumption. In this case, it is assumed that for the product group in question for the standard is about 8 working days. The actual stock in days for each of the last periods can be determined by dividing the available balance by consumption rate during this period.

Consistent use of the chase strategy leads to the necessity of frequent dismissals or, vice versa, urgent recruitment. Although when integrating of sales and operations plans of all aggregated product groups produced by the enterprise, the total operating capacity fluctuations can be reduced, yet the use of the chase strategy in its pure form occurs rarely. Another extreme strategy of the preparation of a production plan is to maintain stable level of production (level strategy). In this case, the actual changes in demand are ignored, and as a result, either stocks increase or consumers' orders are not fulfilled accordingly. This strategy, of course, cannot be used long enough.

Actually, the production plan, as a rule, is a compromise between the desire to produce products in accordance with the forecasted demand and the real capabilities of the enterprise. First of all, when planning sales and production it makes sense to establish a rational relationship between the cost of changes in personnel and the cost of keeping stock balances. To do this, you can consider several plan options and carry out their ranking by value of total expenses. Besides, it is necessary to take into account possible both legal and professional consequences of dismissals of employees, as well as the influence of these factors on the strategic goals of the enterprise.

The production plan option is acceptable if it can be provided by necessary resources. Checking of resources use, for the most part, is reduced to approximate calculations of capacity utilization, but in some cases, it is also necessary to check

Table 10.1 Sales and operations plan for one product group with chase strategy

	Previous periods			Future periods					
Parameter	January	February	March	April	May	June	July	August	September
Sales in pieces									
Forecast	300	280	320	350	360	330	300	300	330
Actual	322	276	295						
Difference	22	−4	−25						
Production in pieces									
Forecast	300	280	320	350	360	330	300	300	330
Actual	260	310	320						
Difference	−40	30	0						
Working days	20	21	23	22	21	22	22	23	23
Workload per plan in hours	3600	3360	3840	4200	4320	3960	3600	3600	3960
Average daily load in hours	180	160	167	191	206	180	164	156	172
Stock at the end of period in pieces									
Forecast	100	100	100	120	120	110	100	100	100
Actual	115	120	96						
Stock in days	7.9	9.2	6.9						

Table 10.2 Workload according to the sales and operations plan in hours

Product group	April	May	June	July	August	September
G1	4200	4320	3960	3600	3600	3960
G2	6300	5100	5500	5400	5400	6200
G3	1800	2100	2100	2300	2300	2400

Table 10.3 Processing time of product groups for divisions

Division	Number of staff	Group processing time, h			Processing time factor *K*		
		G1	G2	G3	G1	G2	G3
A1	23	4.5	1.3	0.9	0.37	0.26	0.19
A2	16	1.8	0.5	1.2	0.15	0.10	0.25
A3	12	2.4	0.6	0.4	0.20	0.12	0.08
A4	14	0.6	1.1	1.2	0.05	0.22	0.25
A5	16	2.7	1.5	1.1	0.23	0.30	0.23

the possibility of providing a plan of working capital, raw materials, qualified personnel, energy, water, vehicles, etc.

In developing the sales and operations plan, the capacity utilization can be verified using relatively crude but simple method of so-called overall factors (Stadtler and Kilger 2008), showing the share of processing time of each division or work center in the total processing time per product unit. Table 10.2 shows an example of the workload calculated for certain version of the sales and operations plan. Table 10.3 shows processing time of each product group for different divisions and work centers of the enterprise, and the processing time share (factor) of each product group, which account for the relevant division.

The capacity load of the *i*-th division in the *j*-th month is determined as

$$W_{ij} = \sum_{k=1}^{n} K_{ik} W_{jk}, \tag{10.1}$$

where n is the number of product group; W_{jk} is the total workload in the j-th month, required for production of the k-th product group; and K_{ik} is the processing time factor of the i-th division when producing the k-th product group.

Assume that the calculation is made for 6 months, the first of which is April, on three groups of products (Table 10.2). Using the data in Table 10.3, we calculate the workload, for example in May, for division A1, which is considered to be the first

$$W_{1,2} = \sum_{k=1}^{3} K_{1k} W_{2k} = 0.37 \times 4320 + 0.26 \times 5100 + 0.19 \times 2100 = 3340.$$

The working time fund of the division in hours is defined as the product of the number of staff by the total number of days each month, specified in Table 10.1, with an 8-h working day. Table 10.4 shows the calculated workload in hours

Table 10.4 Divisions load according to the sales and operations plan

Division	April	May	June	July	August	September
A1	3551/87.7	3340/86.4	3309/81.7	3185/78.7	3185/75.3	3547/83.8
A2	1710/60.7	1683/62.6	1669/59.3	1655/58.8	1655/56.2	1814/61.6
A3	1746/82.7	1651/81.9	1627/77	1560/73.8	1560/70.6	1736/78.6
A4	2046/83	1863/79.2	1933/78.4	1943/78.9	1943/75.4	2162/83.9
A5	3248/115	2983/111	3022/107	2957/105	2957/100	3301/112

according to data of Tables 10.2 and 10.3, as well as after slashing the corresponding percentage of utilization by the periods for each division.

Form Table 10.4 it follows that division A5 in this variant is obviously overloaded and division A2 is underloaded.

The verification of provision with various other resources is usually performed based on the expenditure standards for these resources per product unit of the relevant aggregated group

$$Q_{ij} = \sum_{k=1}^{n} q_{ik} N_{jk}, \tag{10.2}$$

where Q_{ij} is the quantity of resource of the i-th type, required in the j-th month; q_{ik} is the resource quantity of the i-th type per product unit of the k-th group; and N_{jk} is the number of products of the k-th group in the j-th month.

10.2 Sales and Operation Plan Optimization Using Linear Programming

In Sect. 2.2.3, it was indicated that the main criterion for quality of the sales and operations plan is the value of the potential profits to be received. In the examples given in Sects. 2.5.1 and 2.1.2 the use of linear programming methods was considered to draw up various plans with maximization of possible profit.

There are many different statements of such problems in order to optimize sales and operations plans. Here we confine ourselves to the tasks that use the strategy, which is close to the chase strategy, i.e. to the cases when the quantity of products sold should tend to the forecasted demand values. Of course, this does not mean that the quantity of products produced within a certain period should be exactly equal to the quantity sold within this period, although this matching is preferable “on average” for several periods.

If we assume that the number of products sold in all aggregated groups determined by the forecast demand, and prices are set and will not change, the amount of possible income is completely determined by this demand. In this case, the best option plan becomes one in which the values of the production costs are minimized.

10.2.1 Single Aggregated Product Group Optimization

Consider the simplest case, in which only one aggregate group of products is planned. The values of costs are determined by the direct expenditures on raw materials and labour costs, the cost of hire and fire of staff, storing of finished, but not sold products, as well as maintenance of the downtime production in serviceable condition.

$$c = \sum_{t=1}^{h} (c_H H_t + c_F F_t + c_X X_t + c_N N_t + c_O O_t + c_Z Z_t + c_U U_t), \tag{10.3}$$

where h is the planning horizon, c_H is the cost of hire per employee, c_F is the cost of fire per employee, c_X is the cost of materials per product unit, c_N is the average in period salary of one employee in normal conditions, c_O is the cost of overtime hour, c_Z is the cost of storage per product unit, and c_U is the cost of equipment downtime per hour.

Within each period t the following must be planned: H_t—number of hired employees, F_t—number of fired employees, X_t—quantity of product produced with the period, N_t—current number of employees, O_t—number of overtime hours, Z_t—quantity of stocks, and U_t—number of hours of production downtime.

Assume that the quantity of the manufactured products is quite large and accordingly X_t can be considered continuous. At the same time, the values of N_t, H_t, and F_t are integer numbers. Thus, for solving one optimization problem we should use the methods of mixed linear programming. If we consider that the products are also discrete then we have to use fully integer programming, which does not really change but complicates the solution procedure to some extent.

In this case, the plan optimization lies in determination of the minimum of value (formula 10.3) with several constraints below.

Quantity of stocks at the end of period t

$$Z_t = Z_{t-1} + X_t - D_t, \tag{10.4}$$

where D_t is the demand forecast within the planning period.

Balance of staff quantity

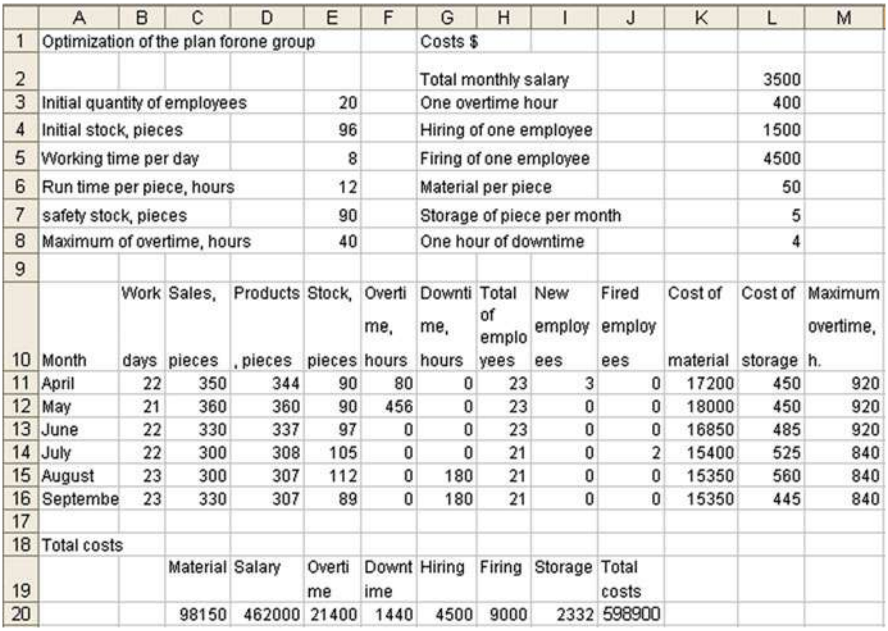
$$N_t = N_{t-1} + H_t - F_t. \tag{10.5}$$

Balance of labour time

$$\text{with} pX_t \geq GN_t,\ O_t = pX_t - GN_t \text{ and } U_t = 0; \tag{10.6}$$

$$\text{with} pX_t < GN_t,\ U_t = GN_t - pX_t \text{ and } O_t = 0. \tag{10.7}$$

where G is the time fund in one employee's hours with regular payment and p is the processing time per product unit in hours.



	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Optimization of the plan forone group						Costs $						
2							Total monthly salary					3500	
3	Initial quantity of employees				20		One overtime hour					400	
4	Initial stock, pieces				96		Hiring of one employee					1500	
5	Working time per day				8		Firing of one employee					4500	
6	Run time per piece, hours				12		Material per piece					50	
7	safety stock, pieces				90		Storage of piece per month					5	
8	Maximum of overtime, hours				40		One hour of downtime					4	
9													
10	Month	Work days	Sales, pieces	Products , pieces	Stock, pieces	Overtime, hours	Downtime, hours	Total of employees	New employees	Fired employees	Cost of material	Cost of storage	Maximum overtime, h.
11	April	22	350	344	90	80	0	23	3	0	17200	450	920
12	May	21	360	360	90	456	0	23	0	0	18000	450	920
13	June	22	330	337	97	0	0	23	0	0	16850	485	920
14	July	22	300	308	105	0	0	21	0	2	15400	525	840
15	August	23	300	307	112	0	180	21	0	0	15350	560	840
16	Septembe	23	330	307	89	0	180	21	0	0	15350	445	840
17													
18	Total costs												
19			Material	Salary	Overtime	Downtime	Hiring	Firing	Storage	Total costs			
20			98150	462000	21400	1440	4500	9000	2332	598900			

Fig. 10.2 Calculation of optimal plan for one product group

Constraint for overtime

$$O_t \leq BN_t, \tag{10.8}$$

where B is acceptable number of overtime hours for each employee within a period.

Necessity for safety stock Z_c at the end of each period

$$Z_t \geq Z_c. \tag{10.9}$$

Consider an example of optimization of the sales and operations plan for the case of forecasted demand, given in Table 10.1. This task is performed using MS Excel spreadsheet similar to the example above in Sects. 2.1.2 and 2.5.1 (Fig. 10.2).

At the top of Fig. 10.2 the initial parameters are given to calculate the optimal plan. They can be divided into three components: the calendar and planning standards, the initial values of the characteristic exponents of the planning process, and cost parameters.

The first includes daily employee's time fund in hours, the processing time per unit of production in hours, standard safety stock at the end of the period, and the maximum allowable overtime hours for the period. As the characteristic parameters of the process in this problem, two parameters are used: the number of employees and the actual stock of product at the end of the period. The cost parameters necessary for solution are listed right at the top of Fig. 10.2.

In the middle of Fig. 10.2 there is the table of designed parameters for all periods in the planning horizon, and the data on sales forecast are taken from Table 10.1.

The variables of the problem here are the values of the monthly output and the number hired and fired employees.

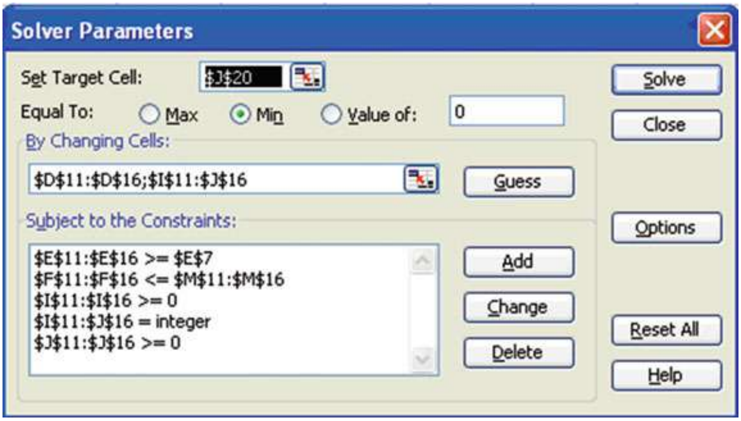
Constraints (10.4)–(10.7) in the spreadsheet are represented by formulas in the relevant cells. For example, in cell E11 the following expression is written =$D$4+$D11–$C11, displaying formula (10.4) for the first month of planning (April). For the next month this dependence in cell E12 has the form =$E11+$D12–$C12, etc.; similarly, the balance of employees for April is expressed in cell H11 as =$D$3+$I11–$J11, and for the subsequent month—entry in cell H12 has the form =$H11+$I12–$J12.

Condition (10.6) is written, for example, in cell F11 in the form =IF($D$6*$D11>=$D$5*$B11*$H11;$D$6*$D11 – $D$5*$B11*$H11;0), and condition (10.7)—cell G11 as =IF($D$6*$D11<$D$5*$B11*$H11;$D$5*$B11*$H11 – $D$6*$D11;0).

Constraints (10.8) and (10.9) should be performed in each of the planning months. To enter these limitations in MS Excel, it is necessary that arrays in the left and right sides of these inequalities have equal dimensions. In particular, to perform conditions (10.8) the elements of the array data in cells F11: F16 must be smaller than the corresponding elements in cells M11: M16. Conditions (10.9) are performed simpler—it is enough that array elements in cells E11: E16 are higher than the value of stock in cell D7.

Figure 10.3 shows the screen for data entry to find a solution for this problem. As can be seen from this form, the calculation result is written in cell J20, and this result should be minimized by adjusting monthly output in cells D11: D16, and the number hired and fired employees in cells I11: J16.

The first two constraints in the form reflect the necessity to perform conditions (10.8 and 10.9), in the following constraints the requirement are recorded that the quantity of hired and fired employees must have the form of non-negative and integer numbers.



Solver Parameters

Set Target Cell: $J$20

Equal To: ○ Max ● Min ○ Value of: 0

By Changing Cells:

$D$11:$D$16;$I$11:$J$16

Subject to the Constraints:

$E$11:$E$16 >= $E$7
$F$11:$F$16 <= $M$11:$M$16
$I$11:$I$16 >= 0
$I$11:$J$16 = integer
$J$11:$J$16 >= 0

Fig. 10.3 Screen for data entry to find the optimal solution

In the problem of linear continuous optimization, the example of which is given above in Sect. 2.1.2, the solution appears to be exact and can be analysed by a sensitivity report (Fig. 2.4). Integer optimization gives an approximate solution, the results of which may depend on the initial values of variables. Unfortunately, the sensitivity report for the problems with integer variables are not provided by MS Excel.

10.2.2 More Complex Case of Optimization of Sales and Operations Plan

Consider the case of optimizing with parallel production of n product groups and the scale of the planned production is limited by capacity of one work center (division). As it was mentioned in the previous paragraph, the objective function, in general, is the received profit. Therefore, we will seek the set of independent variables of the problem at a planning horizon h, which provide the highest possible amount of profit. Instead of expression (10.3) the objective function takes the form

$$\begin{gathered}\varPi=\sum_{i=1}^{n}\sum_{t=1}^{h}\left(c_{qi}q_{i,t}-c_{Xi}X_{i,t}-c_{Zi}Z_{i,t}\right)-hc_p-\\-\sum_{t=1}^{h}(c_H H_t+c_F F_t+c_N N_t+c_O O_t+c_U U_t),\end{gathered}\tag{10.10}$$

where c_{qi} is the product price of the i-th group, $q_{i,t}$ is the sales volumes of products of the i-th group during period t, and c_p is the constant expenditure within the period. The rest of the values match the designations in Sect. 10.2.1; at that the cost of the materials per product unit c_{Xi} and the storage cost of product unit c_{Zi} depend on the i-th product group. So for each group output $X_{i,t}$ and storage $Z_{i,t}$ are different.

The appearance of the capacity constraints can cause a reduction in the planned sales compared with the forecasting demand values. The value of the stock at the end of period t for each group is defined by the dependence

$$Z_{i,t}=Z_{i,t-1}+X_{i,t}-q_{i,t},\tag{10.11}$$

and the requirement for safety stock Z_c at the end of each period

$$Z_{i,t}\geq Z_{ci}.\tag{10.12}$$

Constraint (10.5), related to the staff balance, is maintained. Constraints of labour time balance (10.6) and (10.7) take the form

$$\text{with} \sum_{i=1}^{n} p_i X_{i,t} \geq GN_t, \quad O_t = \sum_{i=1}^{n} p_i X_{i,t} - GN_t \text{ and } U_t = 0; \tag{10.13}$$

$$\text{with} \sum_{i=1}^{n} p_i X_{i,t} < GN_t, \quad U_t = GN_t - \sum_{i=1}^{n} p_i X_{i,t} \text{ and } O_t = 0, \tag{10.14}$$

where p_i is the total processing time of the product unit of the i-th group.

Besides, since it is necessary to provide allowable load on the bottleneck (k-th work center), the additional constraints appear

$$\sum_{i=1}^{n} p_{i,k} X_{i,t} < P_{k,t} \,\text{for all periods}\; 1 \leq t \leq h, \tag{10.15}$$

where $P_{k,t}$ is the capacity of the k-th work center in hours for period t and $p_{i,k}$ is the processing time of the product unit of the i-th group on the k-th work center.

For the purpose of sales it is obvious that the planned sales volume cannot exceed the forecasted values of the relevant demand

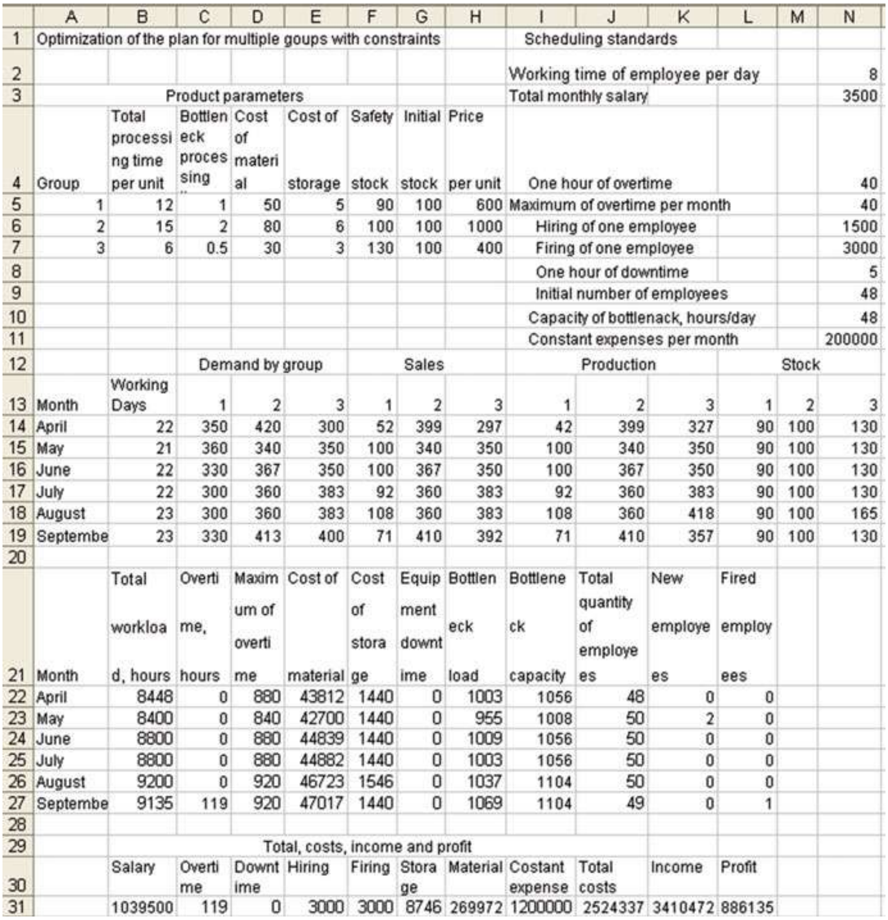
$$q_{i,t} \leq D_{i,t}. \tag{10.16}$$

Figure 10.4 shows the optimized plan of sales and operations for the case. At the top of the spreadsheet there are parameters of three groups of products, as well as calendar and planning standards. All prices and costs are given in conventional units, quantities—in measuring units of products, time funds—in hours, and processing time—in hours.

The next part of the table contains data on demand for each group for periods prior to the planning horizon. The sales and operations are independent variables, the values of which are determined by calculation. Inventories at the end of each period are determined by formula (10.11).

At the bottom of the table there are the main designed parameters that define the values of the constituents in expression (10.10): the cost of materials and the cost of storage and the number of overtime and downtime hours. In addition the total load and the bottleneck load are calculated here, as well as the monthly overtime possibilities and the maximum utilization of equipment. The numbers of hired and fired employees are independent variables of the problem.

The quantity of overtime hours, for example in cell C22, is determined by formula =If($B22>=$N$2*$B14*$J22;$B22-$N$2*$B14*$J22;0), corresponding to expression (10.13). Similarly to determine the equipment downtime according to formula (10.14), the formula =IF($B22<$N$2*$B14*$J22;$N$2*$B14*$J22-$B22;0) is entered in cell G22. The bottleneck load is determined, for example, in cell H22 by expression =$C$5*$I14+$C$6*$J14+$C$7*$K14, and the corresponding capacity—in cell I22 as =$N$10*$B14.



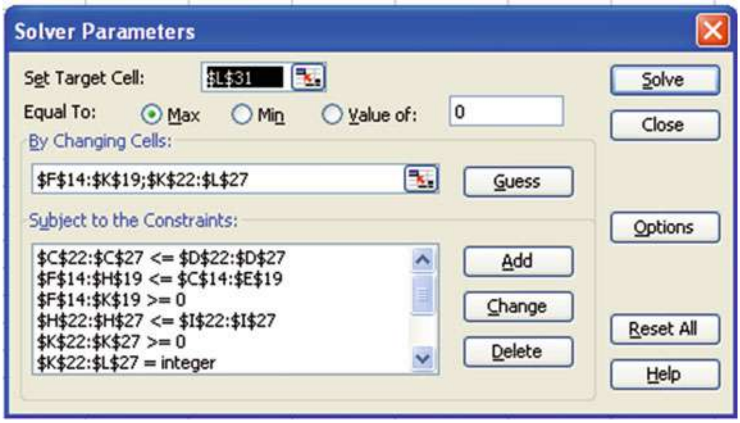
	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	Optimization of the plan for multiple goups with constraints								Scheduling standards					
2									Working time of employee per day					8
3		Product parameters							Total monthly salary					3500
4	Group	Total processing time per unit	Bottleneck processing	Cost of material	Cost of storage	Safety stock	Initial stock	Price per unit	One hour of overtime					40
5	1	12	1	50	5	90	100	600	Maximum of overtime per month					40
6	2	15	2	80	6	100	100	1000	Hiring of one employee					1500
7	3	6	0.5	30	3	130	100	400	Firing of one employee					3000
8									One hour of downtime					5
9									Initial number of employees					48
10									Capacity of bottlenack, hours/day					48
11									Constant expenses per month					200000
12			Demand by group			Sales			Production			Stock		
13	Month	Working Days	1	2	3	1	2	3	1	2	3	1	2	3
14	April	22	350	420	300	52	399	297	42	399	327	90	100	130
15	May	21	360	340	350	100	340	350	100	340	350	90	100	130
16	June	22	330	367	350	100	367	350	100	367	350	90	100	130
17	July	22	300	360	383	92	360	383	92	360	383	90	100	130
18	August	23	300	360	383	108	360	383	108	360	418	90	100	165
19	Septembe	23	330	413	400	71	410	392	71	410	357	90	100	130
20														
21	Month	Total workload, hours	Overtime, hours	Maximum of overtime	Cost of material	Cost of storage	Equipment downtime	Bottleneck load	Bottleneck capacity	Total quantity of employees	New employees	Fired employees		
22	April	8448	0	880	43812	1440	0	1003	1056	48	0	0		
23	May	8400	0	840	42700	1440	0	955	1008	50	2	0		
24	June	8800	0	880	44839	1440	0	1009	1056	50	0	0		
25	July	8800	0	880	44882	1440	0	1003	1056	50	0	0		
26	August	9200	0	920	46723	1546	0	1037	1104	50	0	0		
27	Septembe	9135	119	920	47017	1440	0	1069	1104	49	0	1		
28														
29		Total, costs, income and profit												
30		Salary	Overtime	Downtime	Hiring	Firing	Storage	Material	Costant expense	Total costs	Income	Profit		
31		1039500	119	0	3000	3000	8746	269972	1200000	2524337	3410472	886135		

Fig. 10.4 Optimized sales and operations plan

At the very bottom of the table there are the estimated values of costs, income, and profit. The income equals the first component of expression (10.1); the profit is calculated as the difference between income and costs.

Figure 10.5 shows the screen for data entry to seek the optimal solution. Constraints $C$22:$C$27<=$D$22:$D$27 and $H$22:$H$27<=$I$22:$I$27 reflect conditions (10.8) and (10.15), accordingly, and constraint $F$14:$H$19<=$C$14:$E$19—condition (10.16). The screen has also constraints $L$14:$L$19<=$F$5, $M$14:$M$19<=$F$6, $N$14:$N$19<=$F$7, corresponding to conditions (10.12).

In this example, the complete satisfaction of the demand is impossible due to capacity constraints in the bottleneck. Solution of the optimization problem demonstrates that with the given price and cost structure it is reasonable to ensure demand for group 3 primarily, then for group 2, and lastly for group 1. Since there is great



Set Target Cell: $L$31

Equal To: ● Max ○ Min ○ Value of: 0

By Changing Cells:

$F$14:$K$19;$K$22:$L$27

Subject to the Constraints:

```
$C$22:$C$27 <= $D$22:$D$27
$F$14:$H$19 <= $C$14:$E$19
$F$14:$K$19 >= 0
$H$22:$H$27 <= $I$22:$I$27
$K$22:$K$27 >= 0
$K$22:$L$27 = integer
```

Fig. 10.5 Screen for data entry for optimization

demand, which cannot be satisfied even, the production downtime appears to be insignificant.

It should be noted that the change in the number of working days per month can significantly affect the demand for personnel. For example, the small number of working days in May necessitates to hire two more employees. On the contrary, due to large number of working days in September even a small reduction in load leads to reasonable dismissal of one employee.

The problems of the given type are very diverse. They can use a variety of constraints: energy consumption, availability of certain specialists, working assets during certain periods, etc., depending on the actual situation and the developer's imagination. At the same time, for most of these problems the solution can be obtained using the spreadsheet quite similar to the above.

The data of the sales and operations plan should then be used in the development of the master production plan. The main problem that arises here is specifying a composition of each aggregate group of products. First of all, it is necessary to define the moment of disaggregation as the situation in the market and in the production changes rapidly.

It is usually believed (Vollmann et al. 2005) that disaggregation for a particular group should be performed ahead of sales and operations plan data by the value equalling to the duration of the production cycle of this product group. Determination of output rate for each item in the group should be based on the specifications of existing orders, as well as the forecasts for individual products.

10.3 Customized Reservation of Products

The receipt of proposals for a new order by the contractor requires the latter to perform a series of actions that form the so-called demand fulfilment process. The purpose of this process is to find out the possibility in principle of the order fulfilment, to calculate its cost and definition of the probable due date. In the simplest case, when the order is made on serial products with a known price, the process of accepting the order is reduced to determining the date of complete fulfilment of the order or the schedule of its consistent fulfilment.

The conventional approach to this problem is to check the availability of the product required by the customer at the storage and, then, if the product is available in the required quantity, it is reserved for the new order; otherwise, the deficiency is determined, for which the order must be put into production or placed to external supplier. However, the direct transfer of the order into production is not always possible, because production capability may be limited to a number of reasons—equipment load, materials shortages, lack of staff, etc. As a result, it becomes difficult to get a quick response to a query about the possible due date.

10.3.1 Business Process of Response to New Orders

Modern production management systems provide special business function for determination of the ability to fulfil the incoming order, which is called Available-to-Promise (ATP) function. The result of this function is calculation of the necessary amount of the product for the order and the date of its actual fulfilment.

Depending on the required due date of the new order, in the general case, there may be four different options of a planned situation:

– there is no sales and operations plan for the required due date yet;
– the sales and operations plan is made, but there is no master production plan;
– the master production plan is made, but its implementation is not started yet;
– the master production plan is under way.

In the first case, the feasibility of the order is determined mainly only by the enterprise development strategy and financial capabilities. Consideration of this option applies to business planning and is beyond the scope of this book.

Both the first and the second cases, when the order should be fulfilled in the period, for which the master plan has not yet been developed, are preferred for the manufacturer. Since the sales and operations plan is mainly based on demand forecasts, and covers quite a large horizon, it provides a great opportunity for varying the outputs and due dates of production. In this case, the check of possibility and feasibility of the order can be performed by calculating a new plan considering this order.

If the master plan for the period, which should contain the required due date of the new order, has already been drafted but its implementation has not yet begun, the possibility of the order can also be set by recalculating the plan. However, in this case, the calculation will be imposed with additional constraints—possible quantity of materials, tools, and other resources that were previously ordered for the previous version of the master plan, and cannot be increased.

Finally, if we have the last of the above options, obviously the order can be accepted only if the master plan has been designed as "make-to-stock" and there is still available product reserve in stock.

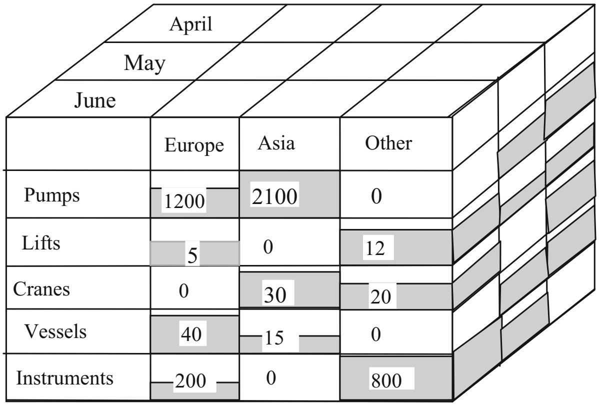
10.3.2 Arrangement of Orders

To analyse the composition, reservation, and fulfilment of orders it is convenient to make preliminary systematization of them by several different aspects: by products, by lead time, by regions, by customers, etc. Let us consider the case of orders grouping by the first three of these aspects. For this grouping makes sense to convert the original transactional orders database (Sect. 5.1.3) into the database of decision-making described in Sect. 5.1.4. The presence of this database makes it possible to use the analytical system operating on technology of multidimensional data analysis OLAP (Sect. 5.2.7).

Application of OLAP in grouping of orders by three specified aspects allows us to present an available set of orders in the form of three-dimensional cube (Fig. 10.6). Here is an example of the supplies analysis of a large engineering company within one-quarter at the top level of products grouping.

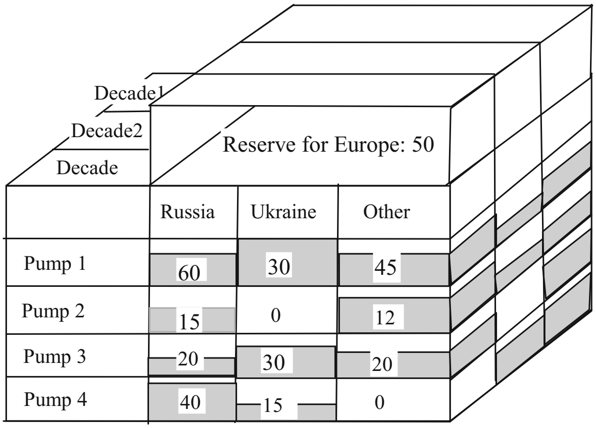
In each cell of OLAP cube the quantity of products of the corresponding group can be specified in physical terms, in cash or in hours. When you select a particular cell the list of the orders should open. The data of these orders are attributable to that cube cell.

In the example of Fig. 10.6 the entire area of the cell in section Products-Regions for a specific period reflects reserved production capabilities of each product by region in this period. In this case, the production load with orders can be displayed by the height of the cell, filled with grey. Similarly, in section Products-Periods, for



	Europe	Asia	Other
Pumps	1200	2100	0
Lifts	5	0	12
Cranes	0	30	20
Vessels	40	15	0
Instruments	200	0	800

Fig. 10.6 Grouping of reserves and orders by three aspects



	Russia	Ukraine	Other
Pump 1	60	30	45
Pump 2	15	0	12
Pump 3	20	30	20
Pump 4	40	15	0

Fig. 10.7 Structure of one cell from the cube in Fig. 10.6

example on the right side of the cube for a specific region you can see the production load in each period. The reserved quantity of each product (product group) in each cell of this cube must be set in accordance with the forecast as a rule.

Each cell of OLAP cube, in turn, can be represented as an OLAP cube with finer division into selected directions. For example, Fig. 10.7 shows an OLAP cube for a cell of group “Pumps” intended in June for Europe. In this example, the entire volume of the cell reserve in Fig. 10.6 set to 1500 pcs. is divided into reserves by decades. The reserve for each decade is divided into two parts: total reserve for Europe and separate reserves for Russia, Ukraine, and other European countries, and in this example, in the total reserve for Europe there is no division by type of product. Figure 10.7 shows that the value of the total reserve for the third decade amounts to 50 pcs.

The existing set of orders in the amount of 1200 pcs. (Fig. 10.6) is divided into the corresponding components shown in Fig. 10.7. The reserve in each cell of the OLAP cube is different. Suppose, for example, for Russia the reserve of pumps of type 1 in decade 3 is set to 90 pcs. and for Ukraine—30 pcs. In this case, as can be seen from Fig. 10.7, if there are orders for Russia in quantity of 60 pcs., the extent of its reserve usage is 2/3, and for Ukraine with the number of orders 30 pcs., the reserve is consumed completely.

Reservation of each element of the cell can also be based on the relevant forecast. However, as the forecast accuracy of a particular type of product is usually small, such reservation is to determine the probable range of production for each specific product.

Three possible rules are used in reserving. According to the first of these the reserve is distributed to customers in accordance with the predetermined hierarchy of customers (or regions, etc.). For example, Russian customers may have priority over customers from other countries.

The second rule of reservation is that the reserve is distributed among the customers in proportion to the volume of their orders. However, in the case of deficiency, the volume of orders may be exaggerated compared to the real needs and therefore it is necessary to focus not so much on the volume of new orders, but rather on the volume previous sales.

In the third case, the reservation volumes for each aspect are fixed, for example, as a percentage. The volume of the established reserve may depend on the profitability of a particular product. For example, although the high-quality products have limited sales opportunities, for their production a large reserve is often set deliberately. A classic example of such reservation is organization of a large number of business class seats in regular flights. A similar situation occurs in production of fashion clothes, expensive cars, etc.

10.3.3 Running ATP Process

When a new order is received we can proceed in different ways. In the first case being best for the customer, the answer about possibility of fulfilment and due dates is given to the customer immediately during ordering. This option is usually performed when purchasing consumer goods and if large stocks are available.

In cases where the feasibility of the order is not obvious, the contractor may take some time for the necessary checks. Since the production planning occurs periodically, the answer to the customer can be given only after development of a new production plan of appropriate level. For small orders, it is possible within the framework of the operational plan without significant changes to the master plan. In most cases, however, new orders must be included in the main production plan, which is usually prepared monthly. Therefore, the decision on the due date of the orders for serial products received during the month can be usually made by the end of this period.

If it is clear beforehand that the order can be processed, necessary to clarify the due date, the customer may be given prior consent to the condition that the final due date will be reported after the planning.

In any of the above options for processing an order the start of ATP process is the search for the ordered product in OLAP cubes of reserves and orders, described in the previous paragraph. The search is carried out using several of the following rules (Stadtler and Kilger 2008).

(A) Availability of reserve of the ordered product at a specified time and for a particular region of supply, for which the corresponding cell in the cube in Fig. 10.7 is found. If such reserve of the required quantity is available, the order can be processed and further search is not necessary.

(B) If the quantity of available reserve in the found cell cube is not enough to fulfil the order, you should check for the available reserve of this product and for the region in the cells with earlier deliveries. If there is such a reserve, it should be used to fulfil the received order.

(C) In the case where after step (B) the existing reserve is insufficient, go to search the ordered product in the required period upward the regions hierarchy, i.e. it is necessary to determine the availability of free reserve within the superior region considering both its special reserve and free reserves in other parts of the region.

(D) If the reserve mentioned in the previous step is used up, check the possibility of replacing the ordered product by other similar products with suitable characteristics. If such product exists, then steps (A)–(C) are repeated.

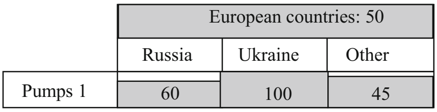
(E) Finally, if the possibilities of using the reserve of required product for delivery within required period have been exhausted, one has to go to searching such opportunities in the next period.

Consider the example of a new order received for Ukraine, which requires for supply 70 pumps of type 1 within the third week of June. By searching in the cube in Fig. 10.7, we see that for Ukraine in this period there are no reserves. We assume that the reserves of the previous periods are also used up. In this case, it is necessary to check the possibility of using reserves of the superior region according to step (C).

From Fig. 10.7, it follows that for the third decade of June a special reserve for Europe in amount of 50 pcs. is provided and this reserve can be used for any type of pump. However, since this amount is not enough, it is necessary to check the possibility of using the free reserve of pumps of type 1, previously allocated to other European countries. In this case, this possibility exists and the missing quantity of the product of 20 pcs. can be redirected to Ukraine either on account of Russia's limit or on account of other countries in the region. There are various solutions here, for example, you can take the missing half quantity from Russia's limit and the other half—on account of other countries.

The obtained result is shown in Fig. 10.8, which shows the upper part of the cube section in Fig. 10.7 for the third decade. After accepting the order, the volume of all orders for Ukraine has risen up to 100 pcs., and further reserve is not provided for Ukraine. At the same time, the total reserve for European countries is used up and the utilization of limit for Russia and other countries of Europe increased. Accordingly, the balance of grey part of the cells for Russia and other European countries in Fig. 10.8 and all the area from these cells is somewhat greater than in Fig. 10.7.

Fig. 10.8 Allocation of the reserve taking into account the new order



	European countries: 50		
	Russia	Ukraine	Other
Pumps 1	60	100	45

10.4 Agreement of Order Specifications with Customers

In search algorithm for available reserve described above in Sect. 10.3.3, step (D) is provided, according to which at some point of searching it is necessary to check possibility to supply the product similar to the ordered one. This task is quite complex, since it is impossible to predetermine which product the customer will consider as full substitute of his requirements. The only possibility in this case is consistent offering to the customer several options of other products supply that the supplier considers close enough to the original in order.

To carry out such an assessment it is advisable to assume that both the ordered product and its potential substitutes have a set of structural and functional features, which can be compared with each other. This set of criteria forms a criteria space with some metric. Section 4.4.1 above described one of the main methods using the space criteria metric—so-called shifted ideal method. In this case, the ideal object is the ordered product, and it naturally becomes a reference point for comparison of actual alternatives with the ideal variant.

In Sect. 4.5.3 there was the example of the shifted ideal method for assessing the quality of a lot size option. In principle, this method can be also used to evaluate different options of replacement of products in an order, but in this case there is a certain difference. The thing is that the options in the example of Sect. 4.5.3 differed only by quantitative values of three characteristic parameters. The products of similar purpose can differ (except for the difference in the quantitative characteristics of the parameters) by the presence or absence of any functional or structural properties.

This kind of property in some cases may be absolutely necessary for the user, or may be desirable, but not mandatory, and it is replaced by any other properties, parameters, or price. At the same time, the presence or absence of any structural unit influences one or more features of the product, i.e. modifies some of its properties. Therefore, any set of product components corresponds to the set of values of its characteristic parameters that can be assessed numerically. If such correlation of components and parameters exists, then it is possible to perform numerical comparison of different product options of similar design.

10.4.1 Problem Criteria and Their Evaluation

For an example of such a comparison, we will try to compare several models of cars of the same class in various designs. Suppose that three types of cars are delivered in parallel to one autocentre: Toyota Corolla (TC), Volkswagen Jetta (VJ), and Chevrolet Lacetti (CL) of different models and configurations, confining to cars of “sedan” type with four doors and petrol engines. Without claiming to generality of the solution, we will take into account six parameters—engine type, transmission type, climate control, set of interior devices, as well as the amount of fuel consumption and price.

Generally speaking, there are a very large number of modifications and configuration of these devices. To be able to analyse the effect of these devices on the functional properties, it is reasonable to aggregate the devices close in parameters and composition into groups. Table 10.5 shows an example of the selected parameters values for several models of the types mentioned above.

In addition to the data in Table 10.5 we will also take into account the established period of warranty, which is 36 months for Toyota Corolla and Chevrolet Lacetti, and 24 months for Volkswagen Jetta. The data on the characteristics of the cars, of course, depend on the vendor and may change over time, so they should be considered as purely indicative.

The type of the listed structural components greatly influences the performance of the vehicle. From the specialists' point of view, the quality of the car is defined by a very large set of parameters: ability to set the speed quickly, steering response, stability, smoothness, safety, etc. However, when supplying to the consumer, it makes sense to evaluate the quality of the product from the point of view of the buyer who is able to evaluate only a few of the qualities of this large list. We assume that these properties (criteria) are quality of the power train (engine and gearbox), comfort, fuel consumption, cost, and warranty.

To assess the quality (utility) in this case it makes sense to use one of so-called psychophysical scales desirability which establishes correspondence between linguistic evaluations (good, bad, etc.) and numerical intervals. The most famous scales are the Likert scale and the Harrington scale. The Likert scale for linguistic evaluations establishes quality scores ranging from 1 to 5. According to the Harrington scale, the qualitative assessment "very good" corresponds to the quantitative values in the range $1.0 \div 0.8$; "Good"—$0.8 \div 0.6$; "Fair"—$0.6 \div 0.4$; "Bad"—$0.4 \div 0.2$; "Very bad"—$0.2 \div 0.0$. The evaluations of criteria of the Harrington scale can be regarded as the utility of these criteria (Sect. 1.7).

We construct a scale of characteristics of the car models that matches the vendor's point of view (Table 10.6). We note here that the first two types of devices in Table 10.5 influence the quality of the power train and the type of device to maintain the climate and the interior equipment set influence the comfort.

We assume that the values of the criteria in Table 10.6 are equal to average values in the Harrington scale. Let us now compare the options of cars in Table 10.5 by the above criteria of consumer properties (Table 10.7) using the Harrington scale. For numeric criteria, the values in Table 10.7 can be determined by interpolation, and for linguistic criteria, the expert evaluation can be used.

10.4.2 Selection of Ordered Product Analogues

According to Table 10.7 we can calculate the distances of different options (alternatives) to the ideal object. Recall (Sect. 4.4.1) that the ideal object does not really exist, but embodies all the best possible qualities of different actually existing alternatives and sets a reference point when comparing them. To compare the diverse criteria it is necessary to go to their normalized values, using formula (4.32). For example, to calculate the normalized values of distance by fuel consumption criterion (criterion 3), first it is necessary to determine the range of possible utility values, which in this case ranges

Table 10.5 Car models and their main characteristics

Model	Line code	Engine capacity, L	Transmission	Climate equipment	Standard interior	Average consumption l/100 km	Standard price, $
TC Comfort	1	1.33	6-sp. mech.	Air conditioner	1	5.8	11,980
TC Comfort Plus	2	1.6	6-sp. mech.	Air conditioner	1	6.9	12,940
	3	1.6	4-sp. auto.	Air conditioner	1	7.0	13,580
TC Elegance	4	1.6	4-sp. auto.	Climate control	2	7.2	14,980
TC Prestige	5	1.6	4-sp. auto.	Climate control	3	7.2	15,940
VJ Trendline	6	1.6	5-sp. mech.	Air conditioner	1	7.6	12,820
	7	1.6	6-sp. auto.	Air conditioner	1	8.4	14,100
	8	1.9	5-sp. mech.	Air conditioner	1	5.9	15,160
VJ Comfortline	9	1.4TSI	6-sp. mech.	Climate control	2	6.6	15,760
	10	1.4TSI	DSG	Climate control	2	6.8	15,800
	11	1.6	5-sp. mech.	Climate control	2	7.6	12,820
	12	1.6	6-sp. auto.	Climate control	2	8.4	14,100
	13	2.0	6-sp. mech.	Climate control	2	8.3	15,160
	14	2.0	6-sp. auto.	Climate control	2	8.7	16,160
CL SE	15	1.4	5-sp. mech.	Air conditioner	1	7.1	9020
	16	1.6	5-sp. mech.	Air conditioner	1	7.3	9940
	17	1.6	4-sp. auto.	Air conditioner	1	8.1	10,680
CL SX	18	1.6	5-sp. mech.	Air conditioner	2	7.3	10,780
	19	1.6	4-sp. auto.	Air conditioner	2	8.1	11,520
	20	1.8	5-sp. mech.	Air conditioner	2	7.5	11,240
	21	1.8	4-sp. auto.	Air conditioner	2	9.1	12,480

Table 10.6 Car characteristics scaling

Assessment	Power train	Comfort	Fuel consumption l/100 km	Price, $	Warranty, months
Very good 1.0 ÷ 0.8	2 L; 6-sp. auto.	Climate control; full set of interior equipment	5	15,000	48
Good 0.8 ÷ 0.6	1.8 L; 5-sp. auto. or 1.4TSI; DSG	Air conditioner; full set of interior equipment	6	18,000	36
Fair 0.6 ÷ 0.4	1.6 L; 5-sp. auto.	Air conditioner; medium set of interior equipment	7	21,000	24
Bad 0.4 ÷ 0.2	1.6 L; 5-sp. mech.	Air conditioner; small set of interior equipment	8	24,000	18
Very bad 0.2 ÷ 0.0	1.4 L; 5-sp. mech.	Small set of interior equipment; no air conditioner	9	27,000	12

Table 10.7 Utility values for various models by quality criteria

Model	Line code	Power train	Comfort	Fuel consumption	Price	Warranty
TC Comfort	1	0.10	0.30	0.75	0.70	0.70
TC Comfort Plus	2	0.22	0.30	0.52	0.60	0.70
	3	0.48	0.30	0.50	0.53	0.70
TC Elegance	4	0.48	0.75	0.46	0.40	0.70
TC Prestige	5	0.48	0.92	0.46	0.30	0.70
VJ Trendline	6	0.40	0.25	0.38	0.58	0.50
	7	0.60	0.25	0.24	0.49	0.50
	8	0.85	0.25	0.72	0.37	0.50
VJ Comfortline	9	0.47	0.75	0.58	0.32	0.50
	10	0.50	0.75	0.52	0.31	0.50
	11	0.30	0.75	0.38	0.58	0.50
	12	0.53	0.75	0.22	0.49	0.50
	13	0.80	0.75	0.24	0.37	0.50
	14	0.90	0.75	0.13	0.28	0.50
CL SE	15	0.12	0.30	0.44	0.98	0.70
	16	0.30	0.30	0.42	0.91	0.70
	17	0.48	0.30	0.28	0.83	0.70
CL SX	18	0.30	0.50	0.42	0.82	0.70
	19	0.48	0.50	0.28	0.75	0.70
	20	0.62	0.50	0.40	0.77	0.70
	21	0.68	0.50	0.10	0.65	0.70

Table 10.8 Normalized values of distances to the ideal object

Model	Line code	Power train	Comfort	Fuel consumption	Price	Warranty	Distance vector
TC Comfort	1	1	0.925	0	0.412	0	1.42
TC Comfort Plus	2	0.85	0.925	0.354	0.559	0	1.42
	3	0.525	0.925	0.385	0.662	0	1.31
TC Elegance	4	0.525	0.254	0.446	0.853	0	1.13
TC Prestige	5	0.525	0	0.446	1	0	1.21
VJ Trendline	6	0.625	1	0.569	0.588	1	1.75
	7	0.375	1	0.785	0.721	1	1.81
	8	0.063	1	0.046	0.897	1	1.68
VJ Comfortline	9	0.538	0.254	0.262	0.971	1	1.54
	10	0.5	0.254	0.354	0.985	1	1.55
	11	0.75	0.254	0.569	0.588	1	1.52
	12	0.463	0.254	0.815	0.721	1	1.57
	13	0.125	0.254	0.785	0.897	1	1.58
	14	0	0.254	0.954	1.029	1	1.74
CL SE	15	0.975	0.925	0.477	0	0	1.43
	16	0.75	0.925	0.508	0.103	0	1.3
	17	0.525	0.925	0.723	0.221	0	1.31
CL SX	18	0.75	0.627	0.508	0.235	0	1.13
	19	0.525	0.627	0.723	0.338	0	1.14
	20	0.35	0.627	0.538	0.309	0	0.95
	21	0.275	0.627	1	0.485	0	1.31

from 0.1 to 0.75. Then, for any line in the Table 10.7, for example, line 15, according to formula (4.32), we obtain

$$a_{15,3} = \frac{\left(f_3^+ - f_{15,3}\right)}{\left(f_3^+ - f_3^-\right)} = \frac{0.75 - 0.44}{0.75 - 0.1} = 0.484.$$

According to the obtained normalized values of criteria a_{ij}, using formula (4.33) we can calculate the distance from the ideal object to each alternative. Table 10.8 shows the normalized values of distance for each criterion and of total distance (distance vector) to the ideal object. In this case, the distance vector for the i-th alternative is defined by formula (4.33) with the Euclidean metric $p = 2$, i.e. based on the dependence

$$L_i = \sqrt{\sum_{j=1}^{k} w_i a_{ji}^2}. \tag{10.17}$$

Since the values of weighting factors w_i are difficult to determine in advance, we assume that their values equal 1.

It is obvious that in this case the shortest distance to the ideal object is obtained for line 20 that corresponds to Chevrolet Lacetti SX with 1.8 L engine and 5-speed manual transmission. This model has the best balance of various quality criteria, which leads to a greater closeness to the ideal. However, it should be borne in mind that the result is an expression of the vendor's views, and it does not necessarily represent the consumer's opinion. In fact, the user can assume that the considered criteria have different importance for him/her, and so may come to quite different conclusions, and order other model.

Theoretically, it is possible to ask the customer to specify weight coefficients for each criterion in order to select similar models and, accordingly, recalculate the models distance vector to the ideal. However, in most cases, for the customer it is difficult to estimate the weighting factors values a priori, and even made estimations may vary over time for various reasons. Therefore, it makes sense not to look for a way to determine the model that is ideal for a customer but rely on his/her order and try to find the closest analogues, using ideas of the vendor. This is usually the case in practice.

Assume that the customer, for whatever his/her reasons, ordered Toyota Corolla Comfort Plus with the parameters given in line with code 3 in Table 10.5, but the autocentre is not able to supply the car within the required period. In order to offer the customer a suitable substitute, we can reconstruct Table 10.8 into the table normalized distances from the ordered object. To do this, we calculate the coefficients of the new table using the dependence

$$a_{ji}^{'} = a_{ji} - a_{3i}, \tag{10.18}$$

we recalculate the distance vectors replacing a_{ji} by $a_{ji}^{'}$ in formula (10.17) and sort the lines ascending for the distance from the order (Table 10.9).

We understand the relative distance in Table 10.9 as ratio $L_j^{'}/L_3$, i.e. the ratio of the distance of the j-th line to the order to the order distance (the third row in Table 10.8) from the ideal, that is, to $L_3 = 1.31$. It is obvious that some models that are sufficiently close to the order, i.e. to line 3 should be presented at the customer's discretion.

The most straightforward is to suggest the customers to consider other models in the sequence described in Table 10.9. But it is unlikely that this suggestion is justified because, as stated above, the priorities of the customer may be quite different from those of the vendor. It's much better to show several models close to the order to the customer, so he/she would have the opportunity to review and select.

Certainly, this raises the question about the number of models in the proposed set. As mentioned above in Sect. 4.4.2, an ordinary person, depending on his/her abilities and objective conditions, can simultaneously analyse 7 ± 2 options. It is, therefore, reasonable during the first approach to suggest the customer to analyse five options, which are the closest to the order from the vendor's point of view. However, if at least some of these options are at large "distance" from the order, the proposal of their analysis will not be understood correctly.

Table 10.9 Normalized values of distances to order

Model	Line code	Power train	Comfort	Fuel consumption	Price	Warranty	Distance to order	Relative distance
TC Comfort Plus	3	0	0	0	0	0	0	0
	2	0.325	0	0.031	0.103	0	0.34	0.261
CL SX	20	0.175	0.299	0.154	0.353	0	0.52	0.395
	19	0	0.299	0.338	0.324	0	0.56	0.424
CL SE	17	0	0	0.338	0.441	0	0.56	0.424
CL SX	18	0.225	0.299	0.123	0.426	0	0.58	0.443
CL SE	16	0.225	0	0.123	0.559	0	0.61	0.469
TC Comfort	1	0.475	0	0.385	0.25	0	0.66	0.504
TC Elegance	4	0	0.672	0.062	0.191	0	0.7	0.535
CL SX	21	0.25	0.299	0.615	0.176	0	0.75	0.572
CL SE	15	0.45	0	0.092	0.662	0	0.81	0.615
TC Prestige	5	0	0.925	0.062	0.338	0	0.99	0.753
VJ Trendline	6	0.1	0.075	0.185	0.074	1	1.03	0.784
	7	0.15	0.075	0.4	0.059	1	1.09	0.833
	8	0.4625	0.075	0.338	0.235	1	1.18	0.899
VJ Comfortline	11	0.225	0.672	0.185	0.074	1	1.24	0.947
	9	0.012	0.672	0.123	0.309	1	1.25	0.954
	10	0.025	0.672	0.031	0.324	1	1.25	0.952
	12	0.0625	0.672	0.431	0.059	1	1.28	0.978
	13	0.4	0.672	0.4	0.235	1	1.35	1.031
	14	0.525	0.672	0.569	0.368	1	1.48	1.128

In the paper (Mauergauz 1989), the concept of “search radius” was introduced, which determines the selection from the database of the objects in the vicinity of the ordered object. The search radius means the hypersphere radius centred at the point of order; the distance vectors of the objects, selected for consideration, are inside the hypersphere. When negotiating with the customer, the vendor has to determine the optimal search radius for this customer with the use of own experience and negotiating skills.

The search radius value can be set as the relative distance in Table 10.9 multiplied by 100 %. In this case, these relative distances are quite large, so that even when the search radius equals 40 %, the area of selection will include only two models (lines 3 and 20). In those cases where the search area of the predetermined seraph radius includes many models, one can initially offer five of them for consideration, and then increase this number to nine.

In this example, weighting factors w_i for all i-th criteria were equated to unity. This led to the fact that all models of Volkswagen Jetta were actually withdrawn from consideration because their warranty period (24 months) is much less than that of the models of other two brands (36 months). If, for example, for criterion “warranty period” ($i = 5$) weighting factor $w_5 = 0.5$ is set, then the utility of review of Volkswagen Jetta models will be significantly increased. In particular, VJ Trendline model with line code 6 and relative distance 0.568 will become the closest to the order, i.e. it will be in the ninth place and may be presented to the customer for consideration.

The experience has shown (Lambert 1994) that the provision the opportunity for the customer to analyse options close to the order dramatically increases the level of service. For example, when the initial service level is 70 %, i.e. only seven orders out of ten are fulfilled, after the customer’s considering the options of the first selection, the service level increased to about 90 %, and after the second selection—to nearly 97 %.

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Lot Sizing 11

11.1 Classification of Lot-Sizing Problems

The chapters above widely use the notion of a batch (lot) as a set of same-type discrete products or as a certain bulk of some material. This interpretation is based on some definitions of the batch adopted in MRP II systems. These definitions are described below:

- Quantity planned for production under a plan of sales and operations, master production schedule, or material requirements schedule;
- Quantity specified in a shop floor schedule and to be processed on a single machine without any setups (production batch);
- Quantity specified in a shop floor schedule for joint shipment (shipment batch);
- Order size in the supply chain;
- Set of identical products subject to a special batch-based inventory control;
- Quantity of products corresponding to the same quantity specified in a respective order (lot for lot).

11.1.1 Lot Properties and Main Problems

As mentioned in Sachko (2008), lot sizes greatly depend on three factors, i.e. economic, process, and social. The classic supply management model (Economic Order Quantity—EOQ) described in Sect. 2.1.1 demonstrates a purely economic method of lot sizing. Purely process approach to lot sizing provides for the use of formula (3.27) described in Sect. 3.4. And finally, social studies (Sachko 2008) demonstrate that a worker needs to produce 100 pieces to fully master to the process. However, once 1000 identical pieces have been processed, a worker will start experience ever-rising fatigue due to monotony of labour.

Sizes and, consequently, quantity of product lots that are in production constantly change as far as they are being processed. As a rule, the so-called release lot

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(start of production process) has the greatest size while the "output" lot containing finished products will be the smallest. Technically, the production process may feature intermediate lot sizes; however, this makes production management quite challenging.

In fact, the production planning process from the master plan to daily/shift targets is to determine start and end time points for different production activities/jobs done to product lots. The size of each lot can be specified either at a higher level of planning before actual production planning starts or directly at the current stage of shop floor scheduling. Here, we will consider lot-sizing process as an activity preceding actual planning. The other approach that is more challenging will be described below in Chaps. 13–15.

Production lots may have some peculiar features that affect the planning process (Tanaev et al. 1998). First of all, lots are described by the method of processing termination. In the first case, processing is finished individually where each discrete product leaves processing site right after its termination. Here, the transfer lot size will be 1. In the other extreme case, the lot leaves the processing site on the whole; so, the transfer lot is equal to the production lot.

Naturally, there are intermediate cases where a production place splits into some transfer lots under a processing job. This is the case, for example, for the most processing time job where the startup lot is divided into a number of output lots. Another example of splitting the initial lot into sub-lots is the so-called parallel-successive processing (Sachko 2008).

Another peculiar property of a lot is the processing procedure for individual lot items. Technically, there are two options as follows: (a) each lot item is processed individually one after another and (b) the whole lot is processed at a time (furnaces, baths, etc.).

And finally, it is important whether there is a maximum lot size limit. Such limit is often caused by a limited size of a pallet, container, buffer, transitional vessel, etc.

The optimal lot sizing can be based on either single-parameter or multiple-parameter approach. The above EOQ model is a classic example of the single-parameter approach. Section 4.2.2 describes lot sizing using two parameters, and three-parameter approach is described in Sect. 4.4.2. Anyway, most lot-sizing problems use the single-parameter optimization approach.

The existing classification of single-parameter optimization problems is described, for example, in Jans and Degraeve (2008). The lot-sizing problem itself is generally reduced on one of the following options:

- One product and no capacity limits:
  - constant demand and constant price;
  - constant demand and variable price that depends on the lot size;
  - variable demand.
- Multiple products manufactured on a single machine; capacity limits; and finite planning horizon including multiple time buckets:
  - multiple different products are manufactured during each time bucket (large);

  - only one type of products is manufactured during a small time bucket (regardless of the machine load);
  - only one type of product is manufactured during each (small) time bucket with the machine load being either full or zero;
  - either one single type or two different types of products can be manufactured during each (small) time bucket.
- The lot of finished products (or their components) is assembled from multiple products of different types.
- Multiple products of different types are ordered simultaneously.

11.1.2 Lot-Sizing Problems with No Capacity Limits

The optimality parameter used in single-product problems with a constant demand and no capacity limits is the cost associated with lot processing and product inventory holding during an indefinite time period. If the product price depends on the lot size, the classic EOQ model gets more complicated as the cost formula needs to be changed. In this case, the formula (2.2) of cost during a demand period is as follows:

$$c = Dc_{pj} + \frac{Q}{2}c_h + \frac{D}{Q}c_o, \tag{11.1}$$

where (as before) c_o represents the costs associated with order performance, c_h represents the costs associated with holding of a product item (in respective measuring units) during a certain time unit, D is the quantity of product consumed during a certain time unit, and Q is the lot size. The product price C_{pj} is different at each j-interval of lot size.

All other cases listed in Sect. 11.1.1 describe dynamic lot-sizing problems; they are characterized by a variable demand and finite planning horizon. The optimality parameter used in dynamic lot-sizing problems with a single product, variable demand, and no capacity limits is the minimum cost associated with pre-production (setup), production, and product inventory holding during a limited time period h:

$$c = \sum_{t=1}^{h} (c_m X_t + c_o \delta_t + c_h Z_t), \tag{11.2}$$

where c_m represents the production cost per product unit, c_o represents the cost of regular setup required to start operations during the established period, c_h represents the cost of holding per product unit during a time period, X_t stands for the product quantity manufactured during period t, δ_t is a binary variable representing setup or no setup during period t, and Z_t is the product quantity in storage by the end of period t.

The stock quantity at the end of period t is as follows:

$$Z_t = Z_{t-1} + X_t - D_t, \tag{11.3}$$

where D_t is the forecasted demand during the planned period.

Product quantity during each period t that features production setups should be less than the cumulative (total) demand for the whole planning period, i.e.:

$$X_t \leq \sum_{t=1}^{h} D_t \times \delta_t \quad \text{at } \delta_t \in \{0.1\}. \tag{11.4}$$

Naturally, X_t and Z_t must be non-negative.

11.1.3 Lot-Sizing Problems with Limited Capacities and Large Planning Periods

Here, we will describe Capacitated Multi-Item Lot-Sizing Problems (CLSP). Under such problems, the time period in question till planning horizon h includes multiple large time buckets. The multi-item production is planned for each of such time buckets. Let us assume that during each time bucket t the machine capacity is P_t hours, and the part of machine capacity equalling to p_i hours is used per measuring unit of i-product item.

Similarly to the previous problem, the optimality parameter is the minimum cost for n manufactured product items.

$$c = \sum_{i=1}^{n} \sum_{t=1}^{h} (c_{mi} X_{it} + c_{oi} \delta_{it} + c_{hi} Z_{it}), \tag{11.5}$$

where all variables and coefficients described above in Sect. 11.1.2 depend on the type of product item i. The stock limit (formula 11.2) gets transformed into a set of inequalities as follows:

$$Z_{it} = Z_{i,t-1} + X_{it} - D_{it} \quad \text{at } 1 \leq i \leq n,\ 1 \leq t \leq h, \tag{11.6}$$

and the product quantity limit for a period (formula 11.3) transforms into the system of inequalities as follows:

$$X_{it} \leq M \times \delta_{it} \quad \text{at } \delta_{it} \in \{0, 1\}. \tag{11.7}$$

In expression (11.7) “the big number M” is generally determined as follows:

$$M = \min\left(P_t/p_i, \sum_{t=1}^{h} D_{it}\right) \quad \text{at } 1 \le i \le n. \tag{11.8}$$

In addition to constraints (11.6) and (11.7) that are similar to constraints listed in Sect. 11.1.2, we get another inequality describing capacity limits:

$$\sum_{i=1}^{n} p_i X_{it} \le P_t. \tag{11.9}$$

11.1.4 Lot-Sizing Problems with Limited Capacities and Small Planning Periods

Here, we will describe Continuous Setup Lot-Sizing Problems (CSLP). Under such problems, time buckets that compose the whole planning period until the planning horizon are small; therefore, each time bucket features only one setup to adjust machine for another product item (such setup takes place at the beginning of the time bucket). So, we have two types of setups, i.e. regular setup that takes place at the job start during each time bucket t with the associated cost of regular production setup c_o and additional setup required to adjust production from one product item to another with the associated cost c_s. Here, the optimality parameter is the minimum cost

$$c = \sum_{i=1}^{n} \sum_{t=1}^{h} (c_{mi} X_{it} + c_{oi}\delta_{it} + c_{si}\gamma_{it} + c_{hi} Z_{it}); \tag{11.10}$$

unlike expression (11.5), the above expression has an additional component, i.e. $c_{si}\gamma_{it}$. Similar to δ_{it}, γ_{it} determines the fact of machine setup from one product item to another and can take on either 0 or 1.

Stock constraints (11.5) remain unchanged:

$$Z_{it} = Z_{i,t-1} + X_{it} - D_{it} \quad \text{at } 1 \le i \le n,\ 1 \le t \le h, \tag{11.11}$$

while the capacity limit (11.9) is replaced by a set of constraints as follows:

$$p_i X_{it} \le P_t \quad \text{at } 1 \le i \le n,\ 1 \le t \le h. \tag{11.12}$$

As under these problems, only one product item can be manufactured during one time bucket t, we need to introduce another constraint as follows:

$$\sum_{i=1}^{n} \delta_{it} \leq 1 \quad \text{at } 1 \leq t \leq h. \tag{11.13}$$

Costs associated production setup from one product item to another during a current time bucket appear if the previous time bucket featured setup for another product item, i.e. $\delta_{it} \neq \delta_{i,t-1}$. This condition can be expressed as inequalities as follows:

$$\gamma_{it} \geq \delta_{it} - \delta_{i,t-1} \quad \text{at } 1 \leq i \leq n,\ 1 \leq t \leq h. \tag{11.14}$$

The problem in question may have an option, i.e. Discrete Lot Sizing and Scheduling Problem (DLSP) where a time bucket *t* can be either fully loaded or fully vacant. In this case, all relations typical of CSPL problem are preserved except for constraint (11.12) that transforms into the equality as follows:

$$p_i X_{it} = P_t \times \delta_{it} \quad \text{at } 1 \leq i \leq n,\ 1 \leq t \leq h. \tag{11.15}$$

The CSLP model has a substantial drawback as it requires setup at the beginning of a time bucket. In order to ensure the possibility of partial lot transfer into an adjacent time bucket, we can use the so-called proportional lot model (Proportional Lot Sizing and Scheduling Problem, PLSP) which allows processing of two lots of different product items during one time bucket *t*.

Technically, DLSP and PLSP models are scheduling models rather then lot-sizing problems. The same applied to the Multi-Stage Lot-Sizing Problems (MSLSP). That's why such problems are described below in chapters that deal with production schedules.

Dynamic lot-sizing problem models described above are quite general. Technically, they can yield solutions for a wide range of practical tasks; however, these models become more and more complicated as far as the number of product items and discrete time buckets increases. For instance, CSLP model for three product items and five time buckets (5 work days) will include 15 equations (11.11), 15 inequalities (11.12), 5 inequalities (11.13), and 15 conditions (11.14), i.e. being 50 constraints in total. If we have 5 product items and 10 time buckets, we'll get 160 constraints.

11.2 Constant Demand Lot-Sizing Problems

Generally, various lot-sizing problems with constant demand use the fundamental EOQ model described in Sect. 2.1.1 and consider different variations of this model. The resulting lot size is usually expressed as (Eq. 2.4) multiplied by the so-called correctional factor χ introduced above in Sect. 4.2.1.

11.2.1 Models with Gradual Inventory Replenishment

The EOQ model assumes that a new product lot comes in full to a storage facility at a certain time point (Fig. 2.1). However, if a product is manufactured directly at an enterprise rather that comes from third parties, the inventory of such product at the enterprise (not necessarily at a storage level only) increases gradually (as the product is being manufactured) till its maximum value $\dot{S}$ and then decreases (Fig. 11.1). After, the production/consumption cycle with length T repeats.

The cycle length is made of two components t_1 and t_2:

$$t_1 = \frac{Q}{P} \tag{11.16}$$

and

$$t_2 = T - t_1 = \frac{Q}{D} - \frac{Q}{P} = Q\frac{P-D}{PD}, \tag{11.17}$$

where Q is the lot size, P is the production output (rate of production), and D is the rate of consumption. The greatest value of inventory is determined as the cross point where two straight lines $Z = Q - Dt$ and $Z = (P - D)t$; this cross point occurs at $t = t_1$. Using expression (11.16), we get:

$$\dot{S} = Q\left(1 - \frac{D}{P}\right). \tag{11.18}$$

To calculate the average size of inventory, we need to sum up areas of two triangles in Fig. 11.1 and divide the sum by the cycle length. Using expressions (11.16)–(11.18), we get:

$$\overline{Z} = \frac{1}{T}\left(\frac{\dot{S}t_1}{2} + \frac{\dot{S}t_2}{2}\right) = \frac{Q}{2}\left(1 - \frac{D}{P}\right). \tag{11.19}$$

Now, in order to determine the optimal lot size, let us use the method described for the EOQ formula: we need to calculate the cost of lot preparations (lot setup)

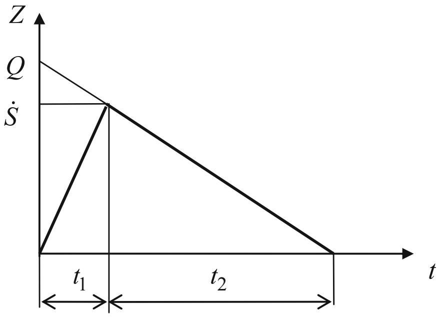


Fig. 11.1 Inventory changes in case of parallel production and consumption

and the cost of holding and then find the minimum value of this expression. Here, the only difference from the solution described in Sect. 2.1.1 is that in the first component of expression (2.2) that describes the holding cost, the average product quantity during the holding period is equal to the value calculated from formula (11.19) (instead of $Q/2$). By inserting this expression into (2.2), we obtain:

$$c = \frac{Q}{2}\left(1 - \frac{D}{P}\right)c_h + \frac{D}{Q}c_o, \tag{11.20}$$

So, the optimal lot formula will be as follows:

$$Q^* = \sqrt{\frac{2c_oD}{c_h}}\sqrt{\frac{P}{P-D}}. \tag{11.21}$$

It is clear that in this example (Economic Production Quantity, EPQ) the correctional factor will be:

$$\chi = \sqrt{\frac{P}{P-D}}. \tag{11.22}$$

Now, let us assume for the example described in Sect. 2.1.1 that the demand rate D is 300 items per month, the order setup cost c_o is \$2000, the holding cost per product unit c_h is \$200 per month, and the production output P is 2000 product units per month. Then, the correctional factor will be $\chi = \sqrt{\frac{2000}{2000-300}} = 1.08$ and the optimal lot size Q^* will increase from 77.5 to 84 product units.

The publication (Sterligova 2006) lays out expressions similar to (formula 11.21) for problems and tasks similar to our example. In all cases, optimal lot expressions are similar and differ only by their correctional factor χ.

Precise calculations of an optimal lot size for a product item that is processed successively on two machines (Fig. 11.2) exist and lead to the expression that is similar to the EPQ. Let us assume that raw stock, intermediate, and finished products are kept at storages 1, 2, and 3; the holding cost per unit is c_{h1}, c_{h2}, c_{h3} dollars per month, respectively.

If $P_1 \geq P_2$, the optimal lot size will be as follows (Axseter 2006):

$$Q^* = \sqrt{\frac{2C_oD}{DC_{h1}/P_1 + DC_{h2}(1/P_2 - 1/P_1) + C_{h3}(1 + D/P_2)}}, \tag{11.23}$$

where c_o is the cumulative cost of order setup for all three storages.

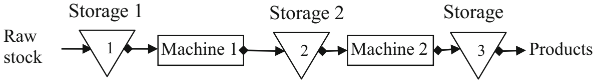


Fig. 11.2 Chain of successive processing of raw stock into products

11.2.2 Model Applicable to the Machinery Industry If No Cost Information Is Available

As in the machinery industry pieces and assembly units are usually quite labour and materials-intensive, their optimal lot sizes are determined by their processing time and the cost of production in progress. Such problem is described in detail in Sect. 4.2.2 where we derived expression (4.33) for the optimal lot size:

$$Q_i^* = A\sqrt{\frac{s_i D_i}{c_i}}\text{measuring units},$$

where s_i stands for the setup time in hours, c_i is the cost of measuring unit, D_i stands for the demand rate in measuring units per month, and A is an undetermined dimensional constant.

In practice, the use of formula (4.22) is quite problematic as it includes the cost of each product item which is not always known at the time of planning. Based on empirical evidence contained in (Guidelines of the USSR Research Institute for Machinery, 1970), the publication (Mauergauz 2007) proposes the following formula for the cost of i-item of machinery production:

$$c_i = a_1\left(K_{pi} n_i M_i^{2/3} + b_i M_i\right), \tag{11.24}$$

where M_i stands for the unit weight in kg, n_i represents the number of process jobs done to such unit, K_{pi} is the production complexity factor, b_i is the coefficient that depends on the type of materials used for production, and a_1 is a certain dimensional constant.

The first component of expression (11.24) represents the treatment cost while the other component stands for the cost of materials. Treatment cost depends on the weight of a unit to the power 2/3 as, on average, the surface area of a unit depends on the weight to this power and processing time substantially depends exactly on such surface area.

Besides, in this publication, (Mauergauz 2007) also assumes that if precise setup time is not known, we can express it as follows:

$$s_i = a_2 K_{si} n_i, \tag{11.25}$$

where K_{si} stands for the setup complexity factor and a_2 is a constant.

By inserting formulas (11.24) and (11.25) into Eq. (4.22) and taking account undetermined constant A that can include a_1 and a_2, we get:

$$Q_i^* = A\sqrt{\frac{K_{si} n_i D_i}{K_{pi} n_i M_i^{2/3} + b_i M}}. \tag{11.26}$$

Table 11.1 Complexity factors

Parameter	Production/setup complexity grades			
	1	2	3	4
Production complexity factor K_{pi}	0.3	1	2	5
Setup complexity factor K_{si}	0.5	1	3	6

Production and setup complexity factors are comparable to the complexity grades of pieces and assembly units and to the complexity grades of processing setups. For example, we can assume complexity grade 2 (refer to Table 11.1) as regular and, consequently, assign complexity factor 1 for such grade. For other grades, complexity factors can be identified empirically. For example, let us assign complexity factor $K_{pi} = 0.3$ for low complexity pieces (complexity grade 1), $K_{pi} = 2$ for medium complexity pieces (complexity grade 3), and $K_{pi} = 5$ for higher complexity pieces. Setup complexity factors K_{si} are assigned similarly.

Please note that b_i that takes account of the cost of materials rises with the increase in the cost of materials. According to Mauergauz (2007) the basic coefficient is $b_i = 5$. It corresponds to ferrous metals. According to the calculation model described in Sect. 4.2.2, A does not depend on the type of pieces; its value can be determined by comparing calculations per formula (11.26) and available empirical data on reasonable lot sizes. For pieces, the basic value can be assumed as $A = 10$; for assembly units, $b_i = 0$ and $A = 30$.

Now, let us consider the example problem to determine the optimal lot size for low-alloyed steel using formula (11.26). The weight $M = 1$ kg; the demand rate = 300 pieces per month. Let us assume that processing includes 12 jobs with the average complexity grade of 2 ÷ 3 grades, i.e. the production complexity factor will be $K_{pi} \approx 1.5$. Setup complexity is assumed as regular, i.e. $K_{si} = 1$. Here, for $A = 10$ we'll obtain the following lot size:

$$Q_i^* = 10\sqrt{\frac{1 \times 12 \times 300}{1,5 \times 12 \times 1^{2/3} + 5 \times 1}} = 125\,\text{pieces}.$$

The resulting value is probably overestimated and needs to be adjusted by adjusting A-constant. For example, let us assume that empirical evidence shows that the optimal lot size for such pieces is 30 pieces. Then, A-constant will be approximate 2.5.

Now then, A-constant is known and we need to determine the optimal lot size for aluminium pieces 1 kg each given that all other processing parameters are the same. Given that aluminium alloy is approximately fivefold more expansive than low-alloyed steel, let us assume $b = 25$. As a result, we get the optimal lot size as follows:

$$Q_i^* = 2.5\sqrt{\frac{1 \times 12 \times 300}{1,5 \times 12 \times 1^{2/3} + 25 \times 1}} = 22\,\text{pieces}.$$

11.2.3 Three-Parameter Models for Machinery Industry

Now let us consider the problem which, in addition to setup time and cost of production in progress described in Sect. 4.2.2, uses the parameter of time required to process a lot. The solution to a similar multi-parameter problem is described above in Sect. 4.4.2 where we considered different options based on a two-parameter lot-sizing problem (refer to Sect. 4.2.2) and a minimum waiting time problem (refer to Sect. 3.4.1). Here, we will attempt to determine a uniform relation between the optimal lot size and three parameters. For this purpose, let us use the relation (4.22) in the following form (4.13):

$$Q_i^* = \hat{\chi}_i\sqrt{\frac{s_i D_i}{c_i}},$$

where $\hat{\chi}_i$ stands for a correctional factor. The cost of production in progress c_i represents direct production costs that are equal to the sum of treatment costs and materials costs. That's why the formula Q_i^* can be expressed as follows:

$$Q_i^* = \hat{\chi}_i\sqrt{\frac{s_i D_i}{\tau_i + b_i M}}, \tag{11.27}$$

where τ_i is the total treatment cost of processing per piece of i-type, b_i stands for the ratio between the price per 1 kg of materials to the cost of 1 labour hour, and M is the weight of a piece.

Now, let us determine the correctional factor in formula (11.27) using available empirical data on the relations between optimal lot sizes and machinery loads. For this purpose, let us use Table 11.2 that has been widely used in the USSR machinery industry.

As we can see, we can use Table 11.2 to determine the time period (cycle) of starting the processing of the next lot based on two parameters as follows: machine load and setup time for a machine. The parameters are determined based on the main operation that actually depends on the design of a piece, i.e. a turning job for rotary body pieces, a milling job for box-type piece, etc.

According to Table 11.2, standard startup periods are calculated by dividing or multiplying the number m of work days in a month by an integer. This ensures an even load during a month or, in case of less frequent startups, during a quarter or 6-month period. Table 11.2 is developed for those pieces whose cost of materials is substantially lower than the cost of processing; therefore, the cost of materials has no direct effect on lot startup standards.

Table 11.2 Startup/output standards for lot sizing with the account of setups (in month fractions)

Average monthly load on a machine performing main operations (in shifts)	Setup time for a main operation (in hours)				
	0.25 ÷ 0.5	0.5 ÷ 1	1 ÷ 2	2 ÷ 4	Over 4
Over 20	m/16	m/8	m/8	m/4	m/2
15 ÷ 20	m/8	m/8	m/4	m/4	m/2
10 ÷ 15	m/8	m/4	m/4	m/2	m
5 ÷ 10	m/4	m/4	m/2	m	m
2.5 ÷ 5	m/4	m/2	m	m	3m
1 ÷ 2.5	m/2	m	m	3m	3m
Under 1	m	m	3m	3m	6m

As an example, let us calculate the lot size for a steel rod-type piece with the monthly target of 300 pieces. This piece has a main turning operation; processing time of a turning job is $\tau_0 = 14\,\text{min}$; setup time is $s_0 = 1.5\,\text{h}$. The respective machine load at an 8-h shift will be:

$$\frac{300 \times 14}{8 \times 60} = 8.8\,\text{shifts}.$$

According to Table 11.2 the startup period will be $m/2$. If a month includes 22 work days, the startup period will be 11 days. Accordingly, the lot size will be:

$$Q = \frac{300 \times 11}{22} = 150\,\text{pieces}.$$

As we can see from Table 11.2, a startup period rapidly increases as far as the scale of production (machine load) decreases and as far as setup time grows. This means that, in general, the behaviour of lot size changes is close to the relation described in expression (11.27). This allows determining a correctional factor with which formula (11.27) yields lot sizes close to standard values according to Table 11.2, i.e.:

$$\hat{\chi} = 6\frac{s_0^{0.1}}{D^{0.2}\tau_0^{0.5}}, \tag{11.28}$$

where s_0 stands for the setup time for a main operation (in hours) and τ_0 represents the processing time of a main operation (in hours).

As we can see from formula (11.28), the correctional factor provides a certain adjustment to the relation between the lot size and basic calculation parameters in formula (11.27). Such adjustment is quite small for setup time s_i and monthly demand rate D_i; however, it is rather significant for processing time τ_i. Now, let us determine the lot size for a rod-type piece using the above example. The total processing time is assumed $\tau_i = 1.2$ h, the cumulative setup time is 8 h, the ration of the cost of 1 kg of material to the cost of 1 labour hour is $b = 0.5$, and the weight of material is $M = 0.5$ kg. Using expressions (11.27) and (11.28), we get the following lot size:

$$Q_i^* = \hat{\chi}_i \sqrt{\frac{s_i D_i}{\tau_i + b_i M}} = 6\frac{1.5^{0.1}}{300^{0,2}\times(14/60)^{0.5}}\sqrt{\frac{8\times300}{1.2+0.5\times0.5}} = 6\times0.67\times40.7 \approx 160\,\text{pieces.}$$

Naturally, the correctional factor expression (11.28) is approximate. For each individual production, we need to adjust the constant multiplier of the correctional factor as well as indices of power for its parameters. Besides, please note that under existing production environment lot sizes tend to decrease; so, startup periods according to Table 11.2 can be overestimated.

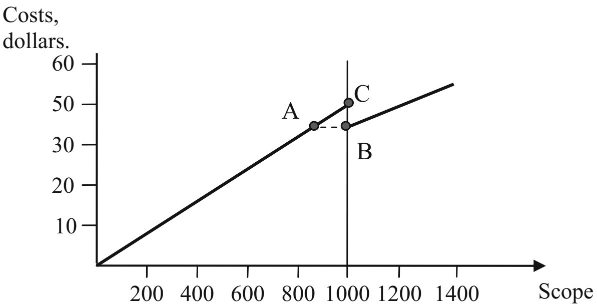
11.2.4 Lot Sizing at Discounted Prices

The typical example of a discount offered on volume purchases is a discounted price per one measuring unit of product. For example, the unit price of \$50 applies to purchases up to 1000 pieces and the unit price is discounted to \$40 for higher purchases. Note that two options are possible: (a) the discounted price applies only to purchased quantity that exceeds the threshold value and (b) the discounted price applies to the whole purchased quantity. Let us consider the second case. Refer to Fig. 11.3.

The cost chart shows that purchases of 800–1000 units (from A to C) are clearly unreasonable as the cost of such purchase exceeds that of the purchase of 1000 units. So, the chart has a “flat” spot between A and B.

Let us assume that the optimal lot size is calculated using the EOQ formula (2.4). If the resulting lot size is higher than the threshold price value, it should be confirmed for production. If the lot size resulting from formula (2.4) is between A and B points, it is clear that we should set the lot size at the threshold level.

Fig. 11.3 Relations between the costs and purchased quantity



Finally, if the optimal lot size is lower than the scope of purchase at A point, we should verify if the order cost is lower than the order cost at B point.

Let us take an example where the demand D is 2000 units per month, the cost of one order c_o is \$2000, and the holding cost c_h is \$100 per month. Using formula (2.4), we get:

$$Q^* = \sqrt{\frac{2c_oD}{c_h}} = \sqrt{\frac{2 \times 2000 \times 2000}{100}} \approx 285.$$

According to expression (11.1) the total cost of orders during the period in question will be as follows:

$$c = Dc_{pj} + \frac{Q}{2}c_h + \frac{D}{Q}c_o,$$

where $c_{p1} = \$50$ per measuring unit and $c_{p2} = \$40$ per measuring unit. Let us compare two options, i.e. an order with an optimal lot size and an order with a threshold lot size.

In the first case, the cost of orders covering monthly demand will be:

$$c_1 = 2000 \times 50 + \frac{285}{2}100 + \frac{2000}{285}2000 = (100 + 14 + 14) \times 10^3 = \$128,000$$

In the second case:

$$c_2 = 2000 \times 40 + \frac{1000}{2}100 + \frac{2000}{1000}2000 = (80 + 50 + 4) \times 10^3 = \$134,000.$$

As we can see, in the latter case holding costs exceed savings that can be gained if the lot size is increased to the threshold value when a discount is offered. Other more complex problems associated with discounted purchases are described in detail in Lukinsky (2007).

11.3 Lot Sizing at Variable Demand and Limited Planning Horizon

In case of variable demand, lot-sizing problems assume that the demand is known for each discrete time bucket until the established planning horizon. Such time bucket usually includes 1 day or 1 week; it is also assumed that the demand arises at the beginning of the time bucket. As a rule, it is assumed that a lot comes to a storage facility in full at the beginning of a certain time bucket; the lot lead time is ignored. This is a classical dynamic lot-sizing problem.

It is proven that the optimal solution to such a problem has two important properties (Axseter 2006):

(A) The product quantity in a lot is exactly equal to the cumulative demand for a number of time buckets, i.e. for a period covered by such lot;
(B) The cost of holding for one time bucket from the whole period covered by a lot shall not exceed the costs associated with lot shipment and delivery.

11.3.1 Exact Solution

The true and exact solution has been proposed in Wagner and Whitin (1958); today, it is known as the Wagner–Whitin (WW) algorithm. Let us assume that demand D_t is assigned to a number of time buckets $t = 1, 2, \ldots h$; the processing cost c_o per one order and holding cost c_h per measuring unit during one time bucket are also known. The solution should determine in which time bucket t every new product lot shall be delivered and which last time bucket k it covers.

Let us denote as $f_{k,t}$ the costs associated with lot delivery and holding till time bucket k, starting from the time period t where the last lot has been supplied. The minimum possible costs required to cover the demand till time bucket k will be as follows:

$$f_k = \min_{1 \le t \le k} f_{k,t}. \tag{11.29}$$

This means that it depends on the delivery time t.

If previous costs already incurred by the time period $t-1$ are known, cover demand at this period and equal to f_{t-1}, then costs $f_{k,t}$ over the next period t will depend on the size of a lot supplied at this period. The more bucket number k is (until which a lot shall cover the demand), the more $f_{k,t}$ will be. It is true that:

$$f_{k,t} = f_{t-1} + c_o + c_h[D_{t+1} + 2D_{t+2} + \ldots + (k-t)D_k]. \tag{11.30}$$

As demand D_t is covered at the beginning of time bucket c_o, i.e. immediately after supply, then, holding costs are partially associated with a lot supplied in such time bucket and intended to cover demand over next time buckets. The more distant such next to-be-covered time bucket is from the supply time bucket t, the longer holding period and the higher associated costs will be.

Let us consider the WW algorithm using the example described in Table 11.3. Let us assume that management costs per one lot $c_o = \$2,000$; holding costs per product measuring unit $c_h = \$10$. Let us also assume that $f_0 = 0$.

Each line in Table 11.3 represents the biggest number of a time bucket covered by a lot supplied in the time period t; cross cells show respective values of $f_{k,t}$. The table starts to be filled with $f_{1,1} = c_o = 2,000$. Such cost will occur if a lot supplied in time bucket $t = 1$ covers only demand over the same time bucket, i.e. $k = t$. This means that the lot immediately leaves the storage and there are no holding costs.

If the lot supplied in time bucket $t = 1$ should also cover the next time bucket $t = 2$, i.e. $k = t + 1$, then according to formula (11.30):

Table 11.3 Dynamic lot size costs

	Number of time bucket t and respective demand D_t									
Finite horizon for a lot	1/60	2/30	3/80	4/60	5/50	6/100	7/60	8/70	9/80	10/70
$k = t$	2000	4000	4300	5900	6900	7900	9900	10,500	11,900	13,300
$k = t + 1$	2300	4800	4900	6400	7900	8500	10,600	11,300	12,600	
$k = t + 2$	3900	6000	5900	8400	9100	9900	12,200	12,700		
$k = t + 3$	5700	7500		10,200			14,300			
$k = t + 4$	7700									

$$f_{2,1} = f_0 + c_o + c_h D_2 = 0 + 2000 + 10 \times 30 = 2300.$$

Similarly, $f_{3,1} = 0 + 2000 + 10 \times 30 + 2 \times 10 \times 80 = 3900$, $f_{4,1} = 5700$, $f_{5,1} = 7700$. For time bucket 6 holding costs are equal to $5 \times 10 \times 100 = 5,000$ and exceed $c_o = 2,000$. So, according to the property (B) described above in Sect. 11.3, it is unreasonable to use the lot supplied in time bucket 1 to cover demand in time bucket 6 and there is no need to further fill column 1 of the table. As we can see, minimum costs associated with lot supplies to cover demand in time period $k = 1$ only will be as follows: $f_1 = f_{1,1} = 2,000$.

Now, let us calculate $f_{k,t}$ when a new lot is supplied during time bucket $t = 2$. Using expression (11.30) we get:

$$f_{2,2} = f_1 + c_o = 2000 + 2000 = 4000.$$

Similar to Column 1 calculations, $f_{3,2} = 4800$, $f_{4,2} = 6000$, $f_{5,2} = 7500$, and there is no need in further calculations.

Now let us consider whether it is reasonable if a lot supplied in time bucket $t = 1$ also covers demand in time bucket $t = 2$, i.e. $k = t + 1$. For this purpose, let us use formula (11.29) or, which is the same, compare two underlined numbers shown in columns for time buckets 1 and 2. Both numbers apply to one and the same value $k = 2$, i.e. they represent the costs that are required to cover demand in time bucket 2. It turns out that if we use the lot supplied in time bucket 1, associated costs will be \$2300, while if we use another lot to cover demand in time bucket 2, associated costs will be \$4000. Accordingly:

$$f_2 = \min_{1 \le t \le 2} f_{2,t} = \min(f_{2,1}, f_{2,2}) = \min(2300, 4000) = 2300.$$

Let us proceed with Column 3 and again use formula (11.30) where $f_{t-1} = f_2 = 2,300$. So, $f_{3,3} = f_2 + c_o = 4,300$, $f_{4,3} = 4,900$, etc. To proceed with Column 4, let us calculate the following value:

$$f_3 = \min_{1 \le t \le 3} f_{3,t} = \min(f_{3,1}, f_{3,2}, f_{3,3}) = \min(3900, 4800, 4300) = 3900.$$

As you can see, compared values of $f_{k,t}$ lie on the diagonal of the table; at the last element on the diagonal is $f_{k,t}$ with equal indices $k = t$. All remaining columns should be completed similarly.

To find the lot size solution, we should move from the last diagonal in Table 11.3. This diagonal will yield us the solution on the minimum cumulative costs as follows:

$$f_{10} = \min(14300, 12700, 12600, 13300) = 12600$$

This means that the last lot should be supplied in time bucket 9.

Table 11.4 Optimal supplies to cover demand under Table 11.3

Lot size	Number of time bucket t and respective demand D_t									
	1/60	2/30	3/80	4/60	5/50	6/100	7/60	8/70	9/80	10/70
	90		190			230			150	

Let us introduce the set of time buckets $J\{J_1, J_2, \ldots J_n\}$, over which lots should be supplied in order to achieve minimum costs. In this set, elements are numbered in the backward order from the planning horizon to the start of planning. So, $J_1 = 9$ and the last time bucket to be covered by a previous lot is $k = 8$. In order to determine J_2 let us consider the diagonal of Table 11.3 whose last element is the first cell in Column 8. For all elements on this diagonal, the last covered time bucket $k = 8$. According to formula (11.29), $f_8 = \min(9900, 10600, 10500) = 9900$ and $J_2 = 6$.

Now, the last time bucket covered by the previous lot is $k = 5$ and we need to consider the diagonal with $f_5 = \min(7700, 7500, 5900, 6400, 6900) = 5900$ that yields $J_3 = 3$. As we can see, the last element of J-set is $J_4 = 1$. So, optimal lot supplies shall follow Table 11.4.

This exact solution ensures minimum supply costs for the pre-defined number of time buckets till the established planning horizon. So, in order to optimally cover demand at further time buckets, we need to do a new planning job. In practice, the supply horizon is flexible, i.e. re-planning takes place after every supply and considers time buckets until the planning horizon. This leads to a natural question—"What minimum planning horizon will ensure precise lot-sizing optimization?".

It turns out (Axseter 2006) that, firstly, such a horizon does not necessarily exist, and even if so, it is quite distant. That's why it is quite challenging to use the WW algorithm in practice, unfortunately. However, such a solution is of great value as it allows verifying different approximated and heuristic algorithms some of which are described below.

11.3.2 Heuristic Silver–Meal Algorithm

This algorithm (Silver and Meal 1973) successively determines if it is reasonable to merge adjacent demand periods to arrange supply of one lot. For example, if a lot is supplied in time bucket 1, we need to increase the lot size from its minimum value (i.e. demand value in time bucket 1) by the demand value for time bucket 2 and then check whether it is reasonable. If yes, the algorithm should be repeated for time bucket 2, etc. If the merge of time buckets is found unreasonable, for example, in time period k, this time bucket should see a new lot supply. For further applications, this time period k is taken as the first time period.

Merging adjacent time periods for one lot supply is feasible if it leads to reduced average costs per period, i.e. adjacent periods should be merged if:

Table 11.5 Demand forecasts over time periods

Period	1	2	3	4	5	6	7	8	9	10
Demand	60	30	80	60	50	100	60	70	80	70

$$\frac{c_o + c_h \sum_{j=2}^{k} (j-1)D_j}{k} \leq \frac{c_o + c_h \sum_{j=2}^{k-1} (j-1)D_j}{k-1}. \tag{11.31}$$

The right-hand side of inequality (11.31) stands for the average costs for one period at $k-1$ of merged time periods. If another time period k is added to already merged time periods, the average cost is presented in the left-hand side of the inequality. Naturally, if the costs do not increase, it is feasible to merge the time periods. Let us consider this algorithm and take the example described above in Sect. 11.3.1 (Table 11.5); the management costs per lot $c_o = \$2,000$ and holding costs per product measuring unit over 1 time period $c_h = \$10$.

If supplied in time period 1, the minimum lot size is 60 pieces. The first check using inequality (11.31) applies to $k = 2$; the right-hand side is $c_o = 2,000$:

$$(2000 + 10 \times 30)/2 = 1150 < 2000.$$

It is feasible to merge time period 1 with time period 2. Let us check further merging. The right-hand side of inequality (11.31) is equal to the left-hand side of the previous inequality:

$$(2000 + 10 \times (30 + 2 \times 80))/3 = 1300 < 1150.$$

As we can see, merging with time period 3 is unfeasible; so, this time period should see a new supply.

Again, let us denote the right-hand side of inequality (11.31) as $c_o = 2,000$ and check the feasibility for $k = 2$ by merging time periods 3 and 4:

$$(2000 + 10 \times 60)/2 = 1300 < 2000.$$

Then let us proceed with merging with time period 5 and check its feasibility by comparing with the right-hand part that is equal to 1300:

$$(2000 + 10 \times (60 + 2 \times 50))/3 = 1200 < 1300.$$

After merging with time period 6, we get:

$$(2000 + 10 \times (60 + 2 \times 50 + 3 \times 100))/4 = 1650 < 1200,$$

and this is unfeasible. So, a new lot should be planned for time period 6.

Again, let us denote the right-hand side of inequality (11.31) as $c_o = 2,000$ and check the feasibility for $k = 2$ by merging time periods 6 and 7:

$$(2000 + 10 \times 60)/2 = 1300 < 2000.$$

Then let us proceed with merging with time period 8 and check its feasibility by comparing with the right-hand part that is equal to 1300:

$$(2000 + 10 \times (60 + 2 \times 70))/3 = 1333 < 1300.$$

As the left-hand side is greater than the right-hand side, merging is unfeasible and we should plan a new supply for time period 8.

Let us check if it is feasible to merge supplies over time periods 8 and 9:

$$(2000 + 10 \times 80)/2 = 1400 < 2000,$$

and with time period 10 for which the left-hand side is equal to the right-hand side of inequality (11.31).

$$(2000 + 10 \times (80 + 2 \times 70))/3 = 1400 = 1400.$$

In this case, merging is possible and feasible. The solution yielded by the algorithm is shown in Table 11.6.

If we compare the results of this approximate solution with the exact solution described in Sect. 11.3.1, we can see that the difference is that the lot size is decreased in period 6 which, it its turn, required the last lot to be transferred from period 9 to period 8. Now, let us calculate planned costs using average costs by periods covered by each lot:

$$c = 1150 \times 2 + 1200 \times 3 + 1300 \times 2 + 1400 \times 3 = 12,700.$$

The resulting value is very close to the exact solution $f_{10} = 12,600$ (Sect. 11.3.1). When comparing two methods please note that the resulting costs $c = 12,700$ (per Table 11.3) lie on the last diagonal in period 8 column which corresponds to the last lot determined based on Silver–Meal algorithm. Generally, the solution resulting from Silver–Meal algorithm leads to 1÷2 % increase in the costs as compared with the exact solution (Baker 1989).

In expression (11.31), costs over several demand periods merged into one lot split among such demand periods. Instead of the number of periods, we can insert the number of product in measuring units required over such periods into the denominator in formula (11.31). All the remaining procedure remains the same as in Silver–Meal method. This modified algorithm is known as the Least Unit Cost

Table 11.6 Lot solution according to Silver–Meal algorithm

Lot size	Number of time period t and respective demand D_t									
	1/60	2/30	3/80	4/60	5/50	6/100	7/60	8/70	9/80	10/70
	90		190			160		220		

heuristic. However, Baker (1989) proves that generally Silver–Meal method yields better solutions.

11.3.3 Part Period Balancing

When the EOQ is calculated using (2.4) the holding costs are exactly equal to the management costs associated with a lot. Such equality is impossible for problems with variable demand; however, we can attempt to size lots in such a way so that the above costs are close to each other. Respective algorithm was proposed in De Matteis and Mendoza (1968) and it is known as the Part Period Balancing (PPB).

According to this method, the first supply covers demand over n-periods whose number is determined with the following inequality:

$$c_h \sum_{j=2}^{n} (j-1) D_j \leq c_o < c_h \sum_{j=2}^{n+1} (j-1) D_j. \tag{11.32}$$

This inequality means that a new supply in period $n+1$ should take place if the cost of holding of the last lot exceeds the cost of organizing a new lot provided that demand is covered.

To illustrate how this method is applied, let us consider the example described in Table 11.5. Let us assume that management costs per one lot $c_o = \$2,000$; holding costs per product measuring unit $c_h = \$10$. As the first lot automatically covers demand in period 1, we will start with period $n = 2$ to check the inequality (11.32).

$$c_h \sum_{j=2}^{2} (2-1) D_2 = 10 \times 30 = 300 < c_o = 2000,$$

and a new lot is not necessary. For the third period, we get:

$$10 \times (30 + 2 \times 80) = 1900 < 2000;$$

again, there is no need in a new lot. For the fourth period, we get:

$$10 \times (30 + 2 \times 80 + 3 \times 50) = 3700 > 2000,$$

and this means that a new lot is necessary.

Again, we take $n = 1$ for period 4 and check the next period (5) starting with $n = 2$.

$$10 \times 50 = 500 < 2000.$$

We continue check to for $n = 3$:

Table 11.7 Lots according to the Part Period Balancing method

Lot size	Number of time period t and respective demand D_t									
	1/60	2/30	3/80	4/60	5/50	6/100	7/60	8/70	9/80	10/70
	170			110		230			150	

$$10 \times (50 + 2 \times 100) = 2500 > 2000;$$

this means that a new lot is necessary.

We take $n = 1$ for period 6 and check the next period (7) starting with $n = 2$.

$$10 \times 60 = 600 < 2000;$$

then, we check period 8 at $n = 3$

$$10 \times (60 + 2 \times 70) = 2000.$$

The last inequality shows that we can plan the same lot for period 8; however, it is clear that period 9 will require a new lot. The solution yielded by the algorithm is shown in Table 11.7.

As mentioned in Axseter (2006), the Part Period Balancing method usually yields worse solutions than Silver–Meal method.

11.3.4 Groff's Heuristic Rule

Unlike other methods described above, Groff's rule assumes that demand is covered evenly during a time period rather than immediately at the beginning. According to this method (Groff 1979), an increase in the lot size to cover demand in an additional period is feasible if resulting reduced costs on lot management exceed respective increased holding costs. Indeed, if we split lot management costs c_o over the number n of demand periods covered by such lot, average costs per one period will be c_o/n. If we increase the number of periods covered by one lot to $n + 1$, lot management savings will be as follows:

$$\frac{c_o}{n} - \frac{c_o}{n+1} = \frac{c_o}{n(n+1)}. \tag{11.33}$$

On the other hand, if the number of demand periods covered by one lot is increased from n to $n + 1$, this will lead to increased holding costs as follows:

$$\frac{c_h D_{n+1}}{2}; \tag{11.34}$$

given the above, Groff's rule has been derived as follows: it is feasible to add a demand period to a lot if:

$$\frac{c_o}{n(n+1)} > \frac{c_h D_{n+1}}{2}, \tag{11.35}$$

and, on the contrary, it is unfeasible if:

$$\frac{c_o}{n(n+1)} \leq \frac{c_h D_{n+1}}{2}. \tag{11.36}$$

To illustrate how this method works, again, let us consider the example described in Table 11.5. Let us assume that organization costs per one lot $c_o = \$2,000$; holding costs per product measuring unit $c_h = \$10$. Calculations start with the second period, i.e. $n = 1$:

$$\frac{c_o}{n(n+1)} = \frac{2000}{1 \times 2} = 1000 > \frac{c_h D_{n+1}}{2} = \frac{10 \times 30}{2} = 150;$$

as we can see, it is feasible to merge the first and the second period into one lot. If we continue calculations for the third period, we get:

$$\frac{c_o}{n(n+1)} = \frac{2000}{2 \times 3} = 333 < \frac{c_h D_{n+1}}{2} = \frac{10 \times 80}{2} = 400.$$

Thus, it is unfeasible to include period 3 demand into a lot.

Let us proceed with the next lot and check period 4 against Groff's rule.

$$\frac{2000}{1 \times 2} = 1000 > \frac{10 \times 60}{2} = 300.$$

Period 4 should be merged with period 3. As for interval 5, we get:

$$\frac{2000}{2 \times 3} = 333 > \frac{10 \times 50}{2} = 250.$$

So, interval 5 should also be included into demand covered by the lot to be supplied in period 3. For interval 6, we get:

$$\frac{2000}{3 \times 4} = 167 < \frac{10 \times 100}{2} = 500,$$

and a new lot is required for this period.

For the next lot and period 7, we get:

$$\frac{2000}{1 \times 2} = 1000 > \frac{10 \times 60}{2} = 300,$$

and for the next period 8, we get:

Table 11.8 Lots according to Groff's rule

Lot size	Number of time period t and respective demand D_t									
	1/60	2/30	3/80	4/60	5/50	6/100	7/60	8/70	9/80	10/70
	170		190			160		150		70

$$\frac{2000}{2 \times 3} = 333 < \frac{10 \times 70}{2} = 350.$$

We should plan a new lot for period 8; then, we move onto period 9 for which we get:

$$\frac{2000}{1 \times 2} = 1000 > \frac{10 \times 80}{2} = 400,$$

and for the last period 10:

$$\frac{2000}{2 \times 3} = 333 < \frac{10 \times 70}{2} = 350.$$

A new lot is required for period 10. The solution yielded by the algorithm is shown in Table 11.8.

It was proven in Nydick and Weiss (1989) that solutions yielded by Groff's rule are as good as those resulting from Silver–Meal heuristic.

11.3.5 Period Order Quantity

As mentioned above in Sect. 8.2.4, small enterprises and shops that receive regular supplies from sustainable suppliers widely use a simple inventory management model that is characterized by a fixed delivery period. If demand is variable though forecasted, it is possible to determine a fixed delivery period that allows reducing costs associated with lot management and holding. In this case, each order will become variable or period (Period Order Quantity, POQ).

Let us assume that demand forecasts are available for 10 time periods according to the input data per Table 11.5. Using these data, we can calculate the average rate of demand over 1 period:

$$\overline{D} = \frac{1}{10}\sum_{t=1}^{10} D_t = 66.$$

If such demand rate were constant, the optimal lot size (EOQ) would be as follows (based on formula 2.4); $c_o = \$2,000$, $c_h = \$10$.

Table 11.9 Lot sizes for two period order options

Delivery period	Number of time period t and respective demand D_t											
	1/ 60	2/ 30	3/ 80	4/ 60	5/ 50	6/ 100	7/ 60	8/ 70	9/ 80	10/ 70	11/ 70	12/ 70
2	90		140		150		130		150		140	
3	170			210			210			210		

$$Q^* = \sqrt{\frac{2c_oD}{c_h}} = \sqrt{\frac{2 \times 2000 \times 66}{10}} = 162.$$

This ratio $\frac{T^*=Q^*}{D}$ represents the so-called economic delivery period. In our example, $T^* = 162/66 = 2.46$. As a fixed delivery period must be an integer, two options are possible, i.e. $T^* = 2$ or $T^* = 3$. In any case, the size of each delivered lot shall cover the forecasted demand from one delivery to the next one delivery.

We can compare resulting options by their cost. However, as you can see, we can't compare options for 10 periods as if $T^* = 3$ the lot size for interval 10 is not clear. So, let us make it 12 periods. We assume that these additional periods will have the same demand rate as period 10, i.e. $D_{11} = D_{12} = 70$ units. Table 11.9 shows both possible options over 12 periods.

If $T^* = 2$, the associated costs will be as follows:

$$c_1 = 6 \times 2000 + 10 \times (30 + 60 + 100 + 70 + 70 + 70) = 16,000.$$

If $T^* = 3$, the associated costs will be as follows:

$$\begin{aligned} c_2 &= 4 \times 2000 + 10 \\ &\quad \times (30 + 2 \times 80 + 50 + 2 \times 100 + 70 + 2 \times 80 + 70 + 2 \times 70) \\ &= 16,800. \end{aligned}$$

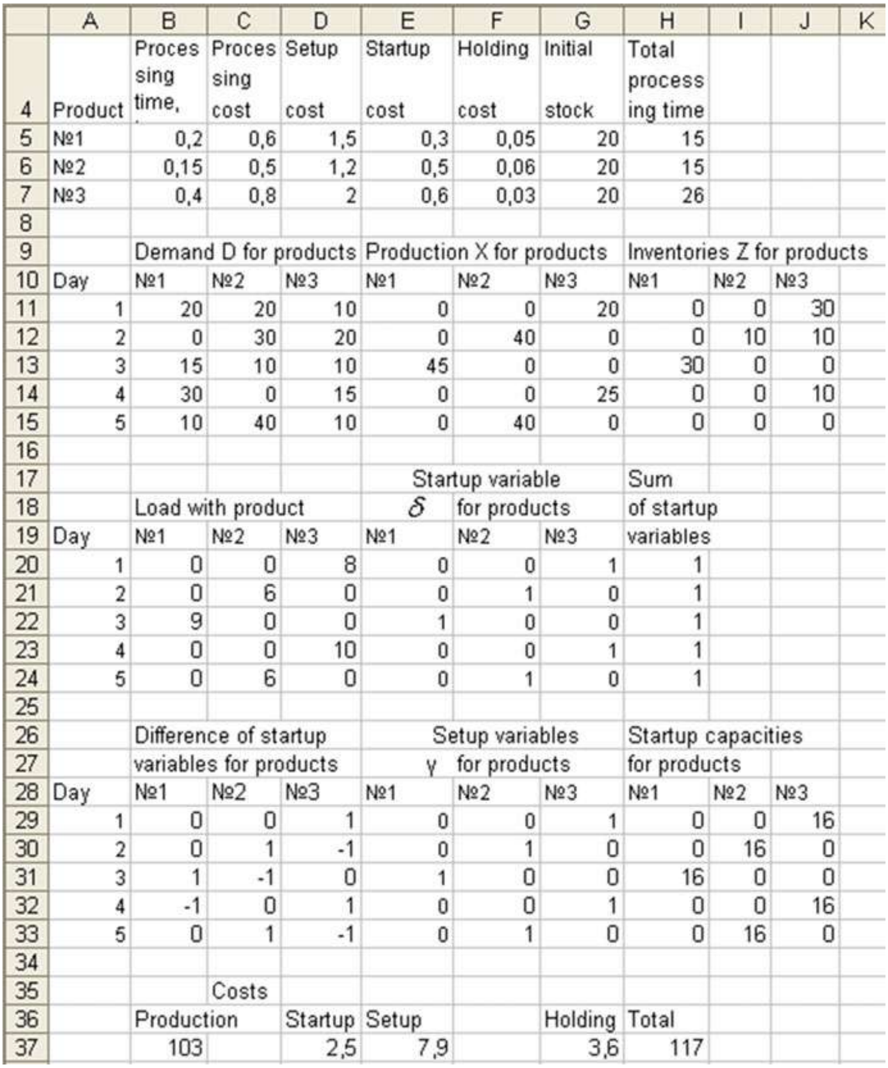
As we can see, the option with $T^* = 2$ is more economic. Now, let us compare period order costs over 10 periods with the exact solution described in Sect. 11.3.1. For ten periods we get:

$$c_1 = 5 \times 2000 + 10 \times (30 + 60 + 100 + 70 + 70) = 13,300,$$

While according to the exact decision, the costs over 10 periods will be$f_{10} = 12,600$.

11.4 Lot Sizing with Constraints

As an example, we'll consider the capacitated lot-sizing problem with a small planning period (CSLP) that is described above in Sect. 11.1.4. To ensure a relative transparency of the solution, we'll attempt to do lot sizing for three different



	A	B	C	D	E	F	G	H	I	J	K
4	Product	Processing time,	Processing cost	Setup cost	Startup cost	Holding cost	Initial stock	Total processing time			
5	№1	0,2	0,6	1,5	0,3	0,05	20	15			
6	№2	0,15	0,5	1,2	0,5	0,06	20	15			
7	№3	0,4	0,8	2	0,6	0,03	20	26			
8											
9		Demand D for products			Production X for products			Inventories Z for products			
10	Day	№1	№2	№3	№1	№2	№3	№1	№2	№3	
11	1	20	20	10	0	0	20	0	0	30	
12	2	0	30	20	0	40	0	0	10	10	
13	3	15	10	10	45	0	0	30	0	0	
14	4	30	0	15	0	0	25	0	0	10	
15	5	10	40	10	0	40	0	0	0	0	
16											
17					Startup variable			Sum			
18		Load with product			δ	for products		of startup			
19	Day	№1	№2	№3	№1	№2	№3	variables			
20	1	0	0	8	0	0	1	1			
21	2	0	6	0	0	1	0	1			
22	3	9	0	0	1	0	0	1			
23	4	0	0	10	0	0	1	1			
24	5	0	6	0	0	1	0	1			
25											
26		Difference of startup			Setup variables			Startup capacities			
27		variables for products			γ	for products		for products			
28	Day	№1	№2	№3	№1	№2	№3	№1	№2	№3	
29	1	0	0	1	0	0	1	0	0	16	
30	2	0	1	-1	0	1	0	0	16	0	
31	3	1	-1	0	1	0	0	16	0	0	
32	4	-1	0	1	0	0	1	0	0	16	
33	5	0	1	-1	0	1	0	0	16	0	
34											
35			Costs								
36		Production		Startup	Setup		Holding	Total			
37		103		2,5	7,9		3,6	117			

Fig. 11.4 Optimal lot-sizing calculations

products to be produced on a single machine within 5 work days. Each day is assumed as a time bucket. We also assume that a lot of one product is produced over one time bucket.

Figure 11.4 shows tables required for calculations using MS Excel tools. First of all, there are input data on machine capacity, parameters of each product, and daily demand for such products. Cell $H$5:$H$7 calculates the total processing time for each product that is required to cover demand. Note that we can check the viability of a solution at a very early stage by comparing the total processing time with machine capacity over the period in question. In our case, we need 56 h that is lower than the machine capacity being 16 × 5 = 80 h.

According to Sect. 11.1.4, the problem includes the independent variables as follows: production lot sizes X_{it}, startup variables δ_{it}, and setup variables γ_{it} for which cells $E$11:$G$15, $E$20:$G$24, and $E$29:$G$33 are reserved, respectively. Product inventories are calculated in $H$11:$J$15 using formulas (11.11).

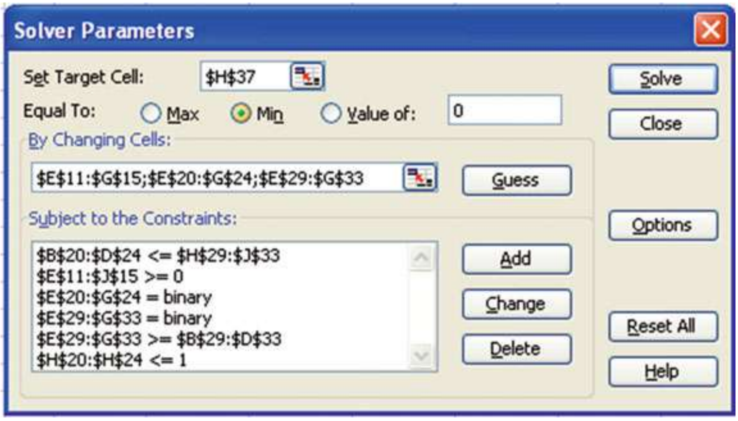
Cells $B$20:$D$24 calculate load for each of the product over each of 5 days. For example, cell $B$20 contains the following expression: =$B$5*$E11. Given constraint (11.15), this load shall correspond to the potential useful capacity available at the startup of a respective product; such available useful capacity is put down into cells $H$29:$J$33. For example, cell $H$29 contains the following expression: =$E$2*E20.

In order to check the solution against constraint (11.13), cells $H$20:$H$24 contain the sum of startup variables δ_{it}. For example, cell $H$20 contains the following formula: =SUM($E20:$G20). Inequalities (11.14) are checked by comparing the table of differences of startup variables in cells $B$29:$D$33 with the table of setup variables γ_{it}.

Cells calculating the costs associated with production, startup, setup, and product holding as well as the target cell of cumulative costs $H$37 are located at the lower part of MS Excel spreadsheet.

Figure 11.5 shows the input screen for problem solving. As we can see from this screen, in order to ensure minimum costs, we need to change product lot sizes in cells $E$11:$G$15 and determine startup variables in cells $E$20:$G$24 and setup variables in cells $E$29:$G$33.

In addition to the above constraints, we also need to specify that variables of lot size X_{it} and inventory size Z_{it} shall be positive and that setup variable γ_{it} and startup variable δ_{it} can take on either 0 or 1.



Solver Parameters

Set Target Cell: $H$37

Equal To: ○ Max ● Min ○ Value of: 0

By Changing Cells:

$E$11:$G$15;$E$20:$G$24;$E$29:$G$33

Subject to the Constraints:

$B$20:$D$24 <= $H$29:$J$33
$E$11:$J$15 >= 0
$E$20:$G$24 = binary
$E$29:$G$33 = binary
$E$29:$G$33 >= $B$29:$D$33
$H$20:$H$24 <= 1

Fig. 11.5 Input screen for optimal solution

As we can see from the resulting solution, it is feasible to arrange two lots for product 3, two lots for product 2, and one lot for product 1. Each startup will have a setup as each subsequent day sees the production of a product other than the product manufactured over the preceding work day.

Please note that under the resulting solution inventories for all products will be depleted by the end of the planning period. This somewhat doesn't match input data according to which there is a certain inventory at the beginning of the planning period. Indeed, lack of inventory at the beginning of the period makes it impossible to cover demand for all products over the first time bucket as, according to CSPL problems, only one product can be produced over one time bucket.

A more correct solution should provide for a certain safety stock that must remain at the end of each time bucket, i.e. we need to set inventory limits in cells $H$11:$J$15. Unfortunately, we can't set such limits in our example due to a low number of time buckets. If we have a bigger number of time buckets, e.g. by splitting each work days into two shifts, we can considerably improve our solution. However, this will lead to a drastic increase in the number of variables.

This problem can also be applied for optimal lot sizing in the situations where jobs are performed by one team of workers over one time period and this team is assigned to multiple assembly or casting machines, etc. In this case we can consider such team as a "machine" moving from one work position to another.

Such machines can be used to produce or assemble multiple product types, but the product type shall remain unchanged over one time bucket. The optimal lot size over a time bucket will be equal to the number of machines. Naturally, in this case we need to set another limit on the maximum size of a lot.

11.5 Multi-product Deliveries and Orders

Quite often, suppliers and customers make multi-product arrangements. In these situations, we face a problem about possible combination of orders and deliveries within one shipment lot.

11.5.1 Optimal Multi-product Lot Sizing

If one shipment lot includes n products of different types, then the associated costs will have two components as follows: c_0 that is mainly determined by shipment and c_i associated with warehousing jobs to select product of i-type, i.e.

$$c_o = c_0 + \sum_{i=1}^{n} c_i. \tag{11.37}$$

Warehousing job costs include, for example, the cost of use for lifting tools, trolleys, containers, etc., and the cost of documentation. Please note that costs associated with movements of each product around the storage facility are not included into c_i as they are classified as direct costs per product unit. In order to calculate lot size-based costs c, let us use formula (2.2) and replace the lot size variable with the delivery period according to formula (8.3).

$$c = \frac{TD}{2} c_h + \frac{1}{T} c_o. \tag{11.38}$$

Expression (11.38) is true for each product included into a joint delivery lot. As in case of joint delivery, all products included into the delivery lot have the same period T, then the total costs associated with such lot will be as follows (formula 11.37):

$$C_\Sigma = \frac{T}{2} \sum_{i=1}^{n} D_i C_{hi} + \frac{1}{T} \left(C_0 + \sum_{i=1}^{n} C_i \right). \tag{11.39}$$

Now, let us find the optimal delivery period. We take the derivative from formula (11.39) by T and set it equal to zero. As a result, we get:

$$T^* = \sqrt{\frac{2\left(c_0 + \sum_{i=1}^{n} c_i\right)}{\sum_{i=1}^{n} D_i c_{hi}}}. \tag{11.40}$$

The optimal product quantity for each type under joint delivery will be calculated as follows:

$$Q_i^* = T^* D_i. \tag{11.41}$$

Let us consider an example joint delivery of four products (refer to Table 11.10).

The optimal delivery period (in days) (at 30-day month) will be:

Table 11.10 Products for joint delivery

Product	Demand, measuring unit per month	Order management costs, dollars		Handling cost C_{hi} per measuring unit, dollars per month	Optimal lot size
		c_0	c_i		
1	1000	2000	400	25	333
2	400	2000	250	50	133
3	100	2000	600	60	33
4	50	2000	600	60	17

$$T^* = 30\sqrt{\frac{2 \times (2000 + 400 + 250 + 600 + 600)}{1000 \times 25 + 400 \times 50 + 100 \times 60 + 50 \times 60}} = 11.3 \text{ days.}$$

Given the 10-day delivery period (three deliveries per month), let us determine the quantity of each product for an optimal delivery lot (formula 11.41). For example, $Q_1^* = T^* D_1/30 = 10 \times 1000/30 = 333$. According to formula (11.39), the total cost of deliveries per month will be as follows: $\frac{10}{2\times 30}(1000 \times 25 + 400 \times 50 + 100 \times 60 + 50 \times 60) + \frac{30}{10}(2000 + 400 + 250 + 600$ $+600) = 20{,}568$.

11.5.2 Multi-product Deliveries over Multiple Periods

The key idea is that it is not necessary to combine all shipping products into one lot for each lot. Indeed, demand is often different for various products. So, if products are independent, optimal delivery periods may be considerably different for different products.

According to the system of multiple periods described in Ryzhikov (2001), a product with the highest demand is delivered at the highest frequency; its delivery period is assumed as basic. All other products are delivered at the frequency that is multiple to the basic delivery period, i.e. such products have delivery periods that are equal to the basic period multiplied with 2, 3, 4, ... In this case, we can plan multi-product lots so that quantities of each product will be as close to optimal lot size as possible. The lot-sizing calculations include multiple iterations; the procedure is described, for example, in Lukinsky (2007).

A simpler though quite accurate version of the multiple periods system is the so-called powers-of-two policies. According to these policies, each delivery period is equal to the product of the length of a certain basic period by 2 in the integer power m (that can be negative or positive). This method offers an important advantage as it provides for a relatively short length of cycle that includes the full set of different deliveries.

To illustrate such advantage, let us consider the following example. Let us assume that a basic 1-day period sees delivery of two products. Such delivery can take place in two ways. In the first case, product 1 is delivered with a period of $2^2 = 4$ days and product 2 is delivered with a period of $2^3 = 8$ days. In the second case, product 1 is delivered with a 3-day period and product 2 with a 7-day period.

Suppose the first day sees the simultaneous delivery of both products. In the first case, subsequent deliveries of product 1 will take place on days 5, 9, 13, etc. and deliveries of product 2 will take place on days 9, 17, etc. As we can see, joint delivery can happen again on day 9; so, the delivery cycle length will be 8 days. In the second case, subsequent deliveries of product 1 will occur on days 4, 7, 10, 13, 16, 19, 22, etc., and deliveries of product 2 on days 8, 15, 22, ... Thus, joint delivery is possible on day 22 only, i.e. the cycle length will be 21 days.

In addition to a reduced full delivery cycle, the power-of-two policies allow calculating minimum possible costs for multi-product delivery problems. To prove this, let us first consider a single-product delivery problem. Let us use relation (2.6) that represents the ratio of the actual cost from formula (2.2) to the minimum cost (2.5). Using relation (11.41), we get:

$$\frac{c}{c^*} = \frac{1}{2}\left(\frac{Q}{Q^*} + \frac{Q^*}{Q}\right) = \frac{1}{2}\left(\frac{T}{T^*} + \frac{T^*}{T}\right). \tag{11.42}$$

According to the power-of-two policies, the delivery cycle length will be as follows:

$$T = 2^m q, \tag{11.43}$$

where q stands for the length of a basic period. If optimal period T^* does not equal $2^m q$, then the actual period T should be either less than T^* or greater than T^* (formula 11.43). Figure 11.6 shows a probable chart of cost changes.

Let T^* be about 6.5 (as shown in Fig. 11.6). If $q = 1$, then possible value of T is equal either to 4 or to 8. On curve $c(T)$, c_4 and c_8 approximately correspond to such period values. It can be proved (Axseter 2006) that the biggest cost increase depending on the optimal period is achieved where $c_4 = c_8$, i.e. when cost values are equal at both possible values of T. Let us denote the lower period by T and the greater value by 2 T. Then, according to formula (11.42) the biggest possible ratio c/c^* will be as follows:

$$\frac{c}{c^*} = \frac{1}{2}\left(\frac{T}{T^*} + \frac{T^*}{T}\right) = \frac{1}{2}\left(\frac{2T}{T^*} + \frac{T^*}{2T}\right) \tag{11.44}$$

$$\text{or} \quad \frac{T^*}{2T} = \frac{T}{T^*}.$$

Then, $T^* = \sqrt{2}T$ and

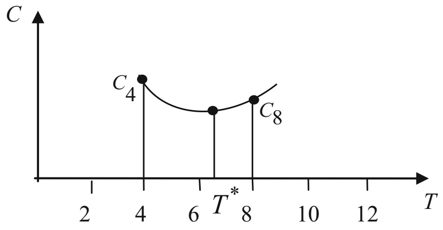


Fig. 11.6 Delivery period-based cost chart

$$\frac{c}{c^*} = \frac{1}{2}\left(\frac{1}{\sqrt{2}} + \sqrt{2}\right) = 1.06. \tag{11.45}$$

As you can see, according to the power-of-two policy, the greatest possible increase in the delivery cost as compared to the optimal cost will only be 6 %. This is true also for multi-product deliveries as the biggest possible increase in the cost for each product will be the same 6 %.

The maximum cost increase by 6 % occurs regardless of the length of a basic period q. It has also been proven (Axseter 2006) that under some values of q such maximum increase can be reduced to 2 %.

11.5.3 Power-of-Two Policies for Multi-product Deliveries

An efficient procedure for the power-of-two policies if applied to multi-product deliveries is proposed in Roundy (1985). Let us consider this application on the example described above in Table 11.10. First of all, let us place multiple products in the ascending order of $\eta_i = C_i/(C_{hi}D_i)$. When applied to Table 11.10, this ratio has the dimension of (month^2); so, it will be 0.016 for product 1; 0.012 for product 2; 0.10 for product 3, and 0.20 for product 4. So, the set of products J is sequenced as 2, 1, 3, 4.

The costs associated with the first lot of product 2 will be $C_0+C_2=2000+250=2250$; $\eta_1^{'}=(C_0+C_2)/(C_{h2}D_2)=2250/(50\times400)=0.112$. Now, let us see if it is feasible to combine the lot of product 2 and that of product 1. Let us compare $\eta_1^{'}$ with $\eta_1=C_1/(C_{h1}D_1)=0.016$. It has been proven in Axseter (2006) that the product going next in J-set should be combined with already created k-lot if for such product η_i is lower than $\eta_k^{'}$.

As for product 1 $\eta_1 = 0.016$ which is lower than $\eta_1^{'} = 0.112$ for lot with $k = 1$, it seems viable to combine such products. For products 2 and 1 combined into one lot, the related costs will be $c_0 + c_2 + c_1 = 2000 + 250 + 2,650$, the holding cost will be $C_{h2}D_2 + C_{h1}D_1 = 50 \times 400 + 25 \times 1000 = 45,000$, and the cost ratio will be $\eta_1^{'} = 2650/45000 = 0.059$.

Now, let us check if is feasible to include product 3 into the first lot. As for product 3 $\eta_3 = 0.10 > \eta_1^{'} = 0.059$, it is not viable to combine product 3 with products 2 and 1 and it's better to create a new lot. In the second lot, related costs will be $c_0 + c_3 = 2000 + 200 = 2200$, holding cost will be $C_{h3}D_3 = 60 \times 100 = 6000$, and $\eta_3^{'} = 2200/6000 = 0.37$. As for product 4 $\eta_4 = 0.2 < \eta_2^{'} = 0.37$, we should include product 4 into the second lot; $\eta_2^{'} = (c_0 + c_3 + c_4)/(c_{h3}D_3 + c_{h4}D_4) = 0.35$.

For each lot, the optimal delivery period is calculated using the formula derived from (11.40).

Table 11.11 Multi-product lots yielded by the power-of-two policy

Product	Parameter η	Lot no.	Delivery period, days	Lot size
2	0.012	1	10	133
1	0.016			333
3	0.10	2	20	66
4	0.20			34

$$T_k^* = \sqrt{2\eta_k'}. \tag{11.46}$$

So, $T_1^* = \sqrt{2\eta_1'} = \sqrt{2 \times 0.059} = 0.34$ month, $T_2^* = \sqrt{2\eta_2'} = 0.83$ month.

Now, let us apply the power-of-two policy. As a basic period, we should take the value close to T_1^* under which a month includes an integer number of deliveries. With three deliveries per month, $q = 10$ days. To calculate other possible delivery periods, we should multiply q by one of the powers of two: 20, 40, 80, etc. So, for the second lot, $T_2 = 20$ days $= 0.66$ month will approach T_2^* as close as possible. Table 11.11 shows the calculated solution.

As we can see from Table 11.11, 2 months will see only six deliveries: four deliveries will include products 1 and 2 only, and two deliveries will include all four products. This cycle will repeat every 2 months. Thus, the total cost of 2-month deliveries includes two components. The first component is calculated using formula (11.39) for products 1 and 2, and the second component represents the holding cost for products 3 and 4 as well as costs c_i associated with required warehousing jobs. Note that shipment costs c_0 are avoided for these products due to the possibility of joint delivery.

So, for products 1 and 2 the 2-month cost will be $2 \times [0.33/2 \times (1000 \times 25 + 400 \times 50) + 1/0.33 \times (2000 + 400 + 250)] = 30{,}750$; for products 3 and 4 the costs will be $2 \times [0.66/2 \times (100 \times 60 + 50 \times 60) + 1/0.66 \times (600 + 600)] = 5{,}700$, and the total 2-month cost of deliveries will be 40,326. If we compare the resulting value with the 2-month cost of simultaneous delivery of all four products calculated above in Sect. 11.5.1, we will see that the power-of-two policy ensures reduced costs.

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Production Planning 12

12.1 Master Production Planning

The objective of the master planning, as is well known, is transformation of existing demand forecasts and orders of an enterprise into a specific plan of action for certain near period. The master plan may be drawn up directly according to the forecasts and orders, and can be elaborated based on the existing sales and operations plan, and the latter is preferred. The master plan unlike the sales and operations plan is much more detailed, i.e. aggregation of individual products here is the exception rather than the rule.

The master plan is a set of planning items that should be produced within several planning periods from the beginning of this plan to the planning horizon. A planning period is as a rule 1 month and with large-scale production—1 week. The horizon of the master planning can be set within 3÷12 planning periods depending on what the product's position in the market is (Sect. 1.6.1). If the market is stable, meaning there is a balance of supply and demand (Fig. 10.1), then it is reasonable to carry out planning with the greatest horizon, and in all other cases, the horizon should be less.

With various types of production, planning stages, and methods of analysis of elaborated plans, it is convenient to use various planning items. The use of computers allows parallel planning simultaneously on several planning items. In this approach, during modelling of the production process the user is able to transfer from analysis of major planning items to a more detailed analysis of their constituents and make a well-reasoned decision.

The planning items used in the planning system can be presented as a hierarchical tree, on the lower level of which apparently there is a technological operation. It is much more difficult to set a rational planning item of the highest level.

In the make-to-stock production, this kind of item is usually represented by a particular type of finished products. With make-to-order strategy, it seems quite natural to use external orders as planning items. Using an external order as a planning item is convenient in terms of production and financial accounting, but

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has a very significant drawback—job-order production leads to small lot sizes and significant increase in cost.

12.1.1 Master Planning as Product Tables

This way of presenting the master plan is mainly used when the demand can be forecasted well for the product and it enables "make-to-stock" strategy (Vollmann et al. 2005). For each such product a table is drawn up that shows the demand forecast, output plan, and current stock level for each planning period within the horizon.

Such a table for the simplest case has the form shown in (Table 12.1). The volume of the product, its receipt at the storage, and issue from the storage are calculated at the end of a period. Table 12.1 shows the case of the standing production of the product and the forecast having a peak demand in the area of the fourth and fifth weeks. The presented production plan tracks the probable peak demand in advance and then the output is reduced in order to restore the initial stock level. This variant is typical for the production with a short production cycle, operating in a stable market subject to seasonal fluctuations.

A more complex version is shown in Table 12.2. Here the demand forecast tends to grow. The product is manufactured in batches of fixed size (Sect. 8.2.1) and the production cycle is 2 weeks. As a result, the stock varies considerably and may even lead to the unsatisfied demand.

When using the model with fixed supply period (Sect. 8.2.2) and duration of the production cycle of 3 weeks the product planning takes the form presented in Table 12.3. The down trend of the demand leads to a reduced lot size in this model.

In developing the master plan the division workload should be checked more in detail than in preparation of the sales and operations plan (Sect. 10.1.2). In the literature, for this purpose it is recommended to use the so-called resource profile method described in Fogarty et al. (1993). This method is quite acceptable; however, to use it, it is necessary to have data on the division workload for each time period until the completion of the batch.

More often, we know the capacity of each division or work center with high workload for each planned product, as well as operation lead in this division and their duration (Table 12.4). In Table 12.4 it is assumed that products B2 and B3 are processed in division A2 right after processing in division A1.

The capacity of a division is understood here as the amount of the product processed or manufactured per day when other products are not produced at this time. A rough estimate of the capacity on the product can be obtained by simply dividing the daily time fund of the production divisions by processing time of the unit of the product measure in this division.

If several products are produced during the same period, the production of each of them uses a relevant share of the total division's capacity. For example, with the planned output of product B1 for week 1 amounting to 1000 UM (Table 12.1) and the length of the working week equal to 5 working days, the consumed part of

Table 12.1 Master plan for product B1

	Volumes in units of measure by planning weeks									
Parameters	1	2	3	4	5	6	7	8	9	10
Demand forecast	1000	1000	1200	1300	1300	1200	1200	1100	1000	1000
Stock (initially 300)	300	500	600	600	600	600	600	500	400	300
Production	1000	1200	1300	1300	1300	1200	1200	1000	900	900

Table 12.2 Master plan for product B2 with fixed size of lots

	Volumes in units of measure by planning weeks									
Parameters	1	2	3	4	5	6	7	8	9	10
Demand forecast	200	200	220	230	250	250	250	270	270	270
Expected receipt		300								
Stock (initially 180)	−20	80	160	230	280	30	80	110	140	170
Planned receipt			300	300	300		300	300	300	300
Planned launch	300	300	300		300	300	300	300		

Table 12.3 Master plan for product B3 with fixed supply cycle

	Volumes in units of measure by planning weeks									
Parameters	1	2	3	4	5	6	7	8	9	10
Demand forecast	60	60	55	55	50	50	50	45	40	40
Expected receipt	160									
Stock (initially 20)	120	60	5	105	55	5	95	50	10	110
Planned receipt				160			150			140
Planned launch	160			150			140			

Table 12.4 Data on capacity of divisions and work centers

Division	Product	Capacity in UM of product per day	Lead in days	Processing duration in days
A1	B1	800	–	–
	B2	300	6	3
	B3	50	12	6
A2	B2	200	3	1
	B3	90	6	2

capacity of division A1 (Table 12.4) is equal to 1000/(5800) = 0.25 of the total capacity. All this capacity is required in the same period, in which the output of product B1 is planned, because the lead for this product is not provided according to Table 12.4.

A different situation occurs in production of products B2 and B3. For example, as the lead of product B2 in division A1 is 6 days, then the processing shall start within the time period with the number one less than the number of the period of the planned receipt and take 1 day of this week. Besides, the processing is partially transferred to the period of the planned receipt and takes 2 days in it. The case with product B3 (Fig. 12.1) is even more complex.

Along the axis of abscissas in Fig. 12.1, the planning weeks with duration of 5 working days each are plotted. The grey rectangles correspond to processing of

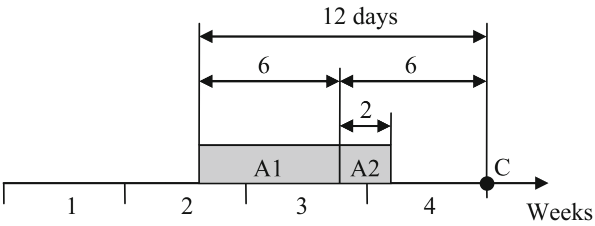


Fig. 12.1 The schedule of processing of product B3 with planned output for week 4

product B3 in divisions A1 and A2 according to the data of Table 12.4. Point C at the end of week 4 is a check point and all leads are referred to it.

It can be seen well in Fig. 12.1 that to supply product B3 at the end of week 4 processing in division A1 should start week 2 and finish week 3. Herewith the processing duration within week 2 is 2 days and within week 3—4 days. In this case for week 2 of product B3 in division A1 it falls within 2/6 and for week 3 4/6 of the total processing time remains. Similarly, 1/2 falls for week 3 and for week 4 also 1/2 of processing time remains for division A2.

Based on these considerations for organizing the receipt of product B2 on week 3 (Table 12.2), it is necessary that for this product during week 2 the partial capacity of division A1 (Table 12.4) is provided, which is equal to $1/3 \times 300/(5 \times 300) = 0.067$ of all its weekly capacity. Accordingly, for the same purpose $2/3 \times 300/(5 \times 300) = 0.133$ should be used during week 3. Furthermore, during week 3, the part of capacity of division A1 equal to $1/3 \times 300/(5 \times 300) = 0.067$ is also used for the same product B2, which is scheduled for week 4. The total requirement for capacity of division A1 for product B2 is thus 0.2 during week 3. The requirement for capacity of division A2 for product B2 is calculated as for product B1, as all processing of product B2 in this division fits in one time period. For example, for period 3 we obtain $300/(5 \times 200) = 0.3$.

Defining the workload for product B3 in a similar way we obtain the data on workload of divisions presented in Table 12.5.

In this case, the calculation results analysis indicates the exceeding workload (0.952) of division A1 during the third week. The above method of defining capacity utilization is very close to the above-mentioned method of resources profile but more crude, as the processing time of the product in the division for each period is defined as grand total. However, it can be assumed that to evaluate the capacity utilization in master planning the method of this closeness is enough while the necessary data for calculation are readily available.

Table 12.5 Workload dynamics of divisions

Division	Product	Capacity utilization coefficients by weeks									
		1	2	3	4	5	6	7	8	9	10
A1	B1	0.25	0.3	0.325	0.33	0.33	0.3	0.3	0.25	0.225	0.23
	B2		0.07	0.2	0.2	0.13	0.1	0.2	0.2	0.2	0.13
	B3		0.21	0.427	0	0.2	0.4	0	0.19	0.373	0
	Total	0.25	0.58	**0.952**	0.53	0.66	0.8	0.5	0.64	0.798	0.36
A2	B2			0.3	0.3	0.3	0	0.3	0.3	0.3	0.3
	B3			0.178	0.18	0	0.2	0.17	0	0.156	0.16
	Total	0	0	0.478	0.48	0.3	0.2	0.47	0.3	0.456	0.46

In this case, the calculation results analysis indicates the exceeding workload (0.952) of division A1 during the third week

12.1.2 Group Master Planning

When operating in the volatile market both the demand forecast and its actual error are subject to great fluctuations. In these cases, of course, every company strives to make-to-order operation. However, with sufficiently long production cycle the operation strategy only under orders increases the waiting period of the order, which reduces the competitiveness of the enterprise.

Therefore, in some cases it is advisable to produce some portion of all products using make-to-order strategy and some portion using make-to-stock or assembly-to-order strategy (Sect. 1.3.2). Anyway, it makes sense to assume that the entire production output is made by orders, but the part of this output is fulfilled as the internal order of the enterprise. Herewith the planning is virtually always performed for a certain jointly processed group of external and internal orders, and in the particular case such a group may consist of only one order.

In many information systems the group of orders intended for joint processing is considered to be a planning item (PI) and called a production order. However, this term is quite overloaded and mostly belongs to the financial and economic activity. For this reason, (Mauergauz 2007) it is suggested to use also term "planning group" for such a group of orders.

We will understand a planning group as a set of some finished products and spare parts launched into production simultaneously. It is reasonable to include products of different types with a close design-engineering composition as well as spare parts similar to some components of these products into this set.

The number of products and parts of each type in the planning group should be established based on rational stock quantity at all times during production of the planning group. The production and marketing time of the planning group and, accordingly, the size of the planning group are determined by the planning horizon of the manufactured products. The output of products belonging to one planning group can be performed gradually within several planning periods. One planning group can provide fulfilment of a number of orders or their separate stages.

Using a planning group as a basic PI allows setting quite large sizes of production lots since each lot of parts, assemblies, components, and finished products is produced not for one order and not even for one planning period but to provide for the established planning group.

The planning group differentiates release lots and output lots of products and their components. On the ground of the planning group definition, there is only one release lot of products of each type, which together form a launch group.

At the same time, there may be several output lots of the same type of products. The completion time of output lots may relate to different time periods, and the output lot sizes are set either by the available capacity of the assembly production or by the expected requirement for products for sales. Each output lot of products in the planning group is a planning item in the hierarchy below the planning group and type of product. Set of output lots of products of the same type within the planning group is a line of these products.

Each output lot in the planning group should be intended for one or more orders, each of which is also a planning item. However, a direct relationship of release lot to orders is impractical because, for example, for one order there can be several output lots and even several planning groups. So it makes sense to introduce another planning item which is hierarchically lower than the order—the so-called Production Process Model (PPM).

The PPM establishes a connection between the manufactured products and customer order. The specification of an external order may contain a number of items with different types of products and different manufacturing time for these products. Moreover, in the order specifications there may be requirements for the periodic delivery, delivery of products of the same type in parts at the relevant time, etc.

The Production Process Model (PPM) includes a set of products and spare parts from the specification of one order with the same fixed delivery date. If the quantity in one delivery of the order specification is more than the assembly shop capacity, then this item of the specification should be divided into several PPM. With this approach, for example with recurrent deliveries, it is necessary to arrange so many PPMs per order as many times this delivery should be performed.

The connection of the planning group and thus the launch group with the external order is provided by referring the PPM to some output lot of the corresponding planned products. The block diagram shown in Fig. 12.2 explains the relations among the production lots and orders.

From Fig. 12.2 it follows that each item in the order specification is associated with the Production Process Model (PPM). The composition of the PPM contains the order configuration, as well as versions of processes used to fulfil the order. One of the output lots provides manufacturing of products in accordance with each PPM.

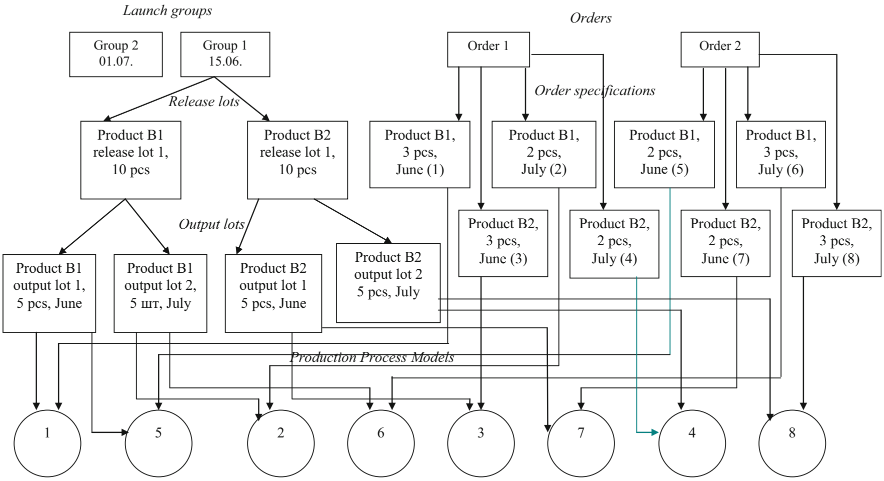

Launch groups
Group 2 01.07.
Group 1 15.06.
Release lots
Product B1 release lot 1, 10 pcs
Product B2 release lot 1, 10 pcs
Output lots
Product B1 output lot 1, 5 pcs, June
Product B1 output lot 2, 5 шт, July
Product B2 output lot 1 5 pcs, June
Product B2 output lot 2 5 pcs, July
Orders
Order 1
Order 2
Order specifications
Product B1, 3 pcs, June (1)
Product B1, 2 pcs, July (2)
Product B1, 2 pcs, June (5)
Product B1, 3 pcs, July (6)
Product B2, 3 pcs, June (3)
Product B2, 2 pcs, July (4)
Product B2, 2 pcs, June (7)
Product B2, 3 pcs, July (8)
Production Process Models
1
5
2
6
3
7
4
8

Fig. 12.2 Block diagram "Lots–Orders"

Table 12.6 Lots–Orders

Planning (launch) group	Product	Release lot	Output lot	Output date	Order	PPM	Quantity
No. 1 15.06	B1	1	1	June	1	1	3
					2	5	2
			2	July	1	2	2
					2	6	3
	B2	1	1	June	1	3	2
					2	7	3
			2	July	1	4	3
					2	8	2
No. 2 01.07	B1	2	3	August	3	9	10

For example, output lot 1 of product B1 includes production process models PPM1 for order 1 and PPM5 for order 2.

Block diagram "Lots–Orders" can be presented as Table 12.6. As shown in Table 12.6, launch group No. 1 includes two types of products that have rather close design-engineering composition. The release lot of, for example, product B1 is divided into two parts during manufacturing—output lots, which are produced during 2 different months. Each of the output lots provides the supply for several, in this case two, orders under the relevant production process modules.

After arranging each new launch group it is necessary to check the situation with equipment workload. For this purpose, the current data on the planned load shall be superimposed by the results of the load calculation on the new launch group, which can be obtained by using, for example, the method described above in Sect. 12.1.1.

12.1.3 Master Production Plan Optimization

With make-to-stock operation the main criterion of optimization for the master production plan, as per Tables 2.5–2.7 in Sect. 2.2.3 is criterion K1 of product cost minimization. For these conditions, the paper of Chung and Krajevski (1984) has the problem laid down of achieving the value minimum

$$c = \sum_{i=1}^{n} \sum_{t=1}^{h} \left(c_{mi} X_{it} + c_{oi} \delta_{it} + c_{hi} Z_{it} \right) + \sum_{t=1}^{h} \left(c_H H_t + c_F F_t + c_N N_t + c_O O_t + c_U U_t \right), \tag{12.1}$$

where n is the number of different manufactured products and h is the planning horizon.

The costs in Eq. (12.1): c_{mi}—the production cost of the unit of measure of the i-th product; c_{oi}—the cost of regular setup of production to start operation in the period; c_{hi}—the storage cost of the unit of measure during one period; c_H—the cost of hiring per person; c_F—the cost of firing per person; c_X—the cost of materials per product unit; c_N—period average salary of one employee in normal; conditions; c_O—the cost of overtime hour; c_Z—the cost of storage per product unit; c_U—the cost of production downtime per hour.

For each i-th product in Eq. (12.1), the following is included: X_{it}—the quantity of product manufactured within period t in hours; δ_{it}—binary variable defining the fact of setup in period t; Z_{it}—the quantity of product in stock by the end of period t. In Eq. (12.1) it is also specified: H_t—the number of newly hired employees; F_t—the number of fired employees; N_t—the current number of employees; O_t—number of overtime hours; U_t—number of downtime hours.

The first constraint is the constraint in stocks which is the set of equations

$$Z_{it} = Z_{i,t-1} + X_{it} - D_{it} \text{ with } 1 \leq i \leq n,\ 1 \leq t \leq h. \tag{12.2}$$

The personnel balance is

$$N_t = N_{t-1} + H_t - F_t \text{ for all } 1 \leq t \leq h. \tag{12.3}$$

The working time balance is

$$GN_t + O_t - \sum_{i=1}^{n} X_{it} - \sum_{i=1}^{n} S_i \delta_{it} \geq 0 \text{ for all } 1 \leq t \leq h, \tag{12.4}$$

where G is the time fund in hours of one employee within the period with regular payment;

S_i the duration of setup for the i-th product.

The constraint for the overtime

$$O_t \leq BN_t \text{ for all } 1 \leq t \leq h, \tag{12.5}$$

where B is the allowable overtime hours for one employee within the period.

The constraint on the quantity of product has the form of inequation system

$$X_{it} \leq M \times \delta_{it} \text{ with } 1 \leq i \leq n,\ 1 \leq t \leq h, \tag{12.6}$$

where “big” number M is defined from expression (11.8).

Variables of setup

$$\delta_{it} = 1 \text{ with } X_{it} > 0 \text{ and } \delta_{it} = 0 \text{ and } X_{it} = 0. \tag{12.7}$$

Requirement for the safety stock Z_{ci} at the end of each period

$$Z_{it} \geq Z_{ci} \text{ with } 1 \leq i \leq n,\ 1 \leq t \leq h\,. \tag{12.8}$$

The variables in expression (12.1)–(12.8) are the output of product X_{it}, the fact of setup for product δ_{it}, the number of fired employees F_t, the number of hired employees H_t, and the number of overtime hours O_t.

The problem stated in the form of objective function (Eq. 12.1) and constraints (Eqs. 12.2 and 12.3) is actually a combination of the problem set for one aggregate group in Sect. 10.2.1 and the problem of the lot sizes with capacity constraints and large planning interval described in Sect. 11.1.3.

The solution of the problems of this type, as was shown above, is possible by MS Excel techniques, but at that, the number of variables becomes considerable. For example, in the book of Shapiro (2001), the problems with the 13-week (3-month) planning for 500 different products are discussed. A problem of this size requires 6500 binary variables 0–1, wherein the amount of the remaining problem's constraints reaches 20,000. Since the direct solution in such cases is difficult to obtain, Shapiro (2001) proposes to use the method of the so-called decomposition in which several possible plans are considered successively and the best of them is selected.

As such possible plans, usually the plans optimizing the initial problem in a small interval of time (for example, 1 week) are accepted. With the set of plans available for several periods ahead, we can agree them consistently and thus obtain a unified plan for the period up to the planning horizon.

With make-to-order operation, the quality criteria of the master plan, as shown in Tables 2.5–2.7, change. In this case, besides the product costs, it is necessary to consider the timing of order fulfilment as well and so the optimization problem becomes multi-objective and cannot be solved by linear programming techniques. The use of group scheduling allows calculating (Mauergauz 2007) the values of the quality criteria such as the full output, profit, necessary working capital, capacity utilization, penalties for non-fulfilment of contracts, duration of the contracts performance, etc., for each elaborated plan option. Using the utility functions of each criterion (Sect. 1.7.2) we can define the integral characteristic of each option and choose the best.

12.2 Material Requirement Planning

To produce complex products consisting of several or many components, the production planning of the latter is carried out in accordance with the developed master plan. These components are objects of dependent demand, i.e. their bill of materials (BOM) can be directly calculated in terms of the master plan. The manufacturing resource planning (MRP2) is an essential element of modern production management, and its core is a set of planning tables of each component similar to Table 12.2 or 12.3 for the master plan (Sect. 12.1.1). Table 2.12 refers to the so-called order policy with a fixed size of an order, and Table 12.3 refers to the order policy with a fixed cycle.

The most difficult issue of material requirement planning is the definition of rational production lot sizes. First of all, the components lots should be sufficient to assemble the planned lots of finished products, i.e. the minimum lot size of components is determined by one lot of finished products. This option is applied with lot-for-lot order policy. However, since with such a relevance the lots of components often get small and thus expensive, the components lot sizes are necessary to be increased.

The second issue is to determine the duration of lot production. While developing the master plan (Table 12.3), the dependence of the lot lead time on its size, in some range, is often overlooked, because the duration of the production cycle of the finished product is taken with a large margin against the duration of the process cycle. However, in manufacturing of components it is no longer possible to neglect this dependence.

Generally speaking, the lot size of components should meet the demand for a certain number of time periods, but as the lead time depends on the size of the lot, the moment of the lot receipt becomes variable. The result is that it is not always clear whether the lot can meet the demand for the selected number of time periods, i.e. when using Table 12.3 we face a vicious circle.

It is possible to eliminate this disadvantage of planning in MRP2, if we use the "planning group" policy of orders. By definition of the planning group in Sect. 12.1.2, each planning group corresponds to one launch group of finished products and spare parts of multiple types. Table 12.6 "Lots–Orders" is a sequence of planning (launch) groups providing fulfilment of external and internal orders. The "planning group" policy of orders as applied to Table 12.6 is a generalization of "lot-to-lot" policy, in which the lot of finished product is a planning group. At the same time, the components release lots corresponding exactly to the launch group and the components output lots must ensure the output lots of the planning group.

12.2.1 Production Lot Duration

The total production time of the i-th product lot (production cycle F_i) is determined by the time of its processing and transfer time between divisions and between operations within the divisions. Furthermore, it is sometimes necessary to provide time for the natural maturing (ageing, drying, etc.). The lot processing time (planned job) is equal to

$$p = s + Q\tau, \tag{12.9}$$

where s is the total setup time for the operations, Q—lot size, and τ—full run time of processing. Duration of processing may differ from value determined by formula (12.9) in the cases of the so-called parallel processing, in which one or more operations of a lot is handled simultaneously on several machines.

In general, with sequential flow of the processed product the production time of a lot with n operations

$$F = s + Q\sum_{j=1}^{n} \tau_j/m_j + A_1 n + A_2(N-1) + T_e, \tag{12.10}$$

where τ_j is the standard run time for performance of the j-th operation, m_j is the number of parallel machines on the j-th operation, A_1—the standard time to transfer a lot from an operation to an operation, A_2—the standard time to transfer a lot from a shop to a shop, N—the number of shops in the shop-to-shop flow sheet, and T_e—the time of natural maturing.

In the book (Sachko 2008) it is indicated that the interoperation time A_1 is usually equal to $0.5 \div 1$ of the shift duration. When a lot is transferred for another operation from a site to a site or when operating with large objects A_1 should be considered equal to 1. For the case if handling small items, the interoperation time shall be accepted equal to 0.5 of shift.

Standard time A_2 shall be established at each enterprise. Moreover, value A_2 may depend on from which shop and to which one the lot is transferred. In this case value A_2 forms the matrix of standard time of transferring from shop to shop.

To reduce the cycle time the lot is often split into several lots in the course of the longest (“main”) operation. In this case, there is the so-called parallel-serial flow, in which the original lot is called a release lot and its parts processed after the main operation separately are called output lots. The duration of the production cycle in this case is determined (Sachko 2008) as

$$F' = F - Q'\left(\frac{Q}{Q'} - 1\right)\sum_{i=1}^{k} \tau'_i, \tag{12.11}$$

where the duration of serial cycle F is determined by dependence (12.10), Q—release lot size, Q'—output lot size, τ'_i—standard run time for an operation, which is overlapped by the main operation, and k—the number of these overlapped operations.

The duration of the production cycle in formulas (12.10) and (12.11) is defined in hours or, respectively, in working days taking into account shifts. In the material requirement planning it is necessary to consider non-working days as well and, of course, a work calendar can be used for this purpose. At the same time, it should be noted that at the time of planning the work calendar may not yet be fully known. In addition, it is definitely clear that the deviations in the implementation of the plan can make one or more days. Therefore, when calculating the plan in calendar days, you can simply increase the estimated duration proportionally to balance calendar and working days in 1 week, i.e. accept calendar duration F_k in the form

$$F_k = F\frac{7}{5} = 1.4F, \tag{12.12}$$

which should be rounded up to integer value.

12.2.2 Optimal Production Lot Sizing

Let us consider the process of arrangement of lots of product components on the basis of the developed master plan, in which output lots were identified for a certain planning group (Table 12.7).

When preparing Table 12.7, unlike the conventional (impersonal) designations of products applied above, we use the so-called specific form of objects designation provided by the Russian system of design documentation. Due to this form of designations it is convenient to track the sequence of building a tree hierarchical structure of the product.

In general, the specific designation of a design object (assembly unit or component) consists of two parts. The first part consists of several letters and numbers constant for all the objects included in the finished products (item). The second part has several sets of numbers separated by dots. If the product is relatively simple, one such group is enough.

For group planning all manufacturing specifications included into the same launch group undergo explosion. Example of this process is given in Table 12.8.

Table 12.8 is made for the launch group (Table 12.7), consisting of the release lot of finished product with designation KK4851.000, the release lot of design-close product KK4853.000, and the lot of spare parts kits KK4851.ZP. Six upper rows of the table contain objects of the first indenture level and in the lower lines there are objects of the second level. The objects of the first level are included in the main assembly units KK4851.000, KK4853.000, or spare parts kit KK4851.ZP. The second-level objects in this case are included in assembly unit KK4851.010. The fourth column of Table 12.8 shows the quantity, which the object, described in the second column, has when entering the object specified in the fourth column.

The most significant difference Table 12.8 from the conventional explosion tables is that for each object it is specified which of the PPMs it relates to (column 3). In this case, four production process modules are in operation PPM 11, PPM 12, PPM 13, and PPM 14, and the first two are composed for the same finished product KK4851.000. The need for two PPM for one product KK4851.000 is due to the fact that there are two different orders for this product.

The quantity of each object for the corresponding PPM is determined in accordance with Table 12.7 and entered in column 5 of Table 12.8. Let us calculate, for example, the number of parts KK4851.104 “Wall” directly included in assembly unit KK4851.010 in the quantity of 2 pcs., for PPM 11 (line 8). To do this, we must note that assembly unit KK4851.010 is included in finished product KK4851.000 in

Table 12.7 Lots–Orders for the planning group in question

Product	Output lot	Output date	Order	PPM	Quantity
KK4851.000	1	June	1	11	3
	2	July	2	12	6
KK4851.SP	1	July	1	13	3
KK4853.000	1	June	3	14	5

Table 12.8 Results of explosion for launch groups

Line number	Designation and description of object	PPM no. and indenture level	Where included: designation and quantity	Quantity per one item and per PPM
1	KK4851.010 Body frame	PPM 11 u = 1	KK4851.000 2	2 6
2	KK4851.104 Wall	PPM 11 u = 1	KK4851.000 3	3 9
3	KK4851.010 Body frame	PPM 12 u = 1	KK4851.000 2	2 12
4	KK4851.104 Wall	PPM 12 u = 1	KK4851.000 3	3 18
5	KK4851.104 Wall	PPM 13 u = 1	KK4851.ZP 4	4 12
6	KK4851.010 Body frame	PPM 14 u = 1	KK4853.000 1	1 5
7	KK4851.102 Base	PPM 11 u = 2	KK4851.010 1	2 6
8	KK4851.104 Wall	PPM 11 u = 2	KK4851.010 2	4 12
9	KK4851.102 Base	PPM 12 u = 2	KK4851.010 1	2 12
10	KK4851.104 Wall	PPM 12 u = 2	KK4851.010 2	4 24
11	KK4851.102 Base	PPM 14 u = 2	KK4851.010 1	1 5
12	KK4851.104 Wall	PPM 14 u = 2	KK4851.010 2	2 10

the quantity of 2 pcs. (line 1), and PPM 11 contains three items KK4851.000 (line 1 of Table 12.7). As a result, we have 2 × 2 × 3 = 12.

Material requirement planning is to define release and output dates for each row in Table 12.8. For this purpose, it is first necessary to determine the moment of assembly start of the output lots in Table 12.7. The book (Mauergauz 2007) describes the possible algorithm of this calculation in detail. In this case, we assume that from the 1st to 12th of June lot 1 of items KK4853.000 is assembled, and then from the 13th to the 30th—lot 1 of items KK4853.000; the assembly of lot 2 of items KK4851.000 is planned for the period from the 10th to 25th of July.

As a rule, all of the components required for assembly should be ready by the beginning of assembly. In some cases, however, it is possible that individual objects, such as spare parts and tools, can be supplied in process or even by the end of assembly, but in this example, we assume that there are no such objects. In addition, we assume that all the components needed for spare parts kits KK4853.SP must be made by 30th July.

Calculation of output and release dates begins with the objects, for which the highest indenture level $u = 1$. If the same object can be included in the finished product at different levels, then for it the planning calculations are made at the greatest level of its indenture. For example, in this case in Table 12.8 all lines with assembly unit KK4851.010 “Body frame” have the first indenture level and therefore are planned in the first place. At the same time, some lines with component KK4851.104 “Wall” refer to the first indenture level, i.e. included directly in main object KK4851.000, and the some have level 2 because they are included in assembly unit KK4851.010. Therefore, planning for this component is made on this second level.

Thus, on the first level it is only necessary to plan for assembly units KK4851.010, which are entered in Table 12.8 in lines 1, 3, and 6. Determining of optimal lot sizes should be based on the property (A) in Sect. 11.3, which in this case may be formulated as the amount of product in the optimal lot is exactly equal to the total demand in those lines in Table 12.8, which should be provided by that lot. Upon finding the optimal lot, the use of this property allows limiting to summation options of the quantity in the lines for the same product. With this combination, the following principle is used: *the moment of production start of the release lot and its division into the output lots should provide the completion of production of the necessary quantity of objects from the given release lot to each moment of requirement for these objects to perform assembling within the planning group*.

To perform planning for assembly units KK4851.010 according to this approach, we shall draw up a table of requirement for this object (Table 12.9) by a planning group. The lines in Table 12.9 are sorted in requirements date ascending order.

To organize optimal lots it is advisable to combine adjacent lines of Table 12.9, just as it is done in heuristic algorithms of Silver-Meal or Groff or others, described above in Sects. 11.3.2–11.3.4. Each of these algorithms is based on a certain rule of stopping combination. In this case, by this rule we will assume the achievement of the optimal lot size determined by dependence (11.27) in Sect. 11.2.3. Table 12.10 shows the data of the planned objects necessary for calculation of optimal lots in the planning group.

Symbols in Table 12.10: D—total requirement per planning group, s—total setup time, τ—total run time of processing, M—object weight, b—the ratio of the cost of 1 kg of material to the cost of an hour, s_0—processing time of the main operation, τ_0—the run time of the main operation, n—number of operations in the process, and N—number of shops in the shop-to-shop routing.

To calculate optimal size Q^* of the component lots we use formulas (11.27) and (11.28). For example for component KK4851.102 “Base” we have

Table 12.9 Required end points for object KK4851.010

No. of the line in Table 12.8	Where included: designation, lot number	Required quantity	Requirements date
1	KK4851.000, lot 1	6	01.06
6	KK4853.000, lot 1	5	13.06
3	KK4851.000, lot 2	12	10.07

Table 12.10 Object parameters for planning

Designation	D (pcs.)	s (h)	τ (h)	M (kg)	b (h/kg)	s_0 (h)	τ_0 (h)	n	N	Q^* (pcs.)
KK4851.010	23	2.5	6.5	12.2	–	–	–	1	1	18.5
KK4851.102	23	8.3	5.0	8.6	0.3	2	2.4	8	4	25
KK4851.104	85	0.7	0.3	0.2	0.3	0.2	0.1	4	1	128

$$Q_i^* = \hat{\chi}_i \sqrt{\frac{S_i D_i}{\tau_i + b_i M}} = 6 \frac{2^{0.1}}{23^{0.2} \times (2.4)^{0.5}} \sqrt{\frac{8.3 \times 23}{5.0 + 0.3 \times 8.6}} = 25.$$

Similarly for component KK4851.104 “Wall” we obtain $Q_i^* = 128$.

For optional lots of assembly units we assume that correction factor $\hat{\chi}_i = 6$ and $b_i = 0$. Then for assembly unit KK4851.010 we have $Q_i^* = 18.5$.

Now we carry out the process of combination for assembly units KK4851.010 in accordance with the data in Table 12.9. Combining the first two lines (lines 1 and 6 of Table 12.7), we get a lot of 11 pcs., which is less than the optimal value. Obviously, this combination is quite reasonable. Adding the next line in Table 12.9 to the obtained lot, we get a lot of 23 pcs., which is more than optimal. We assume that this combination does not make sense, and to ensure this line in Table 12.9 it is necessary to organize a new lot.

Proceeding similarly for object KK4851.102 we get that to process this component it is also advisable to create two lots, the first of which will consist of 11 pcs. and the second one—12 pcs. At the same time, it is obvious that all components KK4851.104 “Wall” should be produced in one lot because the total quantity of these components for one planning group is less than the lot optimal value.

After determining the size of the lots of assembly units KK4851.010 it is possible to calculate their production time using expression (12.10). We assume that standard time $A_1 = 1$ shift (0.5 day), standard time $A_2 = 2$ shifts (1 day), and natural maturing $T_e = 0$. In this case, for example, for the first lot consisting of 11 pcs., assuming that the assembly is carried out in sequence on one stand, we obtain

$$\begin{aligned} F_1 &= s + Q_1\tau + A_1 n + A_2(N-1) + T_e \\ &= 2.5/16 + 11 \times 6.5/16 + 0.5 \times 1 + 1 \times (1-1) = 5.13 \text{ days}. \end{aligned}$$

Duration in calendar days is determined by the formula (12.12). To calculate the required output dates for assembly units KK4851.010 we take into account that they should be ready for the above-mentioned points of start of assembling of relevant finished product lots (Table 12.9). In determining the release date it should be assumed that both the release date and output date are included in the calendar time of production cycle. The results of the calculation are shown in Table 12.11.

To account for the holidays and non-work day established administratively the material requirement plan should automatically shift according to the number of non-work days. After developing of the plan, it is necessary to check workload of

Table 12.11 Lot of objects and their production plan

Designation	Lot	Quantity	Duration, working days	Duration, calendar days	Output date	Release date
KK4851.010	1	11	5.13	7	31.05	25.05
	2	12	5.53	8	09.07	02.07
KK4851.102	1	11	11	15	24.05	10.05
	2	12	11.3	16	01.07	16.06
KK4851.104	1	85	3.64	5	24.05	20.05

divisions and work centers, which may be accomplished using the method set forth above in Sect. 12.1.1, or the method of resources profile (Fogarty et al. 1993).

12.2.3 Analysis of the Material Requirement Plan

For analysis and subsequent check it is advisable to submit the developed production plans of assembly units and components of the planning group in the form of hierarchical sequence—a tree displaying the time connections of production of the objects included in each other. The study of this sequence reveals shortcomings of the planning performed when calculating production time of the components and the time required for assembly with necessary components.

The nomenclature tree should have the list of planning groups at its root. A planning group is described by the number and date of launch into production. At the next level the planning groups are divided into types of planned products. The product type is described by such details as designation, name, and number of release lot.

Each type of products is output to several output lots. The output lot is described by the output lot number in the release lot, the range serial numbers of finished products included in the output lot, and the output date. In each output lot of the planned products, the nomenclature included in this lot opens by the indenture levels up to components.

Material requirement planning may show that the time of implementation of the plan at the executive's disposal is not enough. In such cases, you should try to reduce the total duration of the production cycle. To this end, for each lot of finished products in the current planning group, it makes sense to construct a critical path diagram—the largest by total route length of manufacturing and assembly of components to a finished product. Then you can try to reduce the production time of objects that are on the critical path by means of increasing the shift quantity, using multiple parallel machines, splitting the lots, etc.

After such measures the critical path may change, and to further reduction of the duration it is necessary to pay attention to the corresponding objects of the plan. If it is impossible to fulfil the material requirement plan, one has to shift the output date of the finished product in the master production plan. The issues of critical path construction will be discussed in the next paragraph.

12.3 Project-Based Planning

A typical feature of project manufacturing is that it consists of a number of certain jobs, and the execution of these jobs cannot be initiated before the completion of some others. For these jobs, a network diagram is plotted that represents their relationship. The network diagram is depicted as a direct graph—vertex set connected by directed arcs.

Network models distinguish the process of each job itself and the event associated with its execution. The process is described by duration and the event of execution determines the possibility of starting other jobs. Possible sequence of jobs in the network model is a major constraint on their execution.

12.3.1 Critical Path Method

Let us consider set n of jobs required to complete the project. The execution of each of these jobs may be defined by other job execution; in turn, the execution of each of the jobs may also be a prerequisite to start another job. We assume that the processing time of each i-th job is known and equals p_i. We also assume that the amount of equipment, materials, finance, etc., is not limited, and set the least possible time to complete the project, based only on the duration of the jobs.

At the initial instant that job can be executed, for which there are no constraints on the sequence, and upon its completion the jobs can be performed, for which this job is a constraint. To find the least possible time for project completion $C_{\max}$ the algorithm of "forward" planning is used, in which it is believed that the execution of each possible job begins immediately after execution of the previous job. With this algorithm the smallest possible time $C_i^{'}$ for completion of the i-th job

$$C_i' = S_i' + p_i, \tag{12.13}$$

where $S_i^{'}$ is the least possible start time of the i-th job. It is obvious that the value

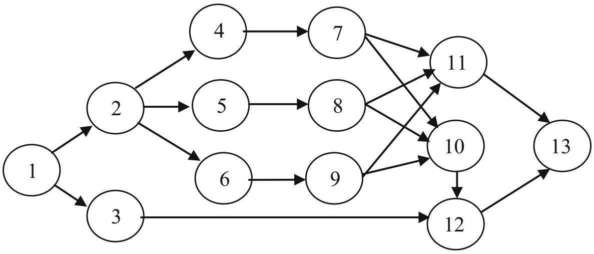
$$C_{\max} = \max\left(C_1', C_2', \ldots, C_n'\right). \tag{12.14}$$

Actually the start time of the i-th job is not necessarily equal to $S_i^{'}$. The job can be started later than $S_i^{'}$, but provided that this delay will not influence $C_{\max}$. To calculate these allowable delays, we use the "backward" planning which sets the largest possible start time $S_i^{''}$ of the i-th job.

Table 12.12 Job list of the project

Job description	Job number	Immediately preceding	Immediately succeeding
Development of test stand specification	1	–	2, 3
General configuration of the test stand	2	1	4, 5, 6
Development and issue of specification on preparing the operation documentation for the test stand	3	1	12
Development of the technology for manufacturing the test stand's electrics	4	2	7
Development of the technology for manufacturing the test stand's mechanical	5	2	8
Placing orders for components required for the test stand assembly	6	2	9
Manufacturing of the test stand's electrics	7	4	10, 11
Manufacturing of the test stand's mechanical	8	5	10, 11
Fulfilling the orders for components	9	6	10, 11
Issuing the data sheet for the operational manual of the test stand	10	7, 8, 9	12
Assembly of the stand	11	7, 8, 9	13
Development of detailed documentation for the test stand operation	12	3, 10	13
Check tests of the stand	13	11, 12	–

Fig. 12.3 Graph of the jobs in the project of the test stand



$$S_i'' = C_i'' - p_i. \tag{12.15}$$

Table 12.12 shows the example of designing, manufacturing, and commissioning of a test stand for equipment suggested in Ross (2006). All the set of jobs under the project can be displayed in the form of the so-called directed graph (Fig. 12.3), the nodes of which correspond to the project's jobs. The sequence of jobs is set by the direction of the arrows in the graph and each of the jobs can be executed only if all the previous jobs are completed.

Assuming that the initial time of jobs is 0, we calculated possible time points of completion for each job $C_i^{'}$ and $C_j^{''}$, using the algorithm of "forward" planning first and then the algorithm of "backward" planning (Table 12.13).

Table 12.13 Duration of the project's jobs in days and the time points of their completion

Job	1	2	3	4	5	6	7	8	9	10	11	12	13
p_j	5	5	3	8	6	10	18	25	15	2	12	8	10
$C_j^{'}$	5	10	8	18	16	20	36	41	35	43	53	51	63
$C_j^{''}$	5	10	45	23	16	26	41	41	41	45	53	53	63
Reserve	0	0	37	5	0	6	5	0	6	2	0	2	0

Completion time point $C_j^{'}$ is defined by expression (Pinedo 2005)

$$C_i' = \max_{\{all\_k \to i\}} C_k' + p_i. \tag{12.16}$$

Expression (12.16) determines completion points $C_k^{'}$ of all k-th jobs immediately preceding job i; among these jobs there is a job with the latest completion and then the processing time of job i is added to the completion point.

For jobs 1 ÷ 9, determination of $C_i^{'}$ is easy as only one arrow goes to the corresponding nodes. For the nodes, to which several arrows go, it is necessary to apply formula (12.16). For example, node 10 has three approaching arrows that indicate that the execution of this job is possible only after completion of jobs 7, 8, and 9. Since the completion of job 8 is the latest then this very job defines the completion of job 10, i.e.

$$C_{10}' = \max_{\{7,8,9 \to 10\}} C_k' + p_{10} = 41 + 2 = 43.$$

Completion time points $C_{11}^{'}$, $C_{12}^{'}$, and $C_{13}^{'}$ are defined similarly. As a result the value of the minimal total duration $C_{\max} = C_{13}' = 63$.

Completion time points $C_i^{''}$, when using the algorithm of "backward" planning, are found from the expression

$$C_i'' = \min_{\{i \to all\_k\}} \left(C_k'' - p_k\right), \tag{12.17}$$

at that value $C_{13}'' = C_{13}' = 63$. One arrow comes from each of nodes 11, 12 and both time points $C_{11}'' = C_{12}'' = C_{13}'' - p_{13} = 63 - 10 = 53$. Similarly $C_{10}'' = C_{13}'' - p_{13} = 53 - 8 = 45$. Two arrows come from node 9, and using formula (12.17), we obtain

$$C_9'' = \min_{\{9 \to 10, 11\}} \left(C_k'' - p_k\right) = \min(45 - 2,\ 53 - 12) = 41.$$

We have the same values for $C_7^{''}$ and $C_8^{''}$. Values $C_4^{''}$, $C_5^{''}$, $C_6^{''}$, and $C_3^{''}$ are defined by the completion time points of jobs 7, 8, 9, and 12, accordingly. Job completion time point $C''_2 = \min_{\{2\to 4,5,6\}}\left(C''_k - p_k\right) = \min(23 - 8,\ 16 - 6,\ 26 - 10) = 10$. Time point $C''_1 = C'_1$, which indicates correct calculation.

The last line of Table 12.13 shows the difference between values $C''_i - C'_i$, which is actually a possible time reserve at the relevant stage of project implementation. The jobs, for which such reserve is equal to zero, lie on the critical path, i.e. in this case the critical path consists of jobs 1, 2, 5, 8, 11, and 13.

12.3.2 Cost Optimization at Various Project Stages

In a number of cases, the duration of the different stages of the project can be reduced by changing financing costs at the stages that are on the critical path. We assume that within certain limits the duration of a stage decreases linearly with increasing costs (Fig. 12.4).

Let us define the value of the relative change of cost per duration unit of the i-th stage as

$$b_i = \frac{c_i^{\max} - c_i^{\min}}{p_i^{\max} - p_i^{\min}}. \tag{12.18}$$

Let us now consider the case of the previous example provided that the values of duration specified in Sect. 12.3.1 represent the greatest possible duration $p_i^{\max}$ (Table 12.14). And the corresponding cost

$$c_i^{\min} = c_i^{\max} - b_i\left(p_i^{\max} - p_i^{\min}\right). \tag{12.19}$$

In this case, the decrease of duration of the stage execution is achieved not by involving additional number of employees but by the use of overtime, bonus system, etc. Value b_i is the average cost of extra pays required to accelerate the i-th stage of the project by 1 day. The size of such extra pays depends on the number of employees simultaneously involved in a stage and their skills. For example, at stages 5 and 6 of the project one engineer of quite high qualification must work, and at stages 7 and 8—3÷4 of semiskilled workers.

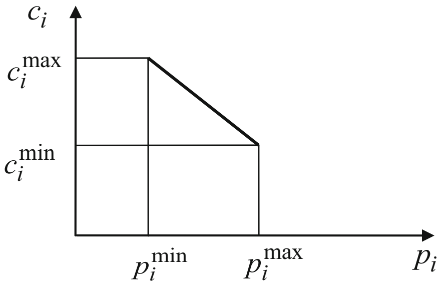


Fig. 12.4 Dependence of the stage duration on the costs

Table 12.14 Duration in days and cost of jobs in conventional units of the project

Job	1	2	3	4	5	6	7	8	9	10	11	12	13
$p_i^{\max}$	5	5	3	8	6	10	18	25	15	2	12	8	10
$p_i^{\min}$	5	5	3	6	5	8	14	18	15	2	10	7	10
$c_i^{\max}$	20	15	9	20	13.2	15	130	256	62	5	64	17.6	38
$c_i^{\min}$	20	15	9	15	11	12	114.8	222.4	62	5	56	15.4	38
b_i	–	–	–	2.5	2.2	1.5	3.8	4.8	1.5	–	4	2.2	–

At some initial stages of the project 1, 2, 3, 4, 5, 6, and at stages 10 and 12, the entire cost of a stage is produced only by labour of engineers and office workers. Typically, acceleration of these stages is either impossible or possible to a very limited extent. In general, the reduced duration can be achieved by means of extra pays at stages 7, 8, and 11 during manufacturing of the test stand.

Suppose that for execution of jobs under the project, a special team of several employees is arranged and total cost of this group per day $b_0 = 6$ conventional units. We also assume that the composition of the team and its expenditures during the project are not changed, but at the end of the project, the team should be disbanded and its financing should be stopped. Of course, an option, when the cost of the project implementation is additional overhead of the enterprise's department, is also possible. As a result, the total cost of the project with total duration $C_{\max} = 63$ days is defined as the sum of all $c_i^{\min}$ in Table 12.14 and overheads during this time, i.e. $595.6 + 6 \times 63 = 973.6$.

The increase of the costs for extra pays to employees at certain stages may be less than the gains from savings on overheads while reducing total duration of the project $C_{\max}$. Reduction of $C_{\max}$ is possible at acceleration of any stage located on the critical path 1, 2, 5, 8, 11, and 13. Let us define the stage at which this change makes sense in the first place.

At stages 1, 2, and 13 $p_i^{\max} = p_i^{\min}$ and they cannot be modified. Now, we compare the average cost of extra pays b_i at the remaining stages 5, 8, and 11. The lowest daily costs occur at stage 5 (designing of technology of mechanical) and is 2.2. Therefore (Baker and Trietsch 2009) the first measure to reduce the total duration of stage 5 is to reduce by 1 day. In this case, the end points on the critical path steps will be $C'_5 = 15$, $C'_8 = 40$, $C'_{11} = 52$, and $C'_{13} = 62$. The project expenses increase at stage 5 by 2.2 and decrease by means of the overheads by 6, i.e. the total cost will be equal to $973.6 + 2.2 - 6 = 969.8$.

Generally speaking, after reducing the duration of stage 5 it is necessary to determine the critical path again because it can vary. In this case, however, the critical path does not change and the optimization of the problem can be continued. A further time reduction of stage 5, according to the conditions in Table 12.14, is not possible. Therefore, let us consider the reasonability of acceleration of stages 8 or 11.

The cost of 1 day at stage 11 is 4, which is less than at stage 8. By means of extra pays stage 11 (test stand assembly) be accelerated by 2 days and herewith no changes will happen in the critical path. So the cost of the project will be

$969.8 + 2945.44 - 2 \times 6 = 945.4$ and the completion time points of the stages on the critical path will be $C'_{11} = 50$ and $C'_{13} = 60$.

The further acceleration can be achieved only by means of stage 8 (manufacturing of mechanical part). According to Table 12.14 the duration of stage 8 can be reduced from 25 to 18 days. If we reduce the duration gradually by 1 day, then the critical path set earlier will remain up to the third days inclusive, and on the fourth day the duration of path 1, 2, 5, 8, 11, and 13 will be equal to the duration of path 1, 2, 4, 7, 11, and 13 (Fig. 12.3). On the fourth day of the reduction of stage 8 we obtain the total cost of the project $945.4 + 4 \times 4.8 - 4 \times 6 = 940.6$.

Now, to further reduce the duration it is necessary to compare values b_i for stage 8 on critical path 1, 2, 5, 8, 11, 13, and stages 4 and 7 on critical path 1, 2, 4, 7, 11, 13. The least one is obviously value $b_4 = 2.5$, and the duration of stage 4 should be reduced by 1 day. However, after acceleration of stage 4, path 1, 2, 5, 8, 11, 13 will become critical again, whereby the duration of stage 8 should undergo reduction. As a result the completion time points will be $C'_4 = 17, C'_7 = 35$, and $C'_8 = 35$. From Fig. 12.3 and Table 12.13, it follows that in this situation there are three critical paths at the same time: 1, 2, 5, 8, 11, 13; 1, 2, 4, 7, 11, 13; and 1, 2, 6, 9, 11, 13.

Obviously, further reducing of the project duration is only possible if it is possible on all three critical paths simultaneously. In this case, in the first of the paths the duration of stage 4 can be reduced, in the second—stage 4, and the third—stage 6. Thereafter, a new critical path 1, 2, 6, 9, 10, 12, 13 appears. Since, however, opportunity to accelerate stage 4 is exhausted, the further process terminates. The result is shown in Table 12.15.

In Table 12.15 p_i is the accepted planned duration in days and c_i is the planned cost of the stage in conventional units. As can be seen from Table 12.15, as a result of optimization stages 4, 5, 6, 8, and 11 underwent acceleration, so the total duration decreased from 63 to 54 days, and the time reserves at the most of stages appeared to be zero. Herewith the total cost of the project is

$$\sum c_i + \ b_0 \times C_{\max} = 641 + 6 \times 54 = 965.$$

The considered problem can be solved by linear programming methods. To do this, we have to find the minimum of expression (Pinedo 2005)

$$b_0 \times C_{\max} - \sum_{i=1}^{n} b_i p_i \tag{12.20}$$

with constraints:

$$x_k - p_i - x_i \geq 0 \text{ for all } i \rightarrow k \text{ and } x_i \geq 0; \tag{12.21}$$

$$p_i \leq p_i^{\max} \text{ and } p_i \geq p_i^{\min} \text{ for all } i; \tag{12.22}$$

Table 12.15 Planned duration and job cost under the project

Job	1	2	3	4	5	6	7	8	9	10	11	12	13
$p_i^{\max}$	5	5	3	8	6	10	18	25	15	2	12	8	10
$p_i^{\min}$	5	5	3	6	5	8	14	18	15	2	10	7	10
p_i	5	5	3	6	5	9	18	19	15	2	10	8	10
$C_i^{'}$	5	10	8	16	15	19	34	34	34	36	44	44	54
$C_i^{''}$	5	10	36	16	15	19	34	34	34	42	44	44	54
Reserve	0	0	28	0	0	0	0	0	0	6	0	0	0
c_i	20	15	9	20	13.2	13.5	114.8	251.2	62	5	64	15.4	38

$$C_{\max} - x_i - p_i \geq 0. \tag{12.23}$$

In expressions (12.21), values x_i are variable time points of beginning of each project stage. The independent variables of the problem are p_i, x_i, and $C_{\max}$. In particular in this example with 13 jobs, the number of variables is $13 + 13 + 1 = 27$. The number of constraints (12.21) is equal to the number of the graph arcs (arrows) in Fig. 12.3, i.e. in this case 18.

The solution of problem (12.20–12.23) exists if at least some of extra pays cost values b_i on the critical path, which ensure the acceleration of the stage by 1 day, are less than the daily overhead b_0. Otherwise, any attempt to reduce the process time will be associated with an increase in total costs, and the problem becomes multi-objective. Such a problem can also be solved, but with the help of methods of the theory of utility and decision-making outlined above in Chap. 4.

12.4 Stability of Planning

The stability of planning has influence on all aspects of the enterprise activity quite remarkably, but, of course, primarily on the production process. The reasons of changes in the developed plans can be divided into primary (independent) and secondary, which are derived from the primary reasons. If we consider the planning and production processes as a single system, it is necessary to consider that it operates in the context of regular exposure to a variety of disturbances that prevent accomplishing the objectives of the system (Fig. 12.5).

In Fig. 12.5 the bold arrows indicate the planned material and information flows in the system, and the thin solid arrows—the disturbances. These disturbances cause changes in the plans, which are automatically transmitted to downstream echelons of the system. If these echelons have reserves sufficient to respond to the disturbances obtained, the information about such disturbances is not transmitted to the echelons of higher level.

Internal reserves of the system, as it follows from Fig. 12.5, consist of time buffer between the material requirement and operations plan and buffer stocks of intermediate products and finished products. If the reserve is not enough, then, as shown by the dashed arrows, the information about the current situation can be transmitted to the next superior level with the request or suggestion of changes in the relevant plans.

Thus, the stability of planning is determined by two groups of factors: (a) external influences and (b) internal reserves. The external factors are random and it is generally believed that their value has a normal distribution. It is obvious that the volumes of internal reserves should be set essentially depending on the amplitude and dispersion of possible changes.

The unstable planning affects the productivity greatly. For example, the study (Pujawan 2004) carried out in the shoe industry showed that the actual changes in the plans of various levels reduce the production output on the shop level by 20 % on average.

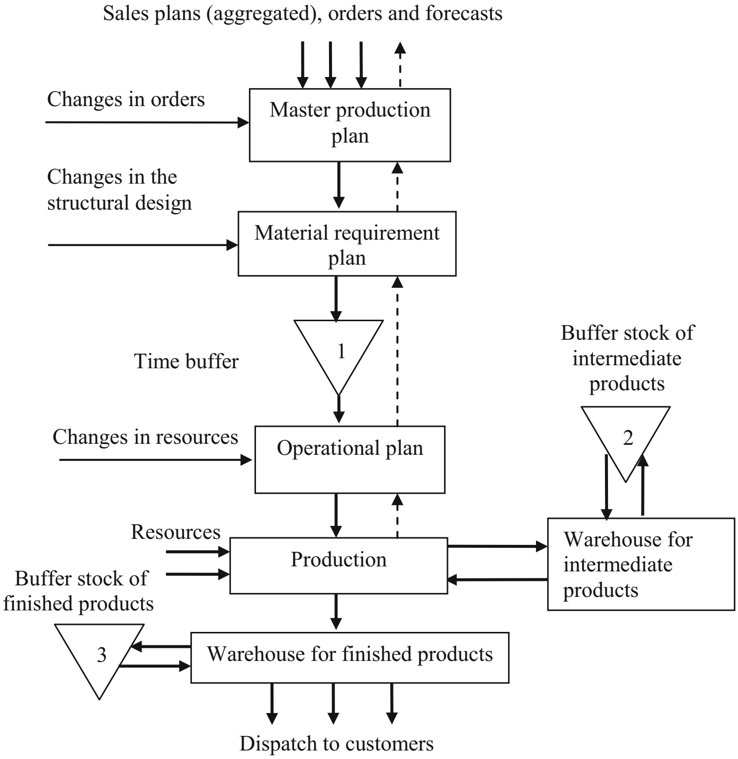

Sales plans (aggregated), orders and forecasts
Changes in orders
Master production plan
Changes in the structural design
Material requirement plan
1
Time buffer
Buffer stock of intermediate products
2
Changes in resources
Operational plan
Resources
Production
Warehouse for intermediate products
Buffer stock of finished products
3
Warehouse for finished products
Dispatch to customers

Fig. 12.5 Material and information flows in the production system

12.4.1 Quantitative Evaluation of Planning Stability

Changes in the production plans, in general, are accompanied by significant additional costs on the very adjustment of the plans, and especially on their implementation. So sometimes in the literature there are attempts to measure the stability of planning in cost values. The book (Heisig 2002), however, shows that the complexity of emerging problems with this approach is too high, and nowadays it is advisable to consider the stability of planning to be an independent technical criteria of quality of the production system.

In the function of this criterion, it is possible to use some concepts that define the stability in many ways. In the theory of dynamical systems, term “robustness” is widely used. In a stable system, small external perturbations cannot cause significant deviations in its normal operation.

When assessing the quality of planning systems performance the so-called nervousness is more often used in the form proposed in Sridharan et al. (1988). Let us consider two series plans with sliding horizon, which at the initial moment of planning equals h. Let the second of these plans (with number j) be made for a set of periods from $t = j$ to $t = h + j$, and, accordingly, the first plan (the plan with number $j - 1$) covers a set of periods from $t = j - 1$ to horizon $h + j - 1$.

In this case, the nervousness for this couple of plans has the form

$$\nu_{j-1} = \frac{1}{n}\sum_{i=1}^{n} \frac{\sum_{t=j}^{h+j-2} w_t \left|Q_{it}^{j-1} - Q_{it}^{j}\right|}{\max\left(\sum_{t=j-1}^{h+j-2} Q_{it}^{j-1}, \sum_{t=j}^{h+j-1} Q_{it}^{j}\right)}, \tag{12.24}$$

where Q_{it}^{j-1}, Q_{it}^{j} is the quantity of the i-th product in period t for the first and second plans accordingly, w_t is the weight factor of a period, and n is the number of product types in the plan.

Expression (12.24) mainly coincides with the similar expression (Sridharan et al. 1988), but expands application of the latter in the case when in some period for one out of two sequential plans, one and the same product is produced, and in the other, it is not produced. Behaviour of the weight factor due to the period of planning is shown in Fig. 12.6 (Schuster et al. 2002).

For reasonable estimate of the planning stability it is necessary to calculate the instability factors for some N of successive values of parameter j, i.e. for $N-1$ of the relevant couples of successive plans, and to define the average nervousness

$$\overline{\nu} = \frac{1}{N}\sum_{j=2}^{N} \nu_{j-1}. \tag{12.25}$$

Let us consider the example of determining nervousness ν_1 for two successive plans prepared for several upcoming periods (weeks) with sliding horizon $h = 8$ weeks. Here we assume that the first of these plans starts from period 1 and the second one starts from period 2, respectively, i.e. $j = 2$ (Table 12.16).

In Table 12.16 number of different products $n = 5$ and according to plan 1 product 5 was not provided at all. The products have different production stability and may be produced in lots with a period more than a week. For example, product

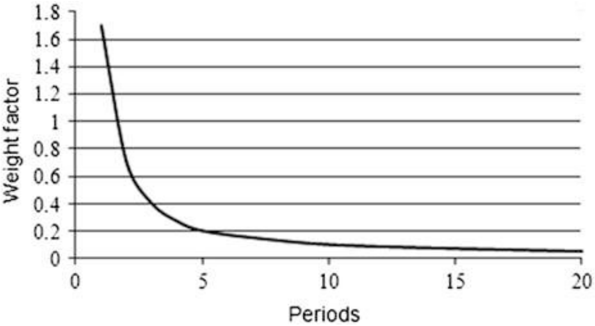


Fig. 12.6 Weight factor of planning periods [based on Schuster et al. (2002)]

Table 12.16 Products output in two compared successive plans

Plan no.	Type of product	Week								
		1	2	3	4	5	6	7	8	9
1	1	100	0	100	0	100	0	100	0	–
	2	50	50	50	50	50	55	55	60	–
	3	0	30	20	20	30	30	40	30	–
	4	65	80	90	90	100	100	70	70	–
2	1	–	100	50	50	100	0	100	0	100
	2	–	50	50	50	50	55	55	60	60
	3	–	30	20	20	40	30	40	30	30
	4	–	90	80	90	100	100	70	70	60
	5	–	0	20	0	0	30	0	20	0

1 is produced in lots of 100 pcs. once in 2 weeks, product 2—50 pcs. stably during 5 weeks, and then its output is planned to be increased. Product 3 is characterized by random fluctuations in demand and product 4—by cyclical. Finally, product 5 is new and the demand for it is not well defined.

For example, let us calculate the component of the nervousness for product 1. The numerator in expression (12.24)

$$\sum_{t=j}^{h+j-2} w_t \left|Q_{it}^{j-1} - Q_{it}^{j}\right| = \sum_{t=2}^{8} w_t \left|Q_{1t}^{1} - Q_{1t}^{2}\right| =$$
$$= (1.7 \times 100 + 0.7 \times 50 + 0.4 \times 50 + 0.27 \times 0 + 0.2$$
$$\times 0 + 0.18 \times 0 + 0.15 \times 0 + 0.13 \times 0) = 225.$$

the common denominator

$$\max\left(\sum_{t=j-1}^{h+j-2} Q_{it}^{j-1}, \sum_{t=j}^{h+j-1} Q_{it}^{j}\right) = \max\left(\sum_{t=1}^{8} Q_{1t}^{1}, \sum_{t=2}^{9} Q_{1t}^{2}\right) = \max(400, 500) = 500.$$

Similarly calculating the components of the nervousness for products 2 ÷5, we obtain

$$\nu_1 = \frac{1}{5} \times \left(\frac{225}{500} + \frac{0}{430} + \frac{2.7}{240} + \frac{24}{665} + \frac{23}{70}\right) = 0.16.$$

12.4.2 Methods of Planning Stability Improvement

Of course, the best way to improve the stability of planning is to reduce urgent changes of any kind disrupting the production process. Generally speaking, the changes in the long-term plans are inevitable and even useful because they reflect the scientific, technological, and social progress. Urgent changes occur in the result of either force majeure or unreasoned decisions, and, for the most part, the latter occurs more often.

Virtually all of the ways to enhance the stability of planning are aimed at damping of disturbance due to urgent changes. There are basically five ways to enhance the stability:

- "Freezing" some part of the master plan;
- Provision of sufficient buffer sizes;
- Efficient sizing of lots;
- Prevention of urgent changes;
- Using of substitutes.

The plan "freezing" consists in abandoning changes to that part of the current plan, implementation of which has already passed the stage of pre-production and blanking operation. In such cases, the production process is usually irreversible already and attempts to introduce changes often lead to the production breakdown. Of course, such freezing has a negative side, because unnecessary or substandard products might be produced. Furthermore, freezing is physically impossible in the event of some resource shortage.

It is natural to use the freezing in group planning. In this case, it makes sense to freeze the plans of components lots launching, because freezing is carried out either after the blanking operations or in process of their performance. At the same time, the plans of output lots of components and assembly units may not be frozen but adjusted. Using the planning group, consisting of both external and internal orders, allows manoeuvring when resizing and even changing of orders due dates by means of transferring some part of products from the external to the internal order and vice versa.

Time buffer 1 in Fig. 12.5 provides some time reserves for workshops, which can be used to adjust operational plans. The availability of material buffers 2 and 3 allows to provide a continuous production process in case of the delays associated with changes in resources.

Reasonable buffer sizing is the main way to maintain stable operation of the production system. To do this, it is advisable to calculate average nervousness of planning $\overline{\nu}$ regularly by the method described in the above paragraph and determine its dependence on the buffer sizes empirically.

In some cases, the plans stability can be improved with proper lot sizing or orders. In the paper (Heisig 2002) the detailed study of the planning stability in the supply chain for the model with fixed order quantity (Sect. 8.2.1) and the two-level model (Sect. 8.2.3) for randomly changing demand was carried out.

Figure 12.7 shows the estimated dependence of panning stability value $1-\overline{\nu}$ on relative lot size Q/D, based on the results in (Heisig 2002), where D is the demand per base forecasting period. As can be seen from Fig. 12.7, in the range of $1 \leq Q/D \leq 3$ the sharp decrease in the planning stability can be observed. Based on these data, the

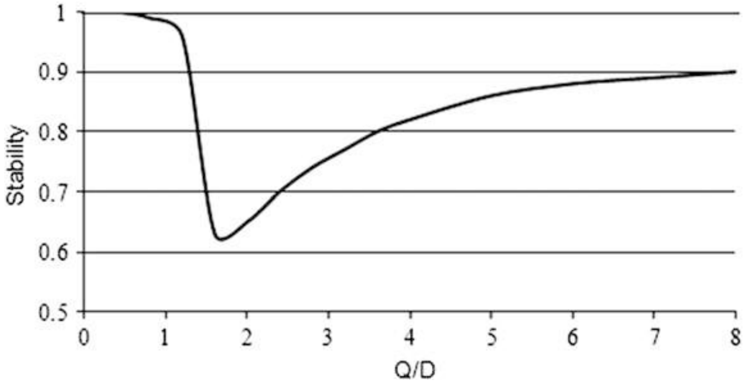


Fig. 12.7 Dependence of planning stability on the relative lot size

order sizing close to the demand for two periods of forecasting should be avoided and rather make use of the orders ensuring meeting the demand for more than three periods.

Methods to prevent urgent changes in supplies fully coincide with methods to reduce fluctuations in supply chains, as described above in Sect. 9.2.3. These methods mainly consist in coordinating the calculation and execution of orders at all echelons of the supply chain, which is achieved by the timely transfer of the necessary information.

As the last way to increase the stability of planning, the use of various substitutes for resources was mentioned. In fact, replacement of one resource by another suitable resource is the object of daily activity of every production manager. A typical example is the selection of the analogue to the product ordered, given above in Sect. 10.4.2.

Generally speaking, the process of using substitutes can be proactive or reactive. For the most part, of course, the substitution of resource is a manager's reactive response to the occurred deviations from the planned course of the production process. In such cases, it is necessary to act quickly and rely mainly on experience and intuition of the production manager.

In many cases, however, the need and possibility of substituting the resource can be anticipated and performed as a preventive measure. For example, when the possibility of replacing the material in the specification by some other material or even a few different materials is known in advance, it is necessary to check the relevance of materials purchasing, considering the available stock of the materials substituting the required one.

This study (Lang 2009) addresses the issue of optimal production lots in the presence of similar product inventory, which can replace the planned products under certain conditions (rework). This problem is significant for the design and technological changes, when the cost of rework of the available stock is comparable to the cost of production of the modified product. For such a decision (Lang 2009) the statements of the problems, described above in Sects. 11.1.2–11.1.4, are used for optimal lot sizes in the presence of constraints; for this purpose the objective functions take into account the rework cost of the product substituting the planned one.

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13 Shop Floor Scheduling: Single-Stage Problems

13.1 Single-Machine Scheduling with Minimized Overdue Penalties

The problem of planning is called single stage, if every job can be fully executed on one of the available machines. In the simplest case, the production system consists of a single machine, on which all planned jobs can be performed. The schedule for such a machine is a sequence of execution of the jobs already received, as well as the jobs that may arrive in the near future.

An elementary or basic version of the problem is based on the following seven assumptions:

(a) All of the planned jobs are equally available at the beginning of the planning.
(b) Only one work is simultaneously executed.
(c) Machine setup time does not depend on the order of execution of jobs and is included in the duration of this job.
(d) Duration of each job is determined and known.
(e) The machine is available and serviceable at any time.
(f) Machine idle time is not allowed if there is a job to be performed.
(g) Each job is executed to the end without interruption for another job.

In accordance with the classification (2.13) above in Sect. 2.2.2, in different cases of scheduling for a single machine the problem formula has the form

$$1|\beta|\gamma, \tag{13.1}$$

where the constraints of various types are recorded in field β. They are superimposed on the dates of job start and end. The designation of the objective function is recorded in field γ. To optimize the schedule of the machine in the described variant of organization, various criteria, described in Sect. 2.2.2, can be used, except for makespan $C_{\max}$,

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Table 13.1 List of order

Order no.	1	2	3	4	5
Processing time	3	5	4	2	6
Due date	3	8	10	11	15

defined by the total time of completion of all scope of scheduled jobs, as this time is just equal to the total of processing time of all jobs.

13.1.1 Schedule with the Minimum of Delayed Jobs

Assume that at the moment of planning there are several orders, for execution of each of which processing time p_i is required. Each order must be fulfilled by relevant due date d_i (Table 13.1).

In Table 13.1 the orders are sorted in order of the required due date. It is necessary to schedule the operation of the machine so that the quantity U of the orders to be fulfilled with a delay regarding to desired date is minimal. Since there are no additional restrictions, the classification formula of the problem in this case has the form

$$1\big|\ \big|U_{\min}. \tag{13.2}$$

Just as above (Sect. 2.3.1), we will write the sequence of jobs execution in the form of a set of due dates, and in parentheses we will indicate the number and processing time of a job. For the first job, we have a sequence of 0 (1/3) 3 and the required execution date $d_1 = 3$ is observed. Similarly, after the second job the sequence has the form 0 (1/3) 3 (2/5) 8, which also provides timely execution of the job $d_2 = 8$.

However, while executing the third job its execution date is violated because here is the following sequence 0 (1/3) 3 (2/5) 8 (3/4) 12 and date $d_3 = 10$. It is proved that the best way to achieve a minimum of delays in such cases is to move with the job of the greatest processing time to the end of the planned sequence. In this example, job 2 with the greatest processing time among already considered jobs 1, 2, 3 should be excluded from the initial sequence. Continuing preparation of the planned sequence, we obtain 0 (1/3) 3 (3/4) 7 (4/2) 9 (5/6) 15 (2/5) 20. As a result, the number of jobs performed with delay is minimized to one. If the sequence of order execution was maintained in accordance with the required date, then it is easy to determine that the number of delays would be equal to three.

13.1.2 Scheduling with Minimum Weighted Tardiness per Each Job

Application of the schedule with the lowest number of delayed jobs can lead to a situation where the execution of jobs with quite high processing time will be delayed for a long time. If the penalty for non-fulfilment is proportional to the delay time, then such failure can cause significant losses. Therefore, during scheduling, optimization of weighted delays for all planned jobs is used. In this case, the criterion is the sum of all n jobs $T^w = \sum_i^n w_i T_i$ delays of each job T_i with weight w_i (2.18), and the corresponding classification formula of the problem has the form

$$1\big|\ \big|T^w. \tag{13.3}$$

The solution to this problem by various priority rules is given above in Sect. 2.3.1 and the example of the precise solution using the branch-and-bound method in Sect. 2.6.1. Analysis of the solutions obtained using the priority rules indicates that such solutions are often far from optimal values; At the same time, the use of the branch-and-bound method with increasing dimension of the problem leads to sharp increase of calculations volume. Therefore, the heuristics for finding the optimal solution is of great significance. Here we present the method described in Sule (2007), which is characterized by fair simplicity and reliability. As an example, we use Table 2.8, which is repeated for convenience as Table 13.2.

The described method consists of two planning stages: "backward" and "forward" planning. On the first stage, total processing time p is calculated and the job with minimal penalty for delaying is defined. This job is accepted as the last one; then the procedure is repeated for the remaining jobs.

In this case for all planned jobs

$$p = \sum_{i=1}^{4} p_i = 20 + 25 + 4 + 18 = 67.$$

If the job is executed as the last one, then it leads to the following penalties.

For job

1: $(67 - 42) \times 1 = 25;$
2: $(67 - 31) \times 2 = 72;$
3: $(67 - 4) \times 1 = 63;$
4: $(67 - 26) \times 2 = 82.$

Since the smallest penalty is observed for job 1, then it is chosen as last. Now, among the remaining jobs we define the job, for which the penalty is minimal. Similar

Table 13.2 List of jobs for planning

Job number i	Processing time in days p_i	Due date d_i	Weighting factor of delay w_i
1	20	42	1
2	25	31	2
3	4	4	1
4	18	26	2

Table 13.3 List for scheduling with modified weighting factors

Job no.	Processing time in days	Due date	Weighting factor of delay
1	20	42	1
2	25	31	5
3	4	4	1
4	18	26	5

to the above, the total of processing time of the remaining jobs is 47; penalty for job 2: $(47-31)\times 2=32$; for job 3: $(47-4)\times 1=43$; for job 4: $(47-26)\times 2=42$. The smallest penalty turns out to be for job 2.

For the remaining two jobs, the total processing time is 22; penalty for job 3: $(22-4)\times 1=18$; for job 4: $(22-26)\times 2=-8$; and the lowest penalty is for job 4. Thus, as a result of "backward" planning stage we have a sequence of scheduling as $\{3, 4, 2, 1\}$. In Sect. 2.6.1, using branch-and-bound method it was proven that this very sequence was optimal in this case. Therefore, for this example, the "backward" planning stage is sufficient to find the optimal solution.

Let us consider the variant of this problem but with other weighting factors (Table 13.3).

While performing the backward planning stage we still find that job 1 should be scheduled as the last one with a penalty amounting to 25. For the remaining three other jobs, the situation, however, changes. Indeed, the penalty for job 2 will be: $(47-31)\times 5=80$; for job $3(47-4)\times 1=43$; for job 4: $(47-26)\times 5=105$. The smallest penalty turns out to be for job 3.

Now, for remaining jobs 2 and 4, the total processing time is equal to 43; penalty for job 2: $(43-31)\times 5=60$; for job 4: $(43-26)\times 5=85$; and the lowest penalty is for job 2. As a result of "backward" planning stage the sequence is as follows: {4, 2, 3, 1}, and since the penalty for the job 4 performed in time is 0, the total penalty is $60+43+25=128$.

Let us verify the possibility to reduce this penalty by forward planning. To do this, we apply the algorithm of permutation with variable "lag" (Sule 2007). "Lag" k refers to the difference between position numbers of the jobs in the sequence. For example, in this case the number of positions $N=4$ and therefore the greatest possible value $k=3$.

The first possible permutation is performed with a maximum lag, and in the sequence {4, 2, 3, 1} found at the first stage there is only one such permutation: between jobs 4 and 1. This permutation leads to sharp increase in the penalty and therefore is impractical.

Let us lower the lag value down to $k=2$. For this lag, two permutations are possible: jobs 4 and 3, as well as jobs 2 and 1. In the first case we obtain:

Job sequence:	0	(3/4)	4	(2/25)	29	(4/18)	47	(1/20)	67
Due date:		(3)	4	(2)	31	(4)	26	(1)	42
Delay:			0		–2		21		25
Value of objective function	$T^w=1\times 0+5\times 0+5\times 21+1\times 25=130.$								

It is obvious that permutation of jobs 4 and 3 is ineffective; the similar result is obtained for jobs 2 and 1 as well.

Let us set lag $k = 1$. Now permutation is possible for jobs 4 and 2; 2 and 3; 3 and 1.

We will consider permutation of jobs 2 and 3:

Job sequence:	0	(4/18)	18	(3/4)	22	(2/25)	47	(1/20)	67
Due date:		(4)	26	(3)	4	(2)	31	(1)	42
Delay:			−8		18		16		25
Value of objective function	$T^w = 5 \times 0 + 1 \times 18 + 5 \times 16 + 1 \times 25 = 123.$								

The obtained result for sequence {4, 3, 2, 1} appeared to be better that the result for initial sequence {4, 2, 3, 1} with penalty equal to 128.

Heuristic algorithm (Sule 2007) suggests to accept this enhanced sequence as a starting point for repeated forward planning. To do this, set the maximum lag $k = 3$ again, test the practicability of possible permutation, and then gradually reduce the lag down to $k = 1$ to ensure no reduction of the penalty by searching through all the emerging opportunities.

In this case we will verify option {3, 4, 2, 1}, which takes place with lag $k = 1$ and permutation of jobs 4 and 3:

Job sequence:	0	(3/4)	4	(4/18)	22	(2/25)	47	(1/20)	67
Due date		(3)	4	(4)	26	(2)	31	(1)	42
Delay:			0		–4		16		25
Value of objective function	$T^w = 1 \times 0 + 5 \times 0 + 5 \times 16 + 1 \times 25 = 105$, which is the optimal value.								

The described algorithm works effectively with a large number of jobs up to 100. It should be noted that when using only the backward planning stage without its confirmation at the forward stage, you can get good approximation to the optimal value, especially when there is no big difference in weighting factors between jobs.

13.1.3 Schedule Optimization with Earliness/Tardiness

Let us consider scheduling for the example (Table 13.4), where the values of penalties are set both for delaying and too early completion of a job. The problem of this type occurs when applying Just-In-Time scheduling, which does not allow accumulation of significant stocks.

The total processing time of all jobs in Table 13.4, as well as in two previous tables, is still 67, but unlike them the execution period of jobs 3 and 4 is extended. In addition, in Table 13.4 the penalties for early and late production are set.

The most significant in Table 13.4 is the fact that the execution period for job 4 is higher than the total processing time of all the jobs. Therefore, there is no point in

Table 13.4 List for planning allowing for penalties for deviations from the due date

Job no.	Processing time in days	Due date	Weighting factor of early production	Weighting factor of late production
1	20	42	1	1
2	25	31	0	2
3	4	14	3	4
4	18	76	3	4

doing this job earlier than others from the list, and obviously it must be the last in the schedule.

Now we can carry out the backward planning stage for the first three jobs in Table 13.4. For them, the total processing time is equal to 49; penalty for job 1: $(49 - 42) \times 1 = 7$; for job 2: $(49 - 31) \times 2 = 36$; for job 3: $(49 - 14) \times 4 = 140$; and, obviously, in the third place in the schedule job 1 should be.

For remaining jobs 2 and 3 the total processing time is equal to 29; penalty for job 2: $(31 - 29) \times 0 = 0$; for job 3: $(29 - 14) \times 4 = 60$; and the smallest penalty is for job 2. As a result, we have sequence {3, 2, 1, 4}:

Sequence of jobs:	0	(3/4)	4	(2/25)	29	(1/20)	49	(4/18)	67
Due dates:		(3)	14	(2)	31	(1)	42	(4)	76
Delay:			−10		−2		7		–9
Value of objective function	$T^w = 3 \times 10 + 0 \times 2 + 1 \times 7 + 3 \times 9 = 66.$								

In the problems of this type, the backward planning stage often gives the result close to the optimal one (Sule 2007) and usually there isn't need for forward planning.

13.2 Common Shipment Date Scheduling

In many cases, it is useful to prepare the schedule for one machine so as to minimize the costs associated with partition of the entire set of jobs into multiple shipping groups. In particular, if the work is carried out for one customer, the latter can fix specific dates or range of time for shipments. If this date should be earlier than the required total processing time, there should be at least two shipments.

In cases where the delivery to different customers can be made by one vehicle, it is important to fix the time of shipment, when the costs of the delay or, vice versa, early delivery may be reduced as much as possible.

13.2.1 Fixed Date Schedule Optimization

In the simplest case, it is necessary to determine the sequence of given set n of jobs on a single machine, which the customer wishes to receive by a certain date. Herewith any

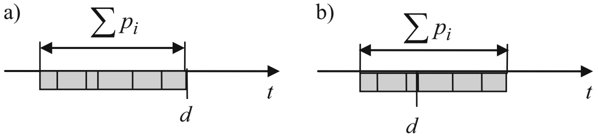


Fig. 13.1 Schedule options with fixed date: (**a**) completion of all set by the fixed date; (**b**) completion of one part of the set before the fixed date, and the other part—after the fixed date

deviation of performance from the fixed date is undesirable and equally penalized for both early and delayed fulfilment.

Figure 13.1 shows two options of execution of the job set with total processing time $\sum p_i$ in regard to fixed date d.

Directly from Fig. 13.1 it follows that the smallest penalty will occur with option b), as the job completion points will be closer to the fixed date, than in option (a). Thus, we can assume that in the considered problem the optimal sequence of jobs execution contains two sets A and B, separated by the fixed date.

There are the following properties of optimal sequences (Baker and Trietsch 2009):

- Optimal sequence should not contain breaks in operation;
- Jobs A, completed before the fixed date, must be positioned according to the rule of the longest processing time LPT (Sect. 2.3.1);
- Jobs B, completed after the fixed date, must be positioned according to the rule of the shortest processing time SPT (Sect. 2.3.1);
- In the optimal sequence, one of the points of job completion matches with the fixed date;
- Quantity of jobs as for the optimal sequence of A is $n/2$, if n is even, and $n/2 + 1/2$, if n is odd.

Let us consider the example (Table 13.5), where the jobs are sorted by processing time increase and fixed date $d = 22$.

The job with the greatest processing time is advisable to include into set A at once. In each successive by reducing processing time pair of jobs, one job belongs randomly to set A, and the other—to set B. The latter job refers to a set with smaller number of jobs. Resulting set A is sorted in processing time descending order and set B in ascending order.

Assume, for example, that at the second step we include job 5 into set B, and job 4—to set A; similarly, at the third step, job 3—to set B, job 2—to set A; finally, we refer job 1 to set B. By sorting in the above order we obtain sequence $\{6, 4, 2, 1, 3, 5\}$.

Obviously job 2 is the last in set A, and according to the above mentioned properties of optimal sequences, its completion has to coincide with fixed date $d = 22$, and for the obtained sequence we obtain the following calculation results:

Table 13.5 List of jobs

Job number	1	2	3	4	5	6
Processing time	2	4	4	5	6	8

$$5\quad(6/8)\quad 13\quad(4/5)\quad 18\quad(2/4)\quad 22\quad(1/2)\quad 24\quad(3/4)\quad 28\quad(5/6)\quad 34.$$

Thus, execution of jobs should be started in day 5 and finished in day 34.

13.2.2 More Complex Cases of Scheduling with Fixed Date

The above result refers to the case when the amount of penalties for too early execution of jobs α is the same as the amount of penalties for delay of jobs β. If this match is absent, the position of the total range of time for execution of job set shifts. According to Baker and Trietsch (2009), in this case, the number of jobs as for optimal sequence A is $Int(n\beta/(\alpha+\beta))+1$, where $Int()$ is the integer part.

For example, assume that in Table 13.5 penalties are $\alpha = 3$, $\beta = 4$. Then the number of jobs in set A should be $Int(6\times 4/(3+4))+1=3+1=4$. In this case in the obtained sequence $\{6, 4, 2, 1, 3, 5\}$ not job 2 but job 1 should match fixed date $d = 22$. As a result, we obtain the following schedule:

$$3\quad(6/8)\quad 11\quad(4/5)\quad 16\quad(2/4)\quad 20\quad(1/2)\quad 22\quad(3/4)\quad 26\quad(5/6)\quad 32.$$

If the fixed date is such that the start of job, determined by the algorithm described in Sect. 13.2.1, is negative, the problem of determining the optimal solution becomes much more complicated, and in this case, it is advisable to use some heuristic solution. We assume that the execution of jobs in this case will begin at the time point equal to 0 (Fig. 13.2).

In Fig. 13.2, value K represents duration of jobs, which can be executed before fixed date d, and value S is the duration of the remaining jobs. It is obvious that $K+S=\sum p_i$. Let us consider the example with data of Table 13.5, but with fixed date $d=12$. Herewith at the starting point $K=d=12, S=\sum p_i-K=29-12=17$.

In the paper (Baker and Trietsch 2009), the following algorithm is suggested:

(a) The data are considered starting with the greatest processing time in descending order.
(b) If $K > S$, the job is put on the closest allowable position, and value K is reduced by the relevant processing time.
(c) If $K \leq S$, the job is put on the last allowable position, and value S is reduced by the relevant processing time.

As an example, the jobs in Table 13.5 are considered starting with the last one. Since in the beginning $K = 12 < S = 17$, then job 6 should be executed last. By reducing S by the processing time of job 6, we obtain new value $S = 17 - 8 = 9$. As

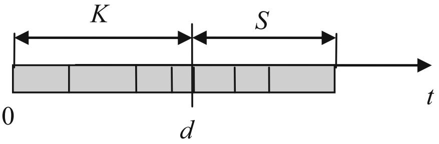


Fig. 13.2 Schedule when starting from zero

Table 13.6 Jobs execution sequencing

Job number	Processing time	K	S	Sequence
6	8	12	17	XXXXX6
5	6	12	9	5XXXX6
4	5	6	9	5XXX46
3	4	6	4	53XX46
2	4	2	4	53X246
1	2	2	0	531246

now $K = 12 > S = 9$, job 5 should be placed in the first place of the sequence, and accordingly new value $K = 12 - 6 = 6$. The further operations can be well traced using Table 13.6.

13.2.3 Selection of Optimal Midpoint Date for Shipping

It is often necessary to execute a particular set of jobs that must be delivered to the same address, but at the same time for each of the jobs a penalty can be set both for delays and too early delivery. Typically, in such cases, it makes sense to divide the entire set of jobs into parts, each of which must be ready by a certain estimated deadline. Optimal definition of this time by minimum costs is a very challenging problem, so different heuristic algorithms are used for its solution.

Let us consider a set of eight jobs (Table 13.7), for which we assume that potential penalties are defined by time counted from the common moment of shipping.

To solve the problem (Sule 2007), it is suggested to build the so-called initial sequence. For this purpose, first of all, we find the minimal penalty of all for early production or delay. If this penalty refers to early production, the corresponding job is placed in the first possible place in the developed sequence; otherwise, this job is placed in the last possible place. In cases where for multiple identical jobs the penalty is the same, then first the job with the greatest processing time is placed into the sequence. This process continues until the complete positioning of all the jobs in the set.

In this case, the least penalties, equalling to 1, have a place for jobs 3 and 5, and they are penalties for early production. As the processing time of job 3 is more than job 5, then job 3 is placed in the first place of the developed sequence, and job 5—in the second. After these jobs, obviously, jobs 4 and 1 should follow with penalties 2 for early shipment in descending order of their processing time.

Table 13.7 Input data for defining the shipping date

Job	1	2	3	4	5	6	7	8
Processing time	15	5	14	16	10	12	3	13
Penalty for early production	2	4	1	2	1	4	5	4
Penalty for delay	5	6	4	5	4	3	5	7

The next penalty in order of size equals 3 and is a penalty for delay; it refers to job 6. It is this job, in accordance with the above algorithm, should be the last in the developed sequence.

For sequencing of jobs 8 and 2, we take into account that their minimum penalty equals 4 and is a penalty for early production, and since the processing time of job 8 is more than of job 2, job 8 is placed before job 2. Remaining job 7 is automatically between jobs 2 and 6. As a result, we obtain sequence $\{3, 5, 4, 1, 8, 2, 7, 6\}$ and schedule

$0(3/14)14(5/10)24(4/16)40(1/15)55(8/13)68(2/5)73(7/3)76(6/12)88.$

Assume that the cost of one shipping corresponds to the penalty of 100 units. Recall that the objective of the considered problem is to define optimal moments for shipping. If all the manufactured products are dispatched together, then it will have to occur at time moment 88 with the total cost

$$(88 - 14) \times 1 + (88 - 24) \times 1 + (88 - 40) \times 2 + (88 - 55) \times 2 + (88 - 68) \times 4 + (88 - 76) \times 5 + 100 = 510.$$

In the case when it is decided to send the products in two parts, the cost associated with transportation doubles. Since the shipping of products, obviously, makes sense only upon completion of one of the jobs, then to find the optimum point of shipping it is necessary to calculate consistently the cost of each of those moments. In particular, at time point 76 we will obtain

$$(76 - 14) \times 1 + (76 - 24) \times 1 + (76 - 40) \times 2 + (76 - 55) \times 2 + (76 - 68) \times 4 + (88 - 76) \times 3 + 2 \times 100 = 496$$

and such partition can obviously give some effect.

Let us consider the shipping at time point 68:

$$(68 - 14) \times 1 + (68 - 24) \times 1 + (68 - 40) \times 2 + (68 - 55) \times 2 + (73 - 68) \times 6 + (88 - 68) \times 3 + 2 \times 100 = 470$$

and the efficiency of the partition increases.

At time point 55 the total cost of penalties and the shipping is

$$(55-14)\times 1+(55-24)\times 1+(55-40)\times 2+(68-55)\times 7+(73-55)$$
$$\times 6+(88-55)\times 3+2\times 100=600,$$

which is obviously not acceptable.

Finally, we find that in the circumstances it makes sense to divide the entire volume of products into two parts, the first of which must be shipped at time point 68, and the second one—at time point 88.

13.3 Some Other Scheduling Problems for Jobs with Fixed Processing Time

The literature contains enormous number of various problems for a single machine. Here we will discuss only several widespread of them.

13.3.1 Schedules for the Case of Several Jobs, the Part of Which Has the Preset Sequence

In some cases, especially in the chemical industry, one and the same machine can be used for production of several different products; herewith any of them may be a product of processing others, previously produced. Referring, for example, to chemical production model described above in Fig. 7.4 in Sect. 7.3.1. In this model, two reactors are used, where three different reactions can be performed, reaction 2 can be carried out only with the product produced by reaction 1. Assuming that at a time only one of the reactors is involved, then its relation between possible jobs has the form of the graph shown in Fig. 13.3.

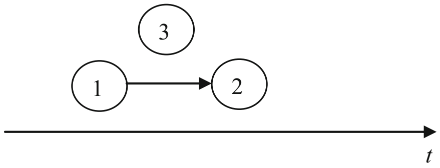
From Fig. 13.3 it follows that job 3 is not related to jobs 1 and 2 and may be executed at any time point t. At the same time job 2 can only be executed strictly after job 1.

Usually in this kind of problems the minimal costs are achieved associated with late completion of jobs. The structural formula of type (13.1) for such cases has the form

$$1|prec|f_{\max}, \tag{13.4}$$

where symbol *prec* means presence of precedence constraints, and objective function $f_{\max}$ is focused to achieving the minimum, for example, the highest penalty for late completion of jobs. In this case, structural formula (13.4) can be replaced by formula

Fig. 13.3 Graph of reactions (jobs) relations for one machine (reactor) in Sect. 7.3.1



$$1\left|prec\right|T_{\max}, \tag{13.5}$$

where objective function $T_{\max}$ must provide the minimal tardiness of each of the jobs.

Problem (13.3) or (13.4) can be solved using the so-called the Lawler algorithm (Lawler 1973). Let us consider this solution as exemplified in www.slidefinder.net. Assume that six computers have to be repaired at the service centre. The computers, received in order 1, 2, and 3, belong to the accounting office and must be repaired strictly in the order they were handed in for repair. Of the three other computers belonging to other departments, computer 4 is a server for computers 5 and 6 and therefore must be repaired before the others. The graph of relations in this case has the form shown in Fig. 13.4 and input data to solve the problem are given in Table 13.8.

At the first step of the algorithm, the total processing time of all jobs $p = 24$ days. Then set of jobs J is generated, which should not precede any other jobs—a set of "leaves" of the relations graph tree. It can be seen in Fig. 13.4 that such "leaves" are jobs 3, 5, and 6.

With completion of all the jobs, the last one will always be one of jobs set J. Therefore, to achieve the minimal delay for all jobs, it is necessary to find an element of set J, the delay of which has a minimum value, and accept it as the last to be completed. In this case, if the last job is job 3, then its tardiness $T_3 = p - d_3 = 24 - 15 = 9$. Similarly, $T_5 = 24 - 17 = 7; T_6 = 24 - 20 = 4$. It is obvious that work 6 should now become the last one to be executed.

Now we eliminate job 6 from the consideration. Herewith value p is reduced by the value of processing time of job 6 and becomes equal to 20. It is obvious that now only jobs 3 and 5 will be free "leaves" of the tree in Fig. 13.4. Arguing as above, we find that the smallest delay occurs for job 5, and it is $T_5 = 20 - 17 = 3$. Therefore, job 5 must be done before job 6.

After excluding job 5 from consideration, remaining processing time $p = 17$, and jobs 3 and 4 become "leaves" of the graph. For job 4, tardiness $T_4 = 17 - 12 = 5$, and for job 3, tardiness $T_3 = 17 - 15 = 2$. Therefore, job 3 must be done after job 4.

By excluding job 3, we get jobs 4 and 2 and total processing time $p = 12$ as "leaves". The minimum delay is the delay of job 4, which is $T_4 = 12 - 12 = 0$, and thus, job 2 must precede job 4. Finally, we obtain the optimal sequence of the form $\{1, 2, 4, 3, 5, 6\}$.

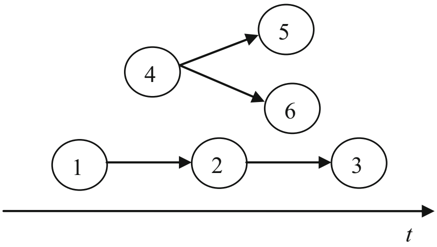


Fig. 13.4 The graph of relations in case of computer repairing

Table 13.8 List of orders for repair

Computer no.	1	2	3	4	5	6
Processing time	3	2	5	7	3	4
Due date	4	9	15	12	17	20

13.3.2 Scheduling of Jobs with Different Arrival Time

If the jobs to be executed on the machine are available at different times, i.e. upon receipt of materials or work pieces, then the optimization problem by any criteria becomes difficult. In these cases, mainly heuristic methods are used to optimally load the machine with those jobs that are possible at the time of the machine release from the previous job or a group of such jobs. Let us consider the example, the data for which are shown in Table 13.9.

Suppose that at the initial moment of planning the machine is free. In this case, job 1 can be started first, which arrives at time point 2. In the absence of external causes, this job can be completed at time point $C_1 = r_1 + p_1 = 2 + 5 = 7$. Since this execution date is less than the required execution date $d_1 = 9$, the penalty for delay of job 1 is absent. Thus, the initial part of the planning sequence has the form

$$2 \quad (1/5) \quad 7\,.$$

At this point, all necessary materials for the job 2 have to be near the machine, and other works are still non-available. Obviously, the start point of job 2 in this case is defined as $max(C_1, r_2) = max(7, 5) = 7$. Job 2 can be completed at time point $C_2 = C_1 + p_2 = 7 + 7 = 14$ with a penalty for delay $(C_2 - d_2) \times c_2 = (14 - 12) \times 1 = 2$. The sequence of planning takes the form

$$2 \quad (1/5) \quad 7 \quad (2/7) \quad 14\,.$$

By time point 14 it appears to be possible to execute three jobs 4, 5, and 6 at once on the machine. Since there are penalties for delay, then the most reasonable way to select the sequence of these jobs is to use the algorithm outlined above in Sect. 13.1.2.

When performing backward planning stage of this algorithm first we define the total processing time p for all possible jobs. In this case, these are jobs 4, 5, and

Table 13.9 Input data of job set

Job no.	1	2	3	4	5	6
Processing time p_i	5	7	10	5	3	4
Due date d_i	9	12	25	16	17	20
Expected arrival date r_i	2	5	15	11	10	12
Penalty for delay c_i per day	1	1	2	1	2	1

6, accordingly, $p = 5 + 3 + 4 = 12$, and the completion time point of these jobs will be $14 + 12 = 26$. Let us determine the possible penalties for delay of these jobs.

If job 4 is the last one of them, the possible penalty will be $(26 - 16) \times 1 = 10$; for job 5, the penalty will be $(26 - 17) \times 2 = 18$; for job 6, the penalty will be $(26 - 20) \times 1 = 6$. Obviously, job 6 will be the last, and before it job 4 is executed, and job 5 must be the first to start with. Considering performance of job 5, we have the following sequence

$$2 \quad (1/5) \quad 7 \quad (2/7) \quad 14 \quad (5/3) \quad 17\,.$$

By time point 17 all remaining jobs 3, 4, and 6 will be available. Their total processing time will make $p_\Sigma = 10 + 5 + 4 = 19$, and completion of the entire set of jobs may occur at time point $17 + 19 = 36$. If job 3 is the last on, then its penalty will be $(36 - 25) \times 2 = 22$; for job 4, the penalty is $(36 - 16) \times 1 = 20$; for job 6, we have $(36 - 20) \times 1 = 16$. Obviously, job 6 should be the last and job 4 the last but one. Eventually, we have the following sequence:

$$2 \quad (1/5) \quad 7 \quad (2/7) \quad 14 \quad (5/3) \quad 17 \quad (3/10) \quad 27 \quad (4/5) \quad 32 \quad (6/4) \quad 36\,.$$

13.3.3 Scheduling of Jobs with Different Arrival Time and Different Shipment Time

In this problem, the delivery is made immediately after processing. Since the delivery time is included in the period of jobs execution, the overall jobs execution time is determined by the last moment of delivery. In the paper (Carlier 1982), it was suggested to consider each job as consisting of three steps: arrival of the job at the machine, processing on the machine, and delivery to the consumer. It is assumed that the first and third steps are provided by unlimited resources, and the second one requires certain throughput.

In this case, each i-th job is characterized by three parameters: moment of arrival r_i, processing time p_i, and delivery time q_i. These parameters are often called head, body, and tail (HBT) of a job, and the objective function of this problem is to minimize makespan $C_{\max}$.

To solve HBT of the problem, the study (Carlier 1982) has developed a heuristic algorithm which among several possible jobs chooses the one that has the largest tail (LT). Let us consider LT job of the algorithm in terms of the example with the data presented in Table 13.10.

Table 13.10 Input data of the job set

Job no.	1	2	3	4	5	6
Processing time p_i	4	3	5	2	6	4
Expected arrival date r_i	0	1	4	0	3	6
Delivery time q_i	9	12	15	16	10	11

At time point $t = 0$ jobs 1 and 4 are available. Since delivery time q_4 is greater than q_1, then job 4 is selected to be the first in order in step 1. By the end point of this job, equalling to 2, jobs 1 and 2 are available. In the next step 2, job 2 is selected with delivery time 12. The sequence after the second step has the form

$$0 \quad (4/2) \quad 2 \quad (2/3) \quad 5\,.$$

By time point 5, non-executed jobs 1, 3, and 5 are available and the greatest delivery time is for job 3. Proceeding with the algorithm until all the jobs are included into the sequence we obtain:

Sequence	0	(4/2)	2	(2/3)	5	(3/5)	10	(6/4)	14	(5/6)	20	(1/4)	24
Delivery time points		4	18	2	17	3	22	6	25	5	30	1	33

The total duration value, obtained in the result of applying LT algorithm, is

$$C_{\max} = r_l + \sum_{n=1}^{n(k)} p_{i(n)} + q_k \tag{13.6}$$

where l is the number of the job, which the sequence starts from, and k is the number of the job called critical, and n is the sequence number of the job in the sequence. The critical job is the job, for which the delivery time point has the greatest value. In this case, the launch starts with job 4 and the latest delivery is made for job 1 and thus $l = 4$, $k = 1$, $n(k) = 6$.

It is proved that in the case, when $q_k \le q_i$ for all i within l to k, the solution is optimal, i.e. $C_{\max}$ equals the minimal possible value. In the presented example, $q_k = q_1 = 9 < q_i$ for any $i \neq 1$, and accordingly in this case we have obtained the optimal result.

13.3.4 Job Sequence-Based Setup Time Scheduling

If the changeover time depends on the jobs execution order, the makespan $C_{\max}$ is no longer equal to the sum-total processing time of jobs and also depends on this sequence. In such cases, $C_{\max}$ may be used as the objective function. Table 13.11 shows an example of time standards for machine changeover from one product to another.

Table 13.11 Time standards for machine changeover for different jobs

	Following job					
Preceding job	1	2	3	4	5	6
1	0	5	6	4	6	3
2	8	0	3	2	4	7
3	4	2	0	6	6	4
4	5	3	4	0	3	2
5	3	7	3	2	0	8
6	5	4	3	4	6	0

An exact solution for the problem of minimum $C_{\max}$ for the job set in Table 13.11 can be obtained by searching through all possible sequence options. If the number of jobs is n, the number of such permutations is $n\,!$, of course. For example, for six jobs in Table 13.11 this number equals $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$. Generally speaking, with modern computer technologies, calculation of this and even much more quantity of options is not very difficult.

However, given the approximate nature of all planning calculations, as well as the possible error of time standards, it makes sense not to search the exact solution of the problem, but to confine oneself to calculation of several possible options, using some heuristic algorithm.

The most common algorithm of this kind is consistent selection of each following job, the changeover for which takes the least time. Suppose, for example, the initial job is job 1. In the line of Table 13.11 corresponding to job 1, the least changeover time is for job 6 and equals 3 units.

If we put job 6 on the second place of the planned sequence, then looking at the relevant line of the Table 13.11, we see that the least time standard corresponds to job 3. Transferring to job 3, we note that the shortest possible transition is possible to job 2. Similarly, for job 2, the transition to job 4 is the closest. From job 4, job 6 gives the shortest distance in time, but it has been included in the planned sequence earlier. So after job 4, we should transfer to the only job 5, not included in the sequence.

Eventually, beginning from job 1, we have heuristic sequence {1, 6, 3, 2, 4, 5} with time for the changeover $\sum_{i=1}^{n} s_i = 3 + 3 + 2 + 2 + 3 = 13$.

If we start the planned sequence with another job we may obtain:

{2, 4, 6, 3, 1, 5} with timing $\sum s_i = 2 + 2 + 3 + 4 + 6 = 17$;
{3, 2, 4, 6, 1, 5} and $\sum s_i = 2 + 2 + 2 + 5 + 6 = 17$;
{4, 6, 3, 2, 5, 1} and $\sum s_i = 2 + 3 + 2 + 4 + 3 = 14$;
{5, 4, 6, 3, 1, 5} and $\sum s_i = 2 + 2 + 3 + 4 + 6 = 17$;
{6, 3, 2, 4, 5, 1} and $\sum s_i = 3 + 2 + 3 + 3 + 3 = 14$.

It is obvious that the first option is the best one with the total time equalling 13.

13.4 Periodic Scheduling with Lots of Economic Sizes

When operating in a stable market and with considerable production scope, as shown above in Sect. 1.3.2, usually "make-to-stock" strategy is commonly used. In cases when several types of products are produced on a single machine, the schedule of the machine operation must provide the necessary stock for each product. This kind of schedule often tends to be repetitive, which greatly simplifies the preproduction.

Since the production of this type is costly both for machine setup and storage of finished products, the question arises about the optimal lot size of each product. In contrast to the problems of the lot size in the presence of capacity constraints, discussed in above Sects. 11.1.3–11.1.4, in this case it is necessary not only to consider these constraints but also to establish the reasonable sequence of the manufacture of different products, i.e. to make an efficient schedule. A problem of this type is called Economic Lot Scheduling Problem (ELSP).

13.4.1 Equal-Time Schedules for All Products

The simplest periodic schedule with several different products is when the production of each product within one period (cycle) of the schedule is carried out once. This schedule is called "rotation schedule" and its construction is entirely determined by the duration of period T. To find optimal value of T, it is possible to use a method similar to that described in Sects. 2.1.1 and 11.2.1: to determine the cost of the lot order (setup for the lot) and the cost of storage for all manufactured products and then to find the minimum of the expression. The relevant cost calculation gives (Pinedo 2005)

$$c = \sum_{i=1}^{n} \left[\frac{c_{hi}}{2}\left(D_i T - \frac{D_i^2 T}{P_i}\right) + \frac{c_{oi}}{T} \right], \tag{13.7}$$

where, as before, c_{oi} is the cost of setup for the i-th product; c_{hi} is the cost of storage per measure unit of the i-th product per time unit, D_i is the product consumption (demand) per time unit; P_i is the machine capacity for the i-th product.

By finding the derivative from Eq. (13.7) for T, and equate the obtained expression to zero we obtain

$$T^* = \sqrt{\sum_{i=1}^{n} c_{oi} \Big/ \sum_{i=1}^{n} \frac{c_{hi} D_i (P_i - D_i)}{2P_i}}. \tag{13.8}$$

Let us consider the example the data of which are given in Table 13.12.

Table 13.12 Products manufactured on the machine

Product no.	Setup cost C_{oi}	Demand D_i pcs. per days	Daily cost of storage C_{hi} of 1 pc.	Machine capacity P_i pcs./day	Optimal lot Q^* pcs.
1	500	40	8	150	98
2	250	10	2	60	25
3	250	20	5	100	49

Let us determine the optimal duration of the schedule cycle in days

$$T^* = \sqrt{(500 + 250 + 250)/(\frac{8 \times 40 \times (150 - 40)}{2 \times 150} + \frac{2 \times 10 \times (60 - 10)}{2 \times 60}}$$
$$+ \frac{5 \times 20 \times (100 - 20)}{2 \times 100} = 2.46$$

and optimal lot sizes in pcs. $Q_1 = D_1 T^* = 40 \times 2.46 = 98$; $Q_2 = 10 \times 2.46 = 25$; $Q_3 = 20 \times 2.46 = 49$. The production time of one lot of product 1 in days amounts to $\frac{Q_1}{P_1} = \frac{98}{150} = 0.65$; for product 2 is $\frac{25}{60} = 0.4$; for the third product—$\frac{49}{100} = 0.49$. So the total operation time of the machine for the period will make 1.54, and the machine downtime is 0.92. Figure 13.5 shows the diagram of stock changing for this example.

In this example, it is assumed that the total setup time is small (less than the estimated time of downtime), and the cost of the setup does not depend on the execution sequence. In this case, the size of the lots does not depend on their order in the schedule. If total setup time $\sum_{i=1}^{n} s_i$ is more than the calculated downtime, the optimal cycle duration increases and is determined by the relation (Dobson 1987)

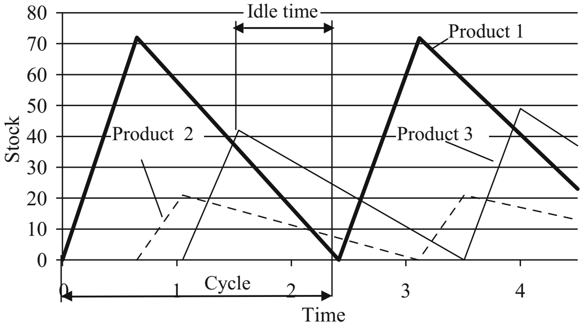
$$T^* = \sum_{i=1}^{n} s_i / \left(1 - \sum_{i=1}^{n} \frac{D_i}{P_i}\right), \tag{13.9}$$

where the time of estimated downtime is 0.

Assume, for example, that with the data in Table 13.12, the total setup time for three types of products in one cycle is 1.4 day. We obtain that $T^* = 1.4/(1 - 40/150 - 10/60 - 20/100) = 3.74$ days, and the optimal lots will be $Q_1 = D_1 T^* = 40 \times 3.74 = 150$; $Q_2 = 10 \times 3.74 = 37$; $Q_3 = 20 \times 3.74 = 75$.

In the case where the time and cost of setup depend on the jobs sequence, then in general case, the problem becomes much more complicated. With a small number of manufactured products (up to four), it is possible just to search through all the options, and with large number of them to use the algorithm described above in Sect. 13.3.4.

Fig. 13.5 Diagram of stocks of the manufactured products with equal cycles



13.4.2 Variable-Time Schedules for Different Products

In general, in one schedule cycle, one and the same product can be produced in several lots. An exact solution of this problem is very difficult to obtain, if possible at all. However, nowadays there is an effective heuristic method developed in the paper (Dobson 1987), which is called Frequency Fixing and Sequencing (FFS). The method consists of three successive stages:

- Theoretical calculation of the relative frequencies of manufacture of each product;
- Establishing efficient frequencies;
- Determining efficient sequence of lots production.

Obviously, that in one schedule cycle with duration T, the lots of each i-th product can be produced a number of times, i.e. frequency ω_i is an integer. In FFS method, for reasons of minimum costs, the optimal (not necessarily integer) frequencies of manufacture of each product are determined, the values of which are rounded then to integer values. Thus, the problem of the first stage is reduced (Pinedo 2005) to finding the minimum cost

$$c = \sum_{i=1}^{n} \frac{a_i T}{\omega_i} + \sum_{i=1}^{n} \frac{c_{oi}\omega_i}{T}, \tag{13.10}$$

with setup time s_i constraint

$$\sum_{i=1}^{n} \frac{s_i \omega_i}{T} \leq 1 - \sum_{i=1}^{n} \frac{D_i}{P_i}. \tag{13.11}$$

In expression (13.10)

$$a_i = \frac{c_{hi}}{2}\left(1 - \frac{D_i}{P_i}\right)D_i. \tag{13.12}$$

Variables for finding the minimum in expression (13.10) are frequency ω_i and cycle duration T. If there is no downtime during the cycle, then inequality (13.11) becomes equality. To find the minimum expression (13.10) is differentiated according to independent variables, and the results of differentiation are equated to zero. With regard to inequality (13.11), this process gives the following for each of the frequencies (Pinedo 2005):

$$\omega_i^* = T^* \sqrt{\frac{a_i}{c_{oi} + \lambda s_i}}, \tag{13.13}$$

where λ is the so-called Lagrange multiplier, the same for all frequencies. If a part of the cycle is downtime, this multiplier is 0, and if there is no downtime, λ is determined from expression

$$\sum_{i=1}^{n} s_i \sqrt{\frac{a_i}{c_{oi} + \lambda s_i}} = 1 - \sum_{i=1}^{n} \frac{D_i}{P_i}. \tag{13.14}$$

As a rule, the frequency determined from expression (13.13) is not an integer. Therefore, at the second stage of the FFS method, the obtained frequency values are rounded to integers, and in this process the so-called Power-of-Two Policy, described in Sect. 11.5.2, is used. At this stage, loading of the machine in the cycle is also verified for constraints (13.11).

The lots of each product, calculated in the way, are evenly distributed and then within one cycle, if possible. To illustrate this method we will use the data of the previous example (Table 13.13).

When determining frequencies ω_i, first of all we assume that machine downtime is possible and accordingly $\lambda = 0$. In this case, we have $\omega_1^* = T^* \sqrt{\frac{a_1}{c_{o1}}} = \sqrt{\frac{117}{500}} T^* = 0.48T^*; \omega_2^* = \sqrt{\frac{8}{250}} T^* = 0.18T^*; \omega_2^* = \sqrt{\frac{40}{250}} T^* = 0.4T^*$.

According to Table 13.12, in this example all the process parameters refer to 1 day of machine operation, and when setting the cycle duration in days as well, the frequencies are dimensionless values. Assume that the cycle duration is equal to the optimal value, obtained in the previous Sect. 13.4.1, i.e. $T^* = 2.46$. In this case, the smallest of the frequencies is $\omega_2^* = 0.18 \times 2.46 = 0.44$.

Since the frequency can be only an integer, the actual frequency ω_2 should be accepted to be equal to unity. As the ratio of frequencies $\omega_1/\omega_2 = 0.48/0.18 = 2.7$ and $\omega_3/\omega_2 = 2.2$, according to Power-of-Two Policy, it should be assumed that $\omega_1 = 2$ and $\omega_3 = 2$. Since for the smallest frequency ω_2 the ratio of $\omega_2 = 0.18T$ should be valid, then we accept the cycle duration in days $T = \omega_2/0.18 = 1/0.18 \approx 5$.

Now we verify the loading of the machine and check for machine downtime (13.11):

Table 13.13 Products manufactured on the machine

Product	C_{oi}	D_i	C_{hi}	P_i	s_i	a_i
1	500	40	8	150	0.5	117
2	250	10	2	60	0.6	8
3	250	20	5	100	0.4	40

$$\sum_{i=1}^{n} \frac{s_i \omega_i}{T} = \frac{0.5 \times 2 + 0.6 \times 1 + 0.4 \times 2}{5} = 0.48;$$

$$1 - \sum_{i=1}^{n} \frac{D_i}{P_i} = 1 - \frac{40}{150} - \frac{10}{60} - \frac{20}{100} = 0.36.$$

Since constraint (13.11) is not fulfilled, then with cycle time $T = 5$ the machine is overloaded. To solve this problem, it is necessary to increase the cycle time and assume that the machine should operate without downtime; for this purpose from expression (13.14) parameter λ must be defined. By trial and error, it is easy to find that in this example, Eq. (13.14) is satisfied with value $\lambda = 1450$.

Now let us define the cycle duration based on expression (13.13). As the set values of ω_i are not equal to optimal ω_i^*, then for each frequency value T_i is various. In this case $T_1 = \omega_1 \sqrt{\frac{c_{o1} + \lambda s_1}{a_1}} = 2 \times \sqrt{\frac{500 + 1450 \times 0.5}{117}} = 6.5$; $T_2 = 1 \times \sqrt{\frac{250 + 1450 \times 0.6}{8}} = 11.8$; $T_3 = 2 \times \sqrt{\frac{250 + 1450 \times 0.4}{40}} = 9.2$ and the average value will make $T = \frac{T_1 + T_2 + T_3}{3} = 9$.

So the lot sizes in pieces will make $Q_1 = D_1 T / \omega_1 = 40 \times 9/2 = 180$; $Q_2 = 10 \times 9/1 = 90$; $Q_3 = 20 \times 9/2 = 90$; the production time of the lots in days $\tau_1 = Q_1/P_1 = 180/150 = 1.2$; $\tau_2 = 90/60 = 1.5$; $\tau_3 = 90/100 = 0.9$.

Now we set the sequence of lots production. The main requirement for this sequence is possibly uniform production of various products. In a uniform sequence, the time interval between two successive lots of product remains constant both within a single cycle and from cycle to cycle.

For example, in possible sequence {1, 3, 2, 1, 3} inside the cycle, the interval between lots of products 1 or 3 makes three positions, and with repetition of this cycle the interval is two positions. When setting sequence {1, 3, 2, 3, 1} for product 1 inside the cycle, the interval is four positions, and between cycles—only one position, i.e. lots of product follow straight one after another. In this case, the stocks volumes vary considerably, which is usually undesirable. Figure 13.6 shows a diagram of stocks of manufactured products for sequence {1, 3, 2, 1, 3}.

The diagram in Fig. 13.6 is noticeably different from the similar diagram in Fig. 13.5. For example, stock of product 1 in the idealized case shown in Fig. 13.5 if always greater than zero and each cycle has the same maximum value. The similar diagram in Fig. 13.6 has the part of the cycle in which the product 1 stock is 0; at the

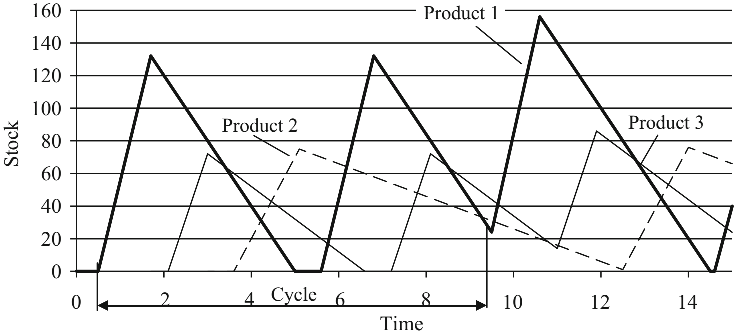


Fig. 13.6 Diagram of stocks of manufactured products with different cycles

same time the going from the first cycle to the second one the maximum stock increases due to some unevenness in manufacture of the product. The same situation takes place for product 3. Product 2 is produced only once during the cycle and its stock schedule is completely similar to the diagram in Fig. 13.5.

13.5 Group Technology in Schedules for a Single Machine

In Chap. 11 above, various ways to determine the lot size based on the optimization of the cost of manufacturing and storage of products (Lot-Sizing Problem) were discussed. As a result of solving such problems, the optimal quantity of each product was determined, which should be produced or ordered in a certain time period. At the same time, however, the impact of the sequence of these lots in the production, i.e. schedules, was not considered at all. Scheduling with Batching can significantly reduce the machine changeovers and, in some cases, the total duration of execution of all jobs. Herewith, it is assumed that each i-th job can be assigned to one of the j-th groups.

Technological affinity of various jobs in the same group provides little setup time of transition from one job to another within the group. If, for example, on the same machine a group of several jobs (orders) must be executed to produce the same product, the setup time between the jobs is zero, and the whole group of such orders becomes a batch. In other case, a group of jobs can become a batch, if the execution of all these jobs is carried out simultaneously on the same machine (furnace).

Schedules with group technology are discussed in many sources of literature and papers, the review of which is in Potts and Kovalyov (2000), for example. Finding the optimal solutions of the problems with group schedules, as shown in Tanayev

Table 13.14 Input data of job set

Job no.	1	2	3	4	5	6	7	8
Processing time p_i	3	5	6	5	2	4	2	3
Due date d_i	9	12	25	16	17	20	15	20
Job group	1	3	1	2	3	2	1	2

et al. (1998), relates to significant computational difficulties, and therefore, as a rule, each of these studies was performed for each individual case. For practical use, even if not very accurate, relatively simple solutions suitable for different problems are required.

13.5.1 Group Scheduling for Series Batches

Let us consider the problem of group scheduling for a single machine with the minimization of the largest deviation of jobs $L_{\max}$ in the schedule from the fixed due dates. We assume that the data in Table 13.14 are planned task for the coming calendar month, which has 25 working days. The processing time of each out of the eight jobs is specified in working days, and each of the jobs belongs to one of three groups. Suppose also that the setup time per group of jobs in working days $s_1 = s_2 = 1$, $s_3 = 2$, and setup time for the job within the group is inconsiderable.

The classification formula of the problem in this case has the form (Appendix C):

$$1\left(\text{batch}, C_{job}\right)\left|s_j\right|L_{\max}, \tag{13.15}$$

where parameter "batch" means that the schedule should be made using the job grouping; parameter C_{job} indicates that after completion of each job the relevant product leaves the single machine (sequential execution of jobs); parameter s_j indicates the necessity to take into account the time for setup with different value for each group.

Let us position the jobs of each group according to EDD (Earliest Due Date) rule, described above in Sect. 2.3.1. For group 1 we obtain sequence {1, 7, 3}, for group 2—sequence {4, 6, 8}, and for group 3—{2, 5}. Suppose the initial time point of the setup of some j-th group equals t_j. When executing the jobs within the group in the specified sequence the deviation from the required date of execution will be

$$L_{ji} = t_j + s_j + p_{j1} + p_{j2} + \ldots + p_{ji} - d_{ji}. \tag{13.16}$$

Let us determine the value of processing time, which remains in the j-th group after completion of the i-th job.

Table 13.15 Estimated parameters of the job sequence

Number of the j-th group	1			2			3	
Total processing time p_j	11			12			7	
Job number	1	7	3	4	6	8	2	5
Due date d_i	9	15	25	16	20	20	12	17
Processing time of the jobs until completion of the i-job	3	5	11	5	9	12	5	2
Remaining processing time q_{ji}	8	6	0	7	3	0	2	0
Parameter a_{ji}	17	21	25	23	23	20	14	17
Family due date d_j	17			20			14	

$$q_{ji} = p_j - \left(p_{j1} + p_{j2} + \ldots + p_{ji}\right), \tag{13.17}$$

where p_j equals the total of processing time of all jobs in the group. If we insert Eq. (13.17) into Eq. (13.16) for each of the jobs in the group, then the most deviation from the fixed deadline will appear to be

$$\max_i \left(L_{ji}\right) = t_j + s_j + p_j - \min_i \left(d_{ji} + q_{ji}\right). \tag{13.18}$$

Let us introduce the concept of the required family due date (Baker and Trietsch 2009)

$$d_j = \min_i \left(d_{ji} + q_{ji}\right) \tag{13.19}$$

and calculate the value $a_{ji} = d_{ji} + q_{ji}$ for all the jobs in Table 13.14, ordered in groups by EDD rule, and corresponding values d_j (Table 13.15). For example, processing time in group 1 until completion of job 7 is $p_1 + p_7 = 3 + 2 = 5$. Herewith the remaining processing time in this group is $q_{1,7} = 11 - 5 = 6$, and accordingly, $a_{1,7} = 15 + 6 = 21$.

In the paper (Baker and Trietsch 2009), it is proved that the minimal deviation from the fixed due date (delay) is reached if the job groups are sorted according to EDD rule against other family due date determined by formula (13.18). In this case, there will be the least delay if job group 3 is executed first and then job group 1 and group 2 after all the others. Eventually the sequence of jobs takes the form {2, 5, 1, 7, 3, 4, 6, 8}.

The record of execution of this sequence for calculation of the dates is reasonable to make as follows (Sule 2007): (job No./group No. | standard for setup/ standard for processing). If the number of the group does not change then the record of this number and time for setup is omitted. In this case, we have sequence

	0	(2/3|2/5)	7	(5/2)	9	(1/1|1/3)	13	(7/2)	15	(3/6)	21	(4/2|1/5)	27	(6/4)	31	(8/3)	34
Due dates		(2)	12	(5)	17	(1)	9	(7)	15	(3)	25	(4)	16	(6)	20	(8)	20
Tardiness			−5		−8		4		0		–4		11		11		14

It is obvious that the set scope of jobs does not fit in the available time fund, and for its execution, for example, the quantity of working shifts should be increased. At that the processing time in working days will decrease accordingly, but the optimal sequence of their execution will not change.

13.5.2 Group Scheduling for Parallel Batches with Minimum Tardiness Criterion

Simultaneous execution of multiple jobs on one machine is typical for treatment in furnace, bath, and like equipment, which can accommodate several batches of different products, for which the processing mode is the same. In such cases, the collection of all such products makes one load batch for which processing is continuous. Herewith, no other jobs are loaded in the machine until the end of the treatment process.

If each job takes about the same space, the machine capacity can be defined as allowable quantity b of simultaneously performed jobs of duration p. Let us consider the problem of the optimal planning for a machine with parallel execution of jobs, for which the processing mode is the same, and for an objective function of minimal total tardiness T for a given set of jobs.

The classification formula of this problem has the form

$$1\,(\text{batch}, C_{batch})\big|g = 1\big|T, \tag{13.20}$$

where parameter ‘batch” indicated that the schedule should be made using the load batches consisting of multiple jobs; parameter C_{batch} shows parallel execution of all loaded jobs; parameter g indicated the quantity of job groups requiring different process settings.

We arrange the jobs in order of expected arrival and consider the example, the data for which are shown in Table 13.16. The time points of arrival and execution can be set in hours, shifts, days, etc. Let time p of the treatment process be 3, and the largest quantity of simultaneous jobs b (machine capacity) is 4.

To solve the problems of this type it is advisable to use a heuristic algorithm with partial searching through options (Sule 2007). At the first step of the algorithm the batch combines the jobs received by the time of load planning in order of increasing due time point. If such jobs are more than the machine capacity, then the joining is carried out until the quantity of jobs in the batch is equal to permissible load quantity b. If the jobs are all used up and all the load batches are of the same size, equalling to b the scheduling stops.

If after scheduling it turns out that any of the batches is less than b, then at the second step of the algorithm we can try to delay the start of processing of the party in order to increase the amount of work in it to an acceptable value. In such cases, it is sometimes possible to obtain reduction of the total delay of the whole set of jobs.

For example, in the case of the data in Table 13.16 at time point 1 only jobs 1 and 2 are ready for processing, which can be combined into one load batch. By the end

Table 13.16 Schedules with jobs executed simultaneously

			Option 1			Option 2			Option 3		
Job no.	Expected arrival time point	Required completion time point	Start	End	Delay	Start	End	Delay	Start	End	Delay
1	1	4	1	4	–	1	4	–	1	4	–
2	1	5	1	4	–	1	4	–	1	4	–
3	3	6	4	7	1	5	8	2	5	8	2
4	4	8	4	7	–	5	8	–	5	8	–
5	5	9	7	10	1	5	8	–	5	8	–
6	6	9	7	10	1	8	11	2	9	12	3
7	8	11	10	13	2	8	11	–	9	12	1
8	8	11	10	13	2	8	11	–	9	12	1
9	9	12	10	13	1	11	14	2	9	12	–
10	10	13	10	13	–	11	14	1	12	15	2
11	10	13	13	16	3	11	14	1	12	15	2
12	12	15	13	16	1	12	15	–	12	15	–
Total delay					10			8			11

of the processing of the batch at time point 4 jobs 3 and 4 are ready for loading, the processing of which is over by the time 7; similarly, jobs 5 and 6 are ready.

In the above cases, the number of possible jobs at the time of machine availability was less than the machine capacity. By time 10 jobs 7, 8, 9, 10, and 11 will be available for processing, i.e. the number of possible jobs exceeds machine capacity $b = 4$. In this case, the load batch should include only the first four jobs in order of increasing required due time point. It is obvious that remaining jobs 11 and 12 must make the last load batch.

For each of the schedule jobs, the value of planned delay is defined as the difference between the time point of processing completion and the required due time point. For example, for the above first option of the schedule, the total planned delay in units of time scheduling is 10.

In order to reduce this delay the machine filling should be increased by enlargement of some load batch. According to the described algorithm, this can be done by delaying loading, e.g. of jobs 3 and 4, in order to combine them with job 5 (option 2). By sequential load batching according to this option, we see that in this case, the total delay is 8 units.

This process can be continued if, for example, in the schedule of option 2 the loading of jobs 6, 7, and 8 is postponed by a unit in order to combine them with job 9 (option 3). However, scheduling under option 3 gives total delay of 11, which obviously does not make sense. It is also possible to delay jobs 9, 10, and 11 by a time unit and combine them with job 12; however, it is also impractical. Thus, in this case, option 2 can be considered to be optimal.

13.5.3 Group Scheduling for Parallel Batches with Maximum Average Utility Criterion

In actual practice of job assignments for the machine with simultaneous processing multiple jobs, the list of these jobs is usually set with a small horizon, but the assignments often include the jobs belonging to several different technological groups. For such problems, it makes sense to use the branch-and-bound method with criterion of average current utility maximum $\overline{V}$ described in above Sect. 2.6.2. In this case, the classification formula is

$$1\,(batch, C_{batch})\big|g\big|\overline{V} \tag{13.21}$$

where parameter g indicates that the number of job groups requiring different process settings can be any. The application of the relevant algorithm will be discussed in terms of the input data in Table 13.17.

The data in Table 13.17 are compiled with a horizon of three working shifts (24 h). Suppose also that occupied machine capacity in litres $b = 100$, the setup time for a load batch of any technological group is inconsiderable, the importance of all jobs is the same and the machine is free at time 0. The arrival time, equalling

Table 13.17 Input data for scheduling

Job no.	Process group	Processing time, h	Capacity taken, L	Expected arrival time point, h	Required due time point, h
1	1	4	30	0	6
2	2	6	50	2	8
3	2	6	40	3	9
4	1	4	30	5	9
5	1	4	60	8	12
6	1	4	30	12	16
7	2	6	50	13	20
8	1	4	30	18	22
9	2	6	40	18	24

to 0, for example, for job 1 in Table 13.17, means that the corresponding product batch may arrive at any time before scheduling starts.

The solution of the considered problem is basically similar to the solution of the problem given in Sect. 2.6.2 but with one major difference. The difference is that the processing of each new load batch may begin only when, firstly, the machine finished processing of the previous batch and, secondly, all the jobs scheduled for the next batch are ready for processing. Let us assume that at some time point C_l the processing of the l-th batch finishes. Processing of any possible k-batches may begin at time point

$$t_k = \max\left(C_l, \max_{i \in J_k}(r_i)\right), \tag{13.22}$$

where J_k is the set of jobs included into the k-th batch and r_i is the time point of arrival of the i-th job.

In this case, average utility of entire available set J of job for all time $t_k + p_k$ from the start of jobs until completion of the k-th job, similarly to formula (2.40),

$$\overline{V}_{l+1,k} = \frac{1}{t_k + p_k}\int_0^{t_k+p_k} V dt = \frac{1}{t_k + p_k}\left(\overline{V}_l \times C_l + \int_{C_l}^{t_k+p_k} V_k dt\right). \tag{13.23}$$

In expression (13.23), value $\overline{V_l}$ is equal to the average utility of all the scope of schedule jobs for the time from initial time t $=0$ to the end of last already scheduled job C_l. For example, at initial time t $=0$, the machine is free, the number of processed batches $l = 0$, and $C_0 = 0$, respectively. Value t_k in this case is determined only by the time of the last job's arrival in the planned k-th batch.

Integral in Eq. (13.23) in this case, as in (2.41), equals

$$\int_{C_l}^{t_k+p_k} V_k dt = \frac{1}{G}\int_{C_l}^{t_k+p_k} \sum_{i\in J-I_l} p_i dt - \int_{C_l}^{t_k+p_k} \sum_{i\in J-I_l} H_i dt, \tag{13.24}$$

where I_l is the set of jobs executed before time point t_k; G is the time fund in hours within a planning period, set by the assignment; p_i is the processing time of the i-th job; and H_i is the current production intensity of the i-th job.

Let us denote the component of the first integral in the right side (Eq. 13.24) from the i-th job as γ_k^i. Given that one k-th batch includes jobs with the same processing time p_k, we have

$$\gamma_k^i = \frac{1}{G}\int_{C_l}^{t_k+p_k} p_i dt = p_i \frac{p_k + t_k - C_l}{G} \text{ for } J_i \neq J_k \tag{13.25}$$

and

$$\gamma_k^i = \frac{1}{G}\left[\int_{C_l}^{t_k} p_k dt + \int_{t_k}^{t_k+p_k} (p_k - (t - t_k))dt\right] = p_k \frac{p_k/2 + t_k - C_l}{G} \text{ for } J_i = J_k. \tag{13.26}$$

Let us introduce parameter a_k^i, which can have one of three values:

0—for the jobs already executed by time point t_k;
1—for the jobs not executed and not included in the k-th batch yet;
0.5—for the jobs not executed yet, but included in the k-th batch.

Thus, with $a^i = 0\ \gamma^i = 0$; with $a_k^i = 1\ \gamma_k^i$ is determined by formula (13.25); with $a_k^i = 0.5$ for calculation of γ_k^i formula (13.26) is used.

The second component in the right side (Eq. 13.24) is represented by integrals calculated by the rules given in Sect. 2.6.2 and specified in Appendix D. The form of those integrals is determined by values of coefficient a_k^i, as well by fulfilment of inequations $d_i - t_k - p_k \geq 0$, $d_i - t_k \geq 0$ and $d_i - C_l \geq 0$.

At the first step of the algorithm the initial utility of jobs execution is determined by formula (2.32):

$$V = \frac{1}{G}\sum_{i=1}^{n} p_i - \sum_{i=1}^{n} H_i,$$

where n is the full number of jobs in the assignment.

In this case, $n = 9, G = 24$, values p_i are given in Table 13.17. Since the required due date of any job has not yet passed, for all the jobs at the initial time, the intensity may be determined by the first out of formulas (2.28), namely

$$H_i = \frac{p_i}{G} \frac{1}{(d_i - t)/\alpha G + 1}.$$

Let us assume that like in examples of Chap. 2, psychological coefficient $\alpha = 0.1$. In this case at time $t = 0$, for example for job 1, we have

$$H_1 = \frac{4}{24} \frac{1}{(6 - 0)/(0.1 \times 24) + 1} = 0.047.$$

By determining the initial intensities for all jobs $\sum_{i=1}^{n} H_i = 0.31$, we obtain the value of initial utility

$$V_0 = \frac{44}{24} - 0.31 = 1.52.$$

At the second step, it is necessary to identify possible load batches in each of two technological groups, the total capacity of which does not exceed the machine capacity in use. These batches are sets consisting of jobs $\{1\}, \{2\}, \{1,4\}, \{2,3\}, \{1,4,6\}$. These sets are the nodes of the first level of the tree of searching for optimal solution that is built starting from the initial stage of jobs execution (Fig. 13.7). The average utility will be calculated using Table 13.18.

For example, in node 1, located on level 1, the set of planned jobs $J_k = J_1 = \{1\}$. Previous level $l = 0$ (Fig. 13.7) and the time of job completion at this level $C_0 = 0$, respectively. The time point of arrival of the only job 1 in set $J_1 = \{1\}$, according to Table 13.17, is $r_1 = 0$. That is why job start time $t_k = t_1 = \max\left(C_l, \max_{i \in Jk}(r_i)\right) = \max\left(0, \max_{i \in J_1}(r_1)\right) = 0$. For job 1 in node 1, according to formula (13.25), parameter value $\gamma_1^1 = p_1^{\frac{p_1/2 + t_1 - C_0}{G}} = 4^{\frac{4/2+0-0}{24}} = 0.33$.

Let us determine the component of average utility in this node from job 2. Since this job is not included in the planned set, then $a_1^2 = 1$ and parameter $\gamma_1^2 = p_2^{\frac{p_1 + t_1 - C_0}{G}} = 6^{\frac{4+0-0}{24}} = 1$. For job 2 in node 1, we have the following values of parameters $d_i - t_k - p_k = d_2 - t_1 - p_1 = 8 - 0 - 4 = 4$; $d_i - t_k = d_2 - t_1 = 8 - 0 = 8$; $d_i - C_l = 8 - 0 = 8$.

By the values of parameters $a_1^2, d_2 - t_1 - p_1, d_2 - t_1$ и $d_i - C_l$ in Appendix 4 for determining the production intensity, we select formula 1, using which we find $H_1^2 = 0.291$. The value of average utility of job 2 in node 1, according to expressions

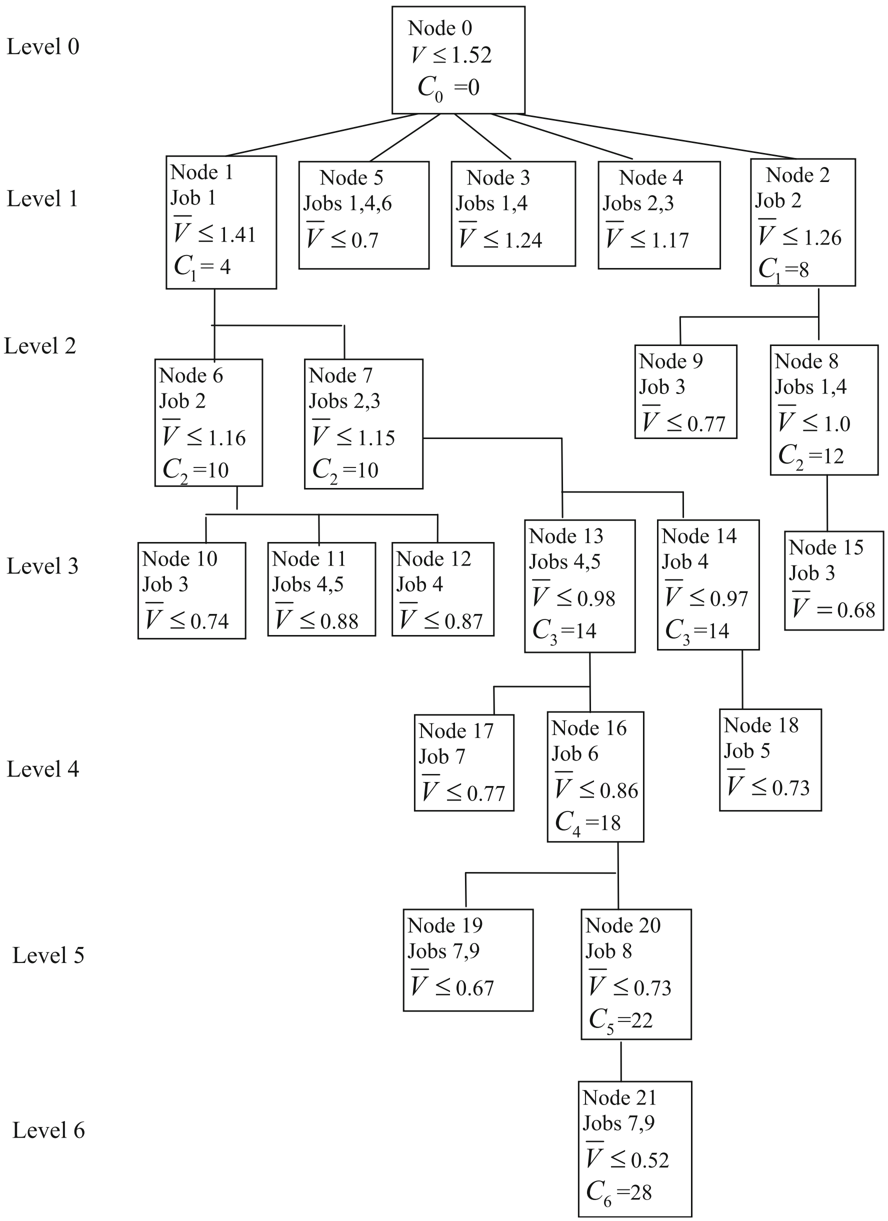


Fig. 13.7 Search tree for the criterion of maximal average utility $\overline{V}$

Table 13.18 Calculation of average utility for nodes of the searching tree

Level $l+1$;
Set I_l
	Node k;
	composition;
	start;
	p_k
		Job i	a_k^i	γ_k^i	$d_i - t_k - p_k$	$d_i - t_k$	$d_i - C_l$	Number of formula for H_k^i	H_k^i	$\overline{V}_k^i$

$l=0$ $I_0=\{\}$ $C_0=0$ $\overline{V}_0=0$	$k=1$ $J_1=\{1\}$ $t_1=0$ $p_1=4$	1	0.5	0.33	2	6	6	2	0.115	0.05
		2	1	1	4	8	8	1	0.291	0.18
		3	1	1	5	9	9	1	0.259	0.19
		4	1	0.67	5	9	9	1	0.173	0.12
		5	1	0.67	8	12	12	1	0.13	0.13
		6	1	0.67	12	16	16	1	0.098	0.14
		7	1	1	16	20	20	1	0.118	0.22
		8	1	0.67	18	22	22	1	0.072	0.15
		9	1	1	20	24	24	1	0.099	0.23
		Average utility in node 1 at level 1 $\overline{V}_{1,1} = 1.41$								1.41
	$k=2$ $J_2=\{2\}$ $t_2=2$ $p_2=6$	1	1	1.33	−2	4	6	1	0.973	0.05
		2	0.5	1.25	0	6	8	2	0.427	0.09
		…								
		9	1	2	16	22	24	1	0.217	0.22
		Average utility in node 2 at level 1 $\overline{V}_{1,2} = 1.26$								1.26
	$k=3$ $J_3=\{1,4\}$ $t_3=5$ $p_3=4$	1	0.5	1.17	−3	1	6	4	0.75	0.04
		2	1	2.25	−1	3	8	3	1.182	0.12
		3	1	2.25	0	4	9	1	0.935	0.15
		4	0.5	1.17	0	4	9	2	0.396	0.09
		…								
		9	1	2.25	15	19	24	1	0.25	0.22
		Average utility in node 3 at level 1 $\overline{V}_{1,3} = 1.24$								1.24

(continued)

Table 13.18 (continued)

Level $l+1$;
Set I_l
	Node k;
	composition;
	start;
	p_k
		Job i	a_k^i	γ_k^i	$d_i - t_k - p_k$	$d_i - t_k$	$d_i - C_l$	Number of formula for H_k^i	H_k^i	$\overline{V}_k^i$

$l = 1$ $I_1 = \{1\}$ $C_1 = 4$ $\overline{V}_1 = 1,41$	$k = 6$ $J_6 = \{2\}$ $t_6 = 4$ $p_6 = 6$	1	0	0	–	–	–	–	0	0
		2	0.5	0.75	−2	4	4	4	0.467	0.03
		...								
		9	1	1.5	15	21	21	1	0.178	0.13
		Average utility in node 6 at level 2 $\overline{V}_{2,6} = 1.16$								0.6
	$k = 7$ $J_7 = \{2, 3\}$ $t_7 = 4$ $p_7 = 6$	1	0	0	–	–	–	–	0	0
		2	0.5	0.75	−2	4	4	4	0.467	0.03
		3	0.5	0.75	−1	5	5	4	0.366	0.04
		...								
		9	1	1.5	15	21	21	1	0.178	0.13
		Average utility in node 7 at level 2 $\overline{V}_{2,7} = 1.15$								0.59
...										
$l = 5$ $I_5 = \{1, 2, 3, 4, 5, 6, 8\}$ $C_5 = 22$ $\overline{V}_5 = 0,77$	$k = 21$ $J_{21} = \{7, 9\}$ $t_{21} = 22$ $p_{21} = 6$	1	0	0	–	–	–	–	0	0
		...								
		7	0.5	0.75	−8	−2	−2	7	2.815	−0.07
		8	0	0	–	–	–	–	0	0
		9	0.5	0.75	−3	3	3	4	0.614	0.005
		Average utility in node 21 at level 6 $\overline{V}_{6,21} = 0.53$								−0.07

(13.24)–(13.27), is $\overline{V}_1^2 = \frac{\gamma_1^2 - H_1^2}{t_1 + p_1} = \frac{1 - 0.291}{0 + 4} = 0.18$. The utility of all jobs in the node 1 located at the level 1, $\overline{V}_{1,1} = \frac{\overline{V}_l C_l}{t_k + p_k} + \sum_i \overline{V}_k^i = 0 = 1.41 = 1.41$.

Let us transfer to calculation of utility in the node 2 at the same level 1, for which $C_0 = 0$ as before. Accordingly, $J_k = J_2 = \{2\}$, $r_2 = 2$ and job start time $t_2 = \max\left(0, \max_{i \in J_2} (r_2)\right) = 2$. For the rest the calculation of the average utility in the node 2 is analogous to that in the node 1. In the node 3 at the first level, the set of planned jobs consists of two jobs $J_3 = \{1, 4\}$, the time of arrival of which $r_1 = 0$, and $r_4 = 5$ and $t_3 = 5$, respectively. Average utility in the node 3 $\overline{V}_{1,3}$ is 1.24. Similarly we determine utilities $\overline{V}_{1,4}$ and $\overline{V}_{1,5}$ (Fig. 13.7).

It is clear that the most utility is in the node 1, which is accepted as initial for branching at the level 2. In this case the end time point of processing of the first batch $C_1 = 4$ and average utility from the beginning of the planning process until this time point is $\overline{V}_l = \overline{V}_1 = 1.41 = \overline{V}_1$. And here the load batches are possible that consist of jobs {2}, {2, 3}, {4}, {4, 5}.

For the node 6 with one planned job 2, received at time point $r_2 = 2$, the possible start time of processing is $t_6 = 4$. For the already executed job 1 $a^1 = 0$, for the being executed job 2 $a_6^2 = 0.5$, and for the rest of the jobs in this node $a_6^i = 1$. By calculating parameters γ_k^i, $d_i - t_k - p_k$, $d_i - t_k$, $d_i - C_l$ for all jobs in the node 6, as described above, we find the values of intensity and utility. Utility of all jobs in the node 6 located at the level 2, $\overline{V}_{2,6} = \frac{\overline{V}_l C_l}{t_k + p_k} + \sum_i \overline{V}_k^i = \frac{1.41 \times 4}{4 + 6} + 0.6 = 1.16$.

After performing similar calculation for the load batch in the node 7 consisting of two jobs $J_7 = \{2, 3\}$, we find the value of utility $\overline{V}_{2,7} = 1.15$. For load batches of jobs {4}, {4, 5} the obtained utilities are much lower, and the corresponding nodes in Fig. 13.7 are not displayed. Thus, when branching from the node 1 at the first level the best result is given by the node 6 at the second level with utility 1.16.

From Fig. 13.7 it can be seen that such utility value on the second level is lower than the utility in nodes 2, 3, and 4 at the first level. Of course, the node with the highest utility is selected first from these nodes, which in this case is the node 2. After branching from the node 2, we obtain possible batches with sets of jobs {1}, {3}, {4}, {4, 5} and Fig. 13.7 shows the nodes 8 and 9 for the first two of these options. The node 8 has the highest utility with such branching, however, its usefulness $\overline{V}_{2,8} = 1.0 < \overline{V}_{2,6} = 1.16$.

Strictly speaking, according to the branch-and-bound method, now it is necessary to perform branching from remaining nodes 3 and 4. However, it can be assumed practically that, if for some branch successively on two levels worse results are observed than for other branches, then further branching of this branch does not make sense.

Going to the third level from node 6 we form nodes 10, 11, and 12, among which node 11 has the highest utility $\overline{V}_{3,11} = 0.88$ with load batch of jobs 4 and 5. The value of this utility is less than the utility in nodes 7 and 8 at the second level, and therefore branching should be performed from them. As the utility is the highest in node 7, then during branching from it, we create nodes 13 and 14 at the third level. The utility of these nodes is greater than that of node 11 obtained when branching from node 6. This shows that here node 7 at the second level is more preferable than node 6, despite the fact that its utility is lower than in node 6.

Since the re-branching from the node at the second level has led to an increase in the potential utility at the third level, then branching at the second level should be continued. However, when branching from node 8 at the second level, we see that in all of such nodes (in Fig. 13.7 only node 15 with the greatest utility is shown) the utility is lower than that when branching from node 7. Therefore, further branching from the nodes at the second level can be stopped. Eventually, node 13 with a load batch consisting of jobs 4 and 5 has the highest utility at the third level.

When branching for the next levels, we obtain the solution of the problem in which all nine jobs are divided into six load batches $J_1 = \{1\}$, $J_2 = \{2, 3\}$, $J_3 = \{4, 5\}, J_4 = \{6\}, J_5 = \{8\}$, and $J_6 = \{7, 9\}$, and all the jobs are planned to be finished by time $C_6 = 28$, which is not much longer than the required due time point, equalling to 24.

Despite some awkwardness of the formulas for intensity given in Appendix D, the algorithm described above is easily realized in MS Excel spreadsheet. Wherein the specified formulas are placed in some cells, and then for each new dataset are simply copied to perform calculations.

13.6 Parallel Machine Scheduling

The theory of schedules differentiates three options of jobs scheduling for several parallel machines:

- Identical machines;
- Uniform machines with different capacities;
- Unrelated machines.

Usually in the optimization problems, the schedules for parallel machines use either the criterion of the minimal total machine operation time $\sum_j C_j$ or the makespan $C_{\max}$. The first case was described above in Sect. 2.5.2, as, for example, the so-called problem of allocation; the second option is discussed below.

13.6.1 Identical Parallel Machine Scheduling

Let us consider the problem of scheduling by criterion of the makespan $C_{\max}$ (Sect. 2.2.2). For this case, the classification formula of the problem has the form

Table 13.19 List of orders

Order number	6	3	7	4	1	5	9	8	2
Processing time p_i	13	11	10	10	8	8	6	5	3

Table 13.20 Attaching the jobs to the parallel identical machines

Machine 1			Machine 2			Machine 3		
Job	p_i	τ_{r1}	Job	p_i	τ_{r2}	Job	p_i	τ_{r3}
6	13	11.7	7	10	14.7	1	8	16.7
3	11	0.7	4	10	4.7	5	8	8.7
			8	5	−0.3	9	6	2.7
						2	3	−0.3

$$P\big|\,\big|C_{\max}, \tag{13.27}$$

where P is the designation of identical parallel machines.

The paper (Sule 2007) describes the algorithm of this problem's solution, which is illustrated in terms of data in Table 13.19. The jobs in Table 13.19 are positioned in order of descending processing time. Let the number of parallel machines $m = 3$.

Let us determine the minimal possible value

$$C^*_{\max} = \frac{1}{m}\sum_{i=1}^{n} p_i = \frac{1}{3} \times 74 = 24.7.$$

We assume that each of the machines is available for scheduling new jobs, if the sum of the previously planned jobs for it is less than $C^*_{\max}$. The difference between $C^*_{\max}$ and total time of previously planned jobs at each step of scheduling is called remaining cumulative time. We denote this value as τ_r.

We attach the next job work in order of descending processing time to the j-th machine, if the obtained here value τ_{rj} is greater than or equal to 0. In the case when τ_{rj} becomes less than 0, you need to begin scheduling for the next machine. Finally, if the next operation results in that is τ_{rj} less than 0 for each of m machines, the job is attached to that of them, for which value τ_{rj} is maximum. The application of this algorithm to the data in the example in Table 13.19 is shown in Table 13.20.

Scheduling begins with attachment of the longest job 6 to the machine, which is considered to be the first (i.e. the most desirable for the job). Since after this attaching the value of remaining cumulative time is sufficient, then we can also attach job 3 to the machine.

Then, obviously, it is necessary to start scheduling for machine 2. Attachment of jobs 7 and 4 to this machine gives remaining time τ_{rj} equal to 4.7, which is not enough to attach jobs 1 following by value of processing time. Therefore, scheduling for machine 3 begins, during which it is possible to attach jobs 1, 5, and 9.

For scheduling job 8 it is suggested to find that of the machines, for which τ_{rj} is maximum—this machine is machine 2. Similarly, job 2 is attached to machine 3.

As a result of scheduling we obtain that for machines 2 and 3 the actual value $C_2 = C_3 = C_{\max} = 25$, which is the closest possible to the minimal value $C^*_{\max} = 24.7$. Simultaneously, for machine 1 value $C_1 = 24$, which indicates quite uniform loading of the equipment.

13.6.2 Schedules for Parallel Unrelated Machines

The classification formula of the corresponding optimization problem for parallel machines of any type has the form

$$R| |C_{\max}, \tag{13.28}$$

where R is the designation of unrelated parallel machines.

In general case, this problem can be stated for m parallel machines, to which n jobs arrive within some period (horizon) at different time points for processing. As an example we use the data in Table 13.21.

Let us consider quite efficient algorithm to solve the stated problem, suggested in (Sule 2007). First of all, we note that execution of any i-th job on the j-th machine can start at any moment

$$t_{j(l+1)} = \max\left(C_j^l, r_i\right), \tag{13.29}$$

where C_j^l is the execution time point on the j-th machine of the previous job, which has sequential number l in the schedule for this machine. Thus, the next planned i-th job with sequential number $l + 1$ can be finished on the j-th machine by time point

$$C_{ji} = t_{j(l+1)} + p_{ji} = \max\left(C_j^l, r_i\right) + p_{ji}. \tag{13.30}$$

We will do scheduling by steps with number k, and at each step in the schedule new jobs are inserted. By the beginning of step k on each of the machines with number j, the execution of the l-th in order job is already in process. Let us denote the maximal time point of availability of all machines at step k through t_k, i.e.

Table 13.21 List of orders

Order number	1	2	3	4	5	6	7	8	9	10
Arrival time r_i	0	0	0	0	2	4	4	7	8	9
Processing time on machine 1 p_{i1}	3	6	4	4	2	3	4	2	5	4
Processing time on machine 2 p_{i2}	2	4	2	5	5	1	4	3	4	3
Processing time on machine 3 p_{i3}	5	6	4	4	4	2	3	3	5	5

Table 13.22 Sequencing of jobs

k					Possible job completion time points C_{ji}									
t_k	j	I_{jk}	l	C_j^l	1	2	3	4	5	6	7	8	9	10
$k = 1$ $t_1 = 0$	1	–	0	0	3	6	4	4	–	–	–	–	–	–
	2	–	0	0	2	4	2	5	–	–	–	–	–	–
	3	–	0	0	5	6	4	4	–	–	–	–	–	–
$k = 2$ $t_2 = 4$	1	1	1	3	–	9	–	–	5	7	8	–	–	–
	2	3	1	2	–	6	–	–	7	5	8	–	–	–
	3	4	1	4	–	10	–	–	8	6	7	–	–	–
$k = 3$ $t_3 = 5$	1	1,5	2	5	–	11	–	–	–	–	9	–	–	–
	2	3,6	2	5	–	9	–	–	–	–	9	–	–	–
	3	4	1	4	–	10	–	–	–	–	7	–	–	–
$k = 4$ $t_4 = 7$	1	1,5	2	5	–	11	–	–	–	–	–	9	–	–
	2	3,6	2	5	–	9	–	–	–	–	–	10	–	–
	3	4,7	2	7	–	13	–	–	–	–	–	10	–	–
$k = 5$ $t_5 = 7$	1	1,5,8	3	9	–	15	–	–	–	–	–	–	–	–
	2	3,6	2	5	–	9	–	–	–	–	–	–	–	–
	3	4,7	2	7	–	13	–	–	–	–	–	–	–	–
$k = 6$ $t_6 = 6$	1	1,5,8	3	9	–	–	–	–	–	–	–	–	14	13
	2	3,6,2	3	9	–	–	–	–	–	–	–	–	13	12
	3	4,7	2	7	–	–	–	–	–	–	–	–	13	14
$k = 7$ $t_7 = 12$	1	1,5,8	3	9	–	–	–	–	–	–	–	–	14	–
	2	3,6,2,10	3	12	–	–	–	–	–	–	–	–	16	–
	3	4,7	3	7	–	–	–	–	–	–	–	–	13	–
$k = 8$ $t_8 = 13$	1	1,5,8	3	9	–	–	–	–	–	–	–	–	–	–
	2	3,6,2,10	4	12	–	–	–	–	–	–	–	–	–	–
	3	4,7,9	3	13	–	–	–	–	–	–	–	–	–	–

$$t_k = \max_j \left(C_j^l\right). \tag{13.31}$$

The set of jobs planned for the machine with number j by the beginning of step k will be denoted as I_{jk}, and the quantity of already scheduled jobs, as it was said before, is l. We will calculate using Table 13.22.

At first step $k = 1$, sets I_{jk} are empty, the quantity of jobs in each set $l = 0$, all the machines are free, and so $C_{1,0} = C_{2,0} = C_{3,0} = t_1 = 0$. By this time jobs $1 \div 4$ (Table 13.21) are available for processing and for these jobs the possible completion time points coincide with the values of their processing time (Table 13.22).

Let us set the option of execution of any possible job giving, which gives its minimum completion. In this case, such options are jobs 1 and 3 on machine 2 with value $C_{2,1} = C_{2,3} = 2$. To select the best of these options (Sule 2007), it is recommended to determine difference δ between the lowest value C_{ji} on other machines and find minimal C_{ji}, compare these values for both options, and choose the option with the highest calculated δ.

In this case for job 1, the lowest value C_{ji} if this job is executed on other machines (except for machine 2) is 3. So $\delta_1 = 3 - 2 = 1$. Similarly, for job 3 $\delta_3 = 4 - 2 = 2$ and we should select job 3 as the first job executed on machine 2. For machine 1, the lowest value $C_{1,1} = 3$ for job 1. In this case for machine 3 jobs 2 and 4 remain possible, of which less $C_{3,4} = 4$.

At the next step $k = 2$ for each machine the quantity of the previously planned jobs $l = 1$, sets of planned jobs $I_{1,2} = \{1\}$, $I_{2,2} = \{3\}$, $I_{3,2} = \{4\}$ (Table 13.22). Accordingly, $t_2 = \max\left(C_1^1, C_2^1, C_3^1\right) = 4$.

Out of four jobs, which can be started at time point 0, at step 2 only job 2 remains unscheduled. Besides, by time $t_2 = 4$ jobs 5, 6, and 7 will be available. For example, for job 6 on machine 1 the possible time of job completion $C_{1,6} = \max\left(C_1^1, r_6\right) + p_{_{1,6}} = \max\left(C_1^1, r_6\right) + p_{_{1,6}} = \max(3, 4) + 3 = 7$. The lowest values of C_{ji} at this step are $C_{1,5} = C_{2,6} = 5$. We add jobs 5 and 6 to the schedule and go to step 3 with $t_3 = 5$.

By time $t_3 = 5$ new jobs do not arrive yet, and that is why only jobs 2 and 7 are available, and the lowest value $C_{ji} = C_{3,7} = 7$. We plan job 7 for machine 3 and go to step 4.

At step 4 $t_4 = 7$, and job 8 becomes available, which is more reasonable for machine 1. When going to step 5, we find that at this step value $t_5 = 7$, i.e. remains the same as at previous step. It is obvious that at step 5 the only available job 2 should be scheduled.

At the sixth step $t_6 = 9$ and two more unscheduled jobs 9 and 10 are available. For job 9 the possible completion time on machine 1 $C_{1,9} = C_1^3 + p_{1,9} = 9 + 5 = 14$; similarly on machine 2 value $C_{2,9} = C_j^l + p_{2,9} == C_2^3 + p_{2,9} = 9 + 4 = 13$. At the same time for machine 3 value $C_{3,9} = r_9 + p_{3,9} = 8 + 5 = 13$. For job 10, values $C_{j,10}$ are found in a similar way. $C_{2,10} = 12$ is the lowest value of C_{ji} at step 6 and job 10 should be added to the schedule for machine 2.

At the next step 7 one job 9 remains which should be scheduled for the machine 3. At the last step 8 the total duration of execution of all jobs $t_8 = 13$ is determined. Figure 13.8 shows Gantt Chart for the made schedule.

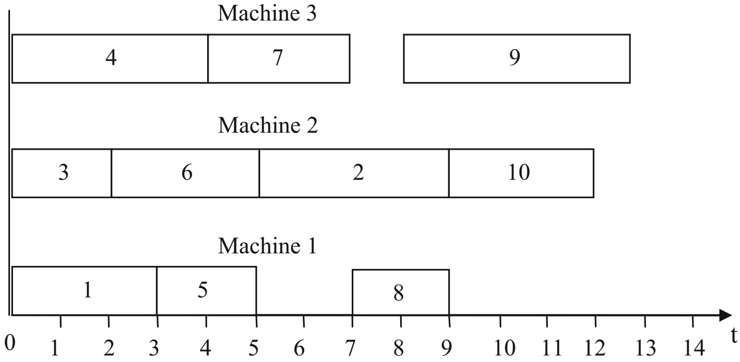


Fig. 13.8 Gantt chart for three parallel machines

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Shop Floor Scheduling: Multi-stage Problems 14

14.1 Synchronized Flowshop Production

This definition applies to three main types of production referred to in Sect. 1.3.1: automated line (type 3a), versatile transfer line that produces only core products (type 3b), and versatile transfer line that produces both core and by-products (type 3c). The main planning parameters of these kinds of repetitive production lines are cycle, rhythm, number of jobs, as well as the size of stock (cycle stock) in the buffers between the machines.

The cycle of repetitive line τ is the time between launches of two adjacent objects (work pieces, materials, raw materials) or outputs from the repetitive line of two adjacent finished products. In cases where the transfer from operation to operation is carried out by transfer quantities, the rhythm of production line is calculated.

$$R = Q\tau, \tag{14.1}$$

where Q is the quantity of the transfer lot.

To provide a unified cycle or rhythm for the repetitive production line it is synchronized, meaning achievement of equality or multiplicity of the time of individual operations of the process to the established rate. Typically, synchronization is performed in two stages. Pre-synchronization with deviation from the cycle time within 10 % is performed during engineering of the production line and final synchronization—during its adjustment.

Full synchronization at all stages of processing is difficult to achieve because it is required to have stepless adjustment of the processing time at each operation. So actually, there is a situation in which the rhythm of the production line is defined by duration of the longest operation and the machines occupied with other operations, are idle for a while. We can suggest to consider the repetitive production to be synchronized, if such idling does not exceed 10 % of the rhythm time. Discussion of the synchronization methods is beyond the scope of this book.

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14.1.1 Discrete Product Lines

Discrete products in the production scale providing a full load of the line with one kind of product can be produced on automated lines. If the scale of production is not so big, then for discrete products versatile transfer lines are used, at the end of which finished products are yielded.

The operation scheduling for an automated line involves only determining duration T of its work in the planning period in accordance with products demand D and line performance P

$$T = D/P. \tag{14.2}$$

For versatile transfer line that produces several kinds of products, it is necessary to schedule its work. In this case, it makes sense to consider this line as a single machine, for which a periodic schedule with economic lots sizes (Sect. 13.4) is prepared. The difference of a production line from a single machine appears mainly as necessity to account the reliability of the machines that make up the line.

Due to the limited reliability of each machine in the line, it is necessary to accumulate some quantity of intermediate products in the buffers between the machines, which ensures operation of the line during repair of one of the machines. Average line performance thus depends not only on the technical performance of the machines used but also on the buffer sizes there between. The main theoretical provisions of such dependence have been developed in Vladzievsky (1950, 1951).

In general, in the synchronized production line (Fig. 14.1), several machines with similar performance $P_1, P_2, P_3 \ldots$ operate, between which there are buffers with capacity $B_1, B_2 \ldots$ Each machine has a limited reliability, which is characterized by mean time between failures $\theta_1, \theta_2, \theta_3 \ldots$. Thus, troubleshooting of each machine requires average time $\zeta_1, \zeta_2, \zeta_3 \ldots$.

If one of the machines fails, in both directions of the line the wave of stoppage propagates in chain order. For example, if machine 2 fails, machine 3 stops due to lack of production work pieces, and machine 1 stops due to buffer overflow between that machine and machine 2. If machine 2 is idle for a long time, the wave reaches the last machine in the line, and product is no longer produced. Similarly, when the first machine in the line stops the supply of raw material or materials for processing should be stopped. Obviously, this process can be slowed down, but certainly not stopped due to sufficiently large buffer capacity.

The design characteristics of the considered process depend on the type of statistical distribution of failure probability. It is most commonly believed that

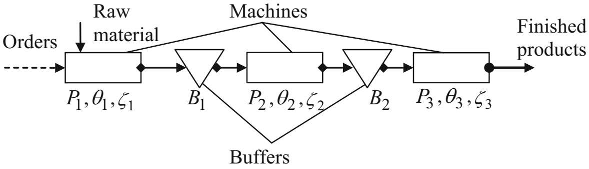


Fig. 14.1 Diagram of synchronized production line

there is a normal (exponential) distribution. However, even in this simplest case, it is rather difficult to calculate the reasonable buffer sizes with different machine reliability. For rough estimates the scheme proposed in Bluemenfeld and Li (2005) is useful, in which all machines have the same reliability with mean time between failures (MTBF) θ and the same average time for troubleshooting ζ.

Suppose that the theoretically highest possible line capacity is $P_{\max}$. In this case, according to Bluemenfeld and Li (2005) average performance of the line of M machines

$$\overline{P} = \frac{P_{\max}}{1 + \zeta/\theta + [(M-1)\zeta/\theta]/[1 + (M/4)(1 + 2\zeta/\theta)(B/\zeta P_{\max})]}. \qquad (14.3)$$

Figure 14.2 shows the calculated by formula (14.3) dependencies of average performance on the buffer size B in pieces with different number of machines in line M for time between failures $\theta = 4$ h, troubleshooting time $\zeta = 0.33$ h, and maximum performance $P_{\max} = 100$ pcs./h. As it follows from the graphs in Fig. 14.2, the influence of buffer capacities on the line performance rises significantly when number of machines in the line increases.

The book of Li and Meerkov (2009) considers in depth various options of failure distribution in the lines that are used mainly in the automotive industry and provides appropriate algorithms for calculating the performance.

Availability of buffers in the line removes the need for continuous fixed rhythm of its operation. At the same time, we can talk of an average rhythm of output corresponding to the average productivity.

14.1.2 Lines for Process Production

In the studies on process production organization and planning, the concept of repetitive line rhythm is almost never used. An exception is the paper of Lobanskaya and Sergina (1969), which defined the rhythm of production in oil refineries. The authors of this article noted the lack of understanding of the concept of process production rhythm and offered in this case to assess the rhythm by the uniformity of

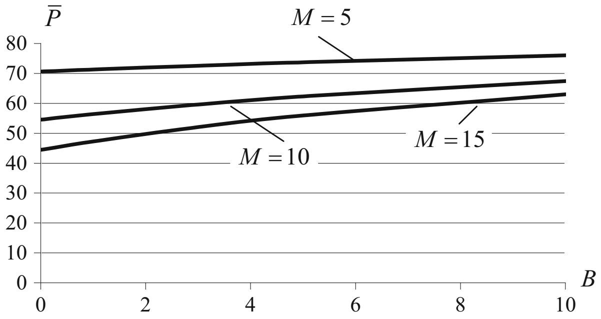


Fig. 14.2 Diagram of the line's average performance

finished product output or raw material consumption. At the same time, this study has shown that at primary crude oil processing, where the raw material is directly consumed, the uniformity of production is usually much higher than at the stage of re-refinery, i.e. finished product output.

As noted above in Sect. 1.3, if various stages of the process production occur simultaneously in different apparatuses (machines), this production is considered to be continuous. The transfer of intermediates in this production is possible only by batches transferred through buffers. Therefore, we can only talk of average (not fixed) rhythm, which is determined by the transfer time of the batches between machines.

In those cases, where the possible time of batch transfer at different stages of the manufacturing process is different, the average rhythm is equal to the maximum of such time, i.e. the finished product output rate is determined by the machine with the lowest throughput—a bottleneck. If one and the same machine can be used for various process operations (Sect. 7.3.1), one of these operations can be a bottleneck. In the example given above in Sect. 7.3, reaction 2 is such bottleneck (Fig. 7.4).

Buffer sizes in process manufacturing are determined, firstly, by capacity of apparatuses and, secondly, by probabilistic parameters of the process of finished product shipment. Obviously, the buffer for each of the products obtained at any stage should not be less than the batch volume of this product Q. In cases where the size of the shipment batch is more than production batches, buffer size B must provide unloading of several production batches. Thus, it should be

$$B \geq nQ, \tag{14.4}$$

where parameter n is determined by product quantity $Q_{\max}$, which can be produced for the longest expected time between shipment of this product, namely

$$n - 1 \leq Q_{\max}/Q \leq n. \tag{14.5}$$

14.1.3 Flexible Flow Lines

If successive operations in a regular line cannot be synchronized adequately and simultaneously it is necessary to get high performance, flexible flow lines are used. In this line, which is sometimes also called a hybrid flowshop line, on one or more operations several parallel identical machines (Fig. 14.3) are installed. Such lines are often used in various industrial fields, especially in process manufacturing (Quadt 2004).

In the hybrid flowshop line a variety of products can be consistently produced in general. The complexity of scheduling for this line is that for each of the parallel machines it is necessary to determine the lot size of the processed product directly in the process of scheduling.

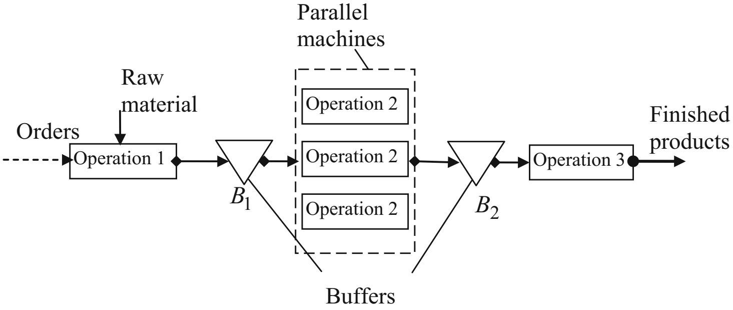


Fig. 14.3 Flexible flow line on one operation

A detailed study of this problem for discrete manufacturing is given in Quadt (2004), and the problem is solved in three successive stages. At the first stage, the planning is done for a group of machines performing the operation, which is the bottleneck of the process. At the second stage, based on the results of the first stage, lot sizing and planning for the rest machines are carried out. As a result of the second stage, the so-called machine/time slots are determined for each product by, which show what time and on what machine each product to be processed.

The third step is required in those cases where one line produces several different groups of similar products. In such cases, at the second stage the planning is made by aggregate groups, and at the last stage the disaggregation of the plan for each product group is confirmed.

As the objective function at the first stage minimization of direct costs is applied, at the second stage—minimization of process time. During the third stage, the changeover cost is minimized while maintaining quite short processing time.

For solution at the first stage, we use the methodology outlined above in Sect. 11.1.3, for Capacitated Multi-Item Lot Sizing Problem (CLSP). To account for the changeover duration for the i-th product s_i in the model described by dependencies (11.5)–(11.9), constraint (11.9) is adjusted, which takes the form

$$\sum_{i=1}^{n} p_i X_{it} + s_i \delta_{it} \leq P_t, \tag{14.6}$$

where P_t is understood as the total capacity in hours for a period of time t of all parallel machines on the operation being a bottleneck.

Since the optimal lot sizes of the products for each planning period are established at the first stage of planning, i.e. by bottleneck capacity, at the second stage we only need to perform the procedure of schedule roll-out for each operation in accordance with its position in the process. As the number of parallel machines on various operations is different, then in this procedure it is necessary to take into account the changes in lot size at the same time.

Description of various approaches to planning of hybrid workshop line operation is contained in the review articles (Ribas et al. 2010; Ruiz and Vázquez-Rodríguez 2010).

14.2 Automated Assembly Lines

The assembly line consists of a number of assembly stations along the conveyor transporting the consistently assembled product. Assembly operations can be carried out directly on the conveyor, and in this case, it is called a working conveyor. If the object to be assembled is removed from the conveyor to perform the assembling operation, and after the operation is returned to the conveyor, it is called a distribution conveyor.

When assembling on the working conveyor all operators move within their work areas and then return to its front end. Distribution conveyor enables the operator to reside at his workplace. Moreover, the continuous motion of the distribution conveyor is not required and this conveyor may be run periodically and thus moved stepwise.

Depending on the number of types of products assembled on the line, and also on the sequence of launching different types, the lines can be differentiated as single-model line, multi-model line, and mixed-model line (Fig. 14.4).

Various geometric figures in Fig. 14.4 represent corresponding assemble products. Mixed assembly of various products (Fig. 14.4b) can be used in cases where there is no need for significant changeover from one product to another. Otherwise, it is necessary to use the diagram shown in Fig. 14.4c, i.e. assemble the products of the same type by lots.

During assembly, the full scope of assembly jobs is made of individual assembly operations—tasks, which must be performed in a certain sequence. For each of these tasks, the standard time for its fulfilment by a semiskilled operator is known. When designing the assembly line the main problem is the correct distribution of

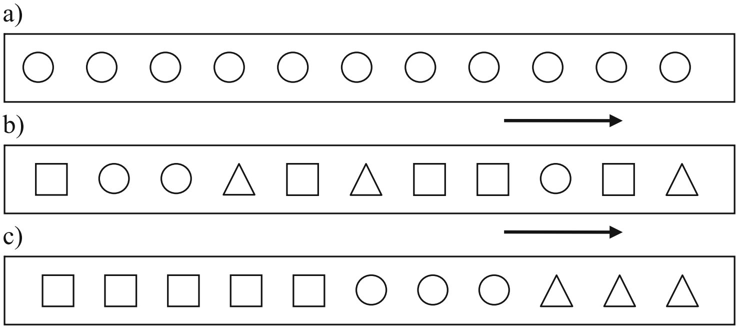


Fig. 14.4 Diagram of assembly lines: (**a**) single-model line; (**b**) mixed-model line; (**c**) multi-model line

operations between assembly stations, providing the so-called Assembly Line Balancing Problem (ALBP).

Assembly lines are divided into paced assembly lines and unpaced assembly lines. Balancing problem exists for both types of lines, but in the latter case, it is also necessary to determine the correct location of the buffers between assembly stations.

Assembly Line Balancing Problem is discussed in a large number of papers, the analysis of which is given in the review (Becker and Scholl 2006). If adequate balance of the assembly line is impossible, as well as for processing lines, sometimes parallel assembly stations are installed, i.e. hybrid workshop lines are used (Sect. 14.1.3). Consideration of the problem of the assembly line synchronization, as well as the similar problem of processing line synchronization in Sect. 14.1, is beyond the scope of this book.

We note only that there is an option of assembly line operation, in which so-called Self-Balancing Assembly Line may take place (Bartholdi and Eisenstein 1996). In this line, operators are located along it in ascending order of individual possible speed of their work. In this case, each successive operator can undertake the functions of the previous operator, if the latter cannot keep up the pace of the line. Of course, for this purpose, the operators must be trained appropriately.

14.2.1 Scheduling for Unpaced Assembly Lines

Let us consider the scheduling of the assembly line with buffers between assembly stations. If any of the buffers is full, then the previous station is forced to stop working because it becomes blocked; if the buffer is empty, and at the previous station the assembly is not finished yet, the work on the next station is ceased. Therefore, in the single-model line, the line throughput is ultimately determined by the station with the lowest throughput.

In operation of the line with mixed launching sequence for various products, the situation depends on the launch sequence. It is obvious that planning should provide such a launch sequence for assembly of various products that ensures minimal downtime of the line.

If the demand for the products produced on the line is constant, it makes sense to develop a cyclic-repeated schedule of these products output. Recurring schedule greatly simplifies the supplying of the line with necessary materials and other assembly elements.

Assume that l kinds of different products should be assembled on the line, and the demand for the k-th product during a plan period, such as a month, is D_k. We introduce the vector

$$D^* = \left(\frac{D_1}{a}, \frac{D_2}{a} \dots \frac{D_l}{a}\right), \tag{14.7}$$

for consideration with components D_k/a, where a is the most common divider. This vector is represented by the smallest set of the assembled products having the same proportions as many other products produced within the entire planning period. This set is called Minimum Part Set (MPS), and the number of parts to be performed in this set is equal to the total of the components of this set, i.e.

$$n = \frac{1}{a}\sum_{k=1}^{l} D_k. \tag{14.8}$$

If one makes an optimal assembly schedule for the minimal set, this schedule can be repeated during the plan period and will be optimal. An optimality criterion is usually the minimum duration of the assembly of all the parts in the minimal set. Let us assume that the processing time of the i-th job on the j-th machine is p_{ij}. When scheduling for the minimal set, the processing time of some jobs can be repeated if these jobs are an assembly of identical products.

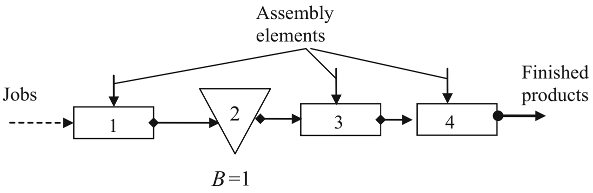
As an example, we consider the assembly scheduling for the line with diagram that is shown in Fig. 14.5. The line consists of three assembly stations 1, 3, 4, and buffer 2 containing one assembling product. The line is designed for assembly of several types of products, and the setup time from one type to another is assumed to be inconsiderable, i.e. it is a mixed assembly line (Fig. 14.4b).

An effective algorithm of optimal scheduling for an unpaced assembly line is suggested in the paper (McCormick et al. 1989) and is called “Profile Fitting” (PF). The block diagram of the algorithm is shown in Fig. 14.6.

Let us consider the algorithm operation when scheduling for the line in Fig. 14.5, which should collect the products of three types in quantity $D_1 = 24$ pcs./month, $D_2 = D_3 = 12$ pcs./month. Since the most common divider of these values $a = 12$, then by using formulas (14.7) and (14.8), we obtain the vector of minimal part set $D^* = (2,1,1)$ and the number of part in the set $n = 4$. Table 14.1 shows the standards of processing time of assembling different types of products on each of three assembly stations.

In accordance with the block diagram in Fig. 14.6, first we find a job with the highest total processing time, and in this case, job 1 can be considered as this kind of jobs. Then (block 3), we consistently consider the practicability of including the rest of the jobs into the schedule as the second job in order. To do this, we construct the Gantt chart for the assembly line with every possible sequence option of two jobs $\{1, 2\}$, $\{1, 3\}$, $\{1, 4\}$.

Fig. 14.5 Diagram of assembly line



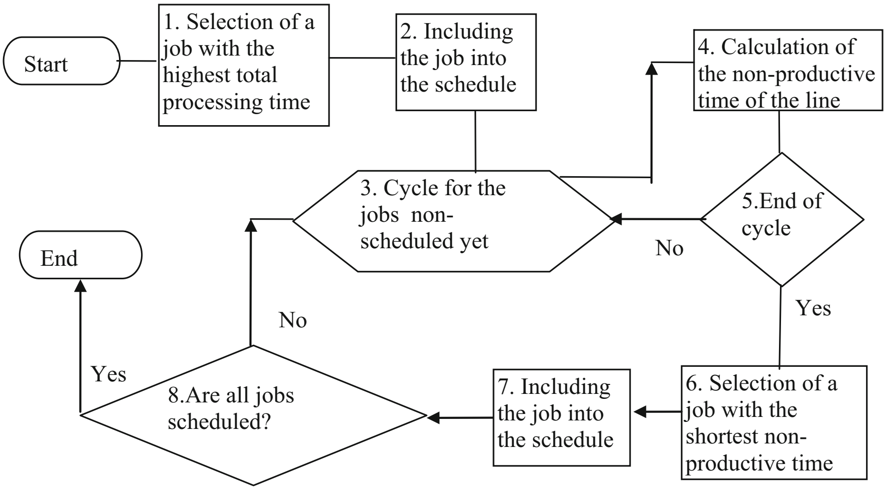

Start
1. Selection of a job with the highest total processing time
2. Including the job into the schedule
4. Calculation of the non-productive time of the line
3. Cycle for the jobs non-scheduled yet
5.End of cycle
No
Yes
End
No
Yes
8.Are all jobs scheduled?
7. Including the job into the schedule
6. Selection of a job with the shortest non-productive time

Fig. 14.6 Block diagram of Profile Fitting algorithm

Table 14.1 Time standards on three assembly stations

Product type	1		2	3
Job no.	1	2	3	4
p_{i1}	4	4	3	2
p_{i2}	3	3	2	3
p_{i3}	3	3	3	2

For example, Fig. 14.7a shows the sequence option of jobs 1 and 2. When constructing, we initially believe that job 1 is performed without delays. If job 2 is performed as the following in order, then at the second and third stations downtimes of 1 h are unavoidable. Thus, the non-productive time of the line (block 4 in Fig. 14.6) in this case is 2 h.

If after job 1 job 3 is performed (Fig. 14.7b), then after the first station job 3 can be transferred to station 2 immediately. After assembling on station 2, however, job 3 should wait for station 3 to be freed from previous job 1, i.e. station 2 appears to be blocked. In this case, the non-productive time of the line is 1 h.

When performing job 4 after job 1 (Fig. 14.7c) the assembly on the first station finishes before the station 2 is free. However, in this case, job 4 can be moved into the buffer, and accordingly, does not create non-productive time. Therefore, the case of sequence {1, 4} is the best possible and should be recorded in the schedule (blocks 6 and 7 in Fig. 14.6).

With continuing operation of the algorithm until the end (block 8), the sequence for the minimum set becomes {1, 4, 3, 2}. Figure 14.8 shows this sequence for the first cycle and a section of this sequence in the second cycle. Obviously, in this case the average rhythm of the line can be understood as the assembly period of the minimum set, i.e. the schedule cycle time, which is equal to 13 h.

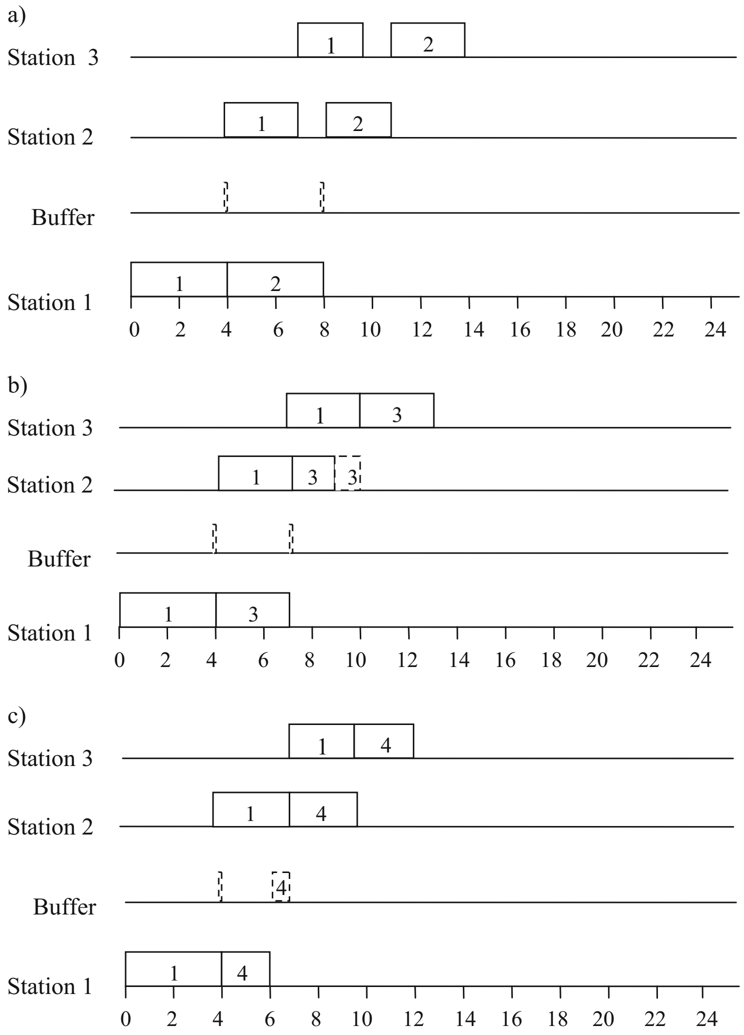


Fig. 14.7 Various options of assembly sequence at the second step of the algorithm: (**a**) sequence {1, 2}; (**b**) sequence {1, 3}; (**c**) sequence {1, 4}

14.2.2 Scheduling for Paced Assembly Line

In this line, the rhythm refers to time interval between two adjacent finished products assembled on the line. Paced assembly line is a smoothly moving conveyor, along which the working areas are located. Each zone is assigned to a certain number of assembly operations. In each area, one or more operators can work simultaneously, and an average production load on each of the operators in all areas

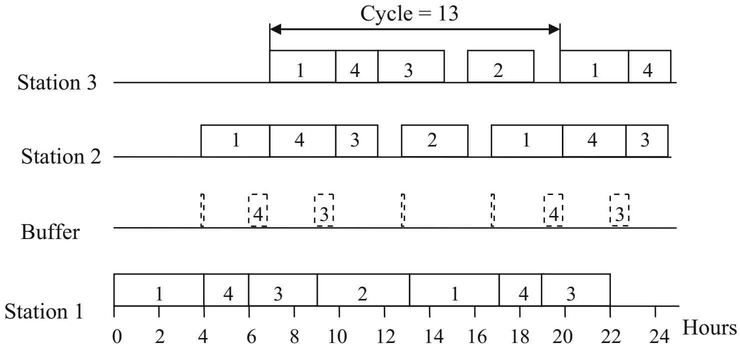


Fig. 14.8 Gantt chart for minimum part set assembly

should be the same. The equality of workload on operators is that what provides a constant rhythm of product output.

The length of the working area depends on the processing time of the assigned operations. If the line outputs one product of constant composition, the processing time of operations in the area must be multiple of the optimal value of the load for a single operator. If the operators are located sequentially along the area, then the area length is proportional to the number of operators.

However, in most cases even homogeneous products manufactured on the assembly line have different composition. For example, in the assembling of cars of the same brand, to the so-called base model various additional device are often installed or instead of the devices of the basic model other devices with additional features are used. Therefore, even in the absence of the need for significant readjustment, continuous operation of the line can be ensured only by reasonable scheduling, i.e. with the correct launch sequence of differing products.

Let us consider the simplest case, when in addition to the base model another modified product, defined by the presence of some additional device, is produced. In accordance with the assembly process this device must be installed in some *k* working area. Let the assembly processing time in this area for the base model is equal to *x* and for the model with modification—*ax*, i.e. *a* times more than the initial processing time. Suppose also that the models with modification occupy part of overall total demand *D* equalling to *D*/*b*.

The presence of modification in area *k* can be considered as a capacity constraint imposed on this area and is called "criticality index". The criticality index is equal to

$$I = a/b, \tag{14.9}$$

and length of working area *k* with this constraint increases by *a* times.

The higher the criticality index is, the more difficult the assembly line balancing. Basically this can be done in two ways. In the first case it is possible to focus manufacturing of modified products in one group and to increase the number of

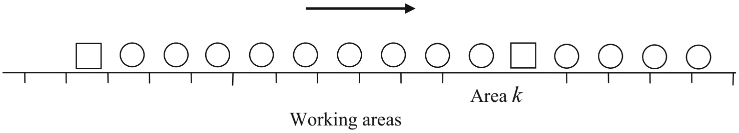


Fig. 14.9 Mixed-model line with one modified product

operators in area k by a times for the assembling time of the entire group. However, here it is necessary to have enough operators and assembly elements at the required moment, which is difficult to accomplish.

A simpler and more common way is to distribute the modified products on the line as evenly as possible. For definiteness, we set $a = 3$; $b = 10$. In this case, the line operation can be presented schematically in Fig. 14.9.

Let us assume that the processing time in each of the working areas in Fig. 14.9, except for area k, is 1. To maintain the rhythm in area k, the average processing time should be equal to 1, i.e.

$$\frac{(b-1)x + ax}{b} = 1, \tag{14.10}$$

$$\text{from which} \quad x = \frac{b}{b + a - 1}. \tag{14.11}$$

When the modified product appears in area k the operator performs assembling of a necessary additional device moving from the beginning to the end of the area. Upon completion of this product assembling, most of the area k is occupied by the regular product, but since processing time x is less than a unity according to expression (14.11), then the operator of area k gradually shifts to the beginning of the area. When choosing the length of the area more than regular by a times, the operator definitely manages to return to the beginning before the modified product approaches.

The above example describes the case when the products assembled on the same line differ by only one parameter. As a rule, the situation on the serial production assembly lines is much more complicated. For example, on the assembly lines of cars of the same brand some specific instances can vary by the type of engine and gearbox, interior configuration, colour, etc.

The line changeover for the colour change is costly; at the same time, changing of the engine type leads to notable but not very large changes in the processing time within one working area. In some cases, for example, when changing the type of chairs or instruments, the processing time of the assembly may vary slightly.

Since it is necessary to consider such a variety of parameters, actually we have to use both multi-model and mixed-model planning on the assembly line simultaneously (Fig. 14.4). Furthermore, although “make-to-stock” strategy is mostly used on assembly lines some instances are assembled in accordance with the placed special order. In these cases, to some extent it is necessary to take into account the shipping date specified in the order. Sometimes the date of shipment may also be defined by the need to co-shipment of several instances of the product under the terms of transportation.

It is not always possible to meet all these requirements at the same time and in full. Therefore, usually in planning these requirements are ranked, i.e. some hierarchy is established: colour can be considered as a more important requirement, the date of shipment often comes next, etc. The book (Pinedo 2005) presents the algorithm of the type, which is called “Grouping and Spacing” developed for the automotive industry.

At the first step, the algorithm sets the planning horizon and the number of assembled products for which planning is carried out. This horizon is determined by the scheduler’s experience as the time, during which significant fluctuations in the requirements for line performance and assembled nomenclature are hardly probable. Typically, the value of the planning horizon is from 1 day to a working week.

At the second stage, the planned jobs are divided into groups to perform an operation that requires significant expenditures on changeover. The duration of this operation for each group depends mainly on the number of items in the group. If some group appears to be much more than other groups, it can be divided into parts. Such division slightly increases the cost of changeover, but can accelerate the performance of individual controlled orders.

At the third step, the fulfilments of obtained groups is planned in such a way so to fulfil given orders with smaller delay. In addition, as close as possible to these orders those jobs are scheduled, which must be shipped together with important orders.

Finally, at the last stage, the remaining jobs are distributed along the conveyor so that various modifications of the basic configuration of the product were placed as even as possible.

Let us consider the example (Table 14.2). Let us assume that within an 8-h shift ten instances of the product different by colour and motor unit type are assembled. The line cycle in this case is 0.8 h. Besides, jobs 2 and 9 must be shipped together with desired shipping point 3 h after the operation start.

In this example, the planning horizon is set, and the jobs are divided into groups with the same colour. In order to bring completion of job 9 into proximity to the possible time of shipment together with job 2, first the assembly of products with colour 3 should be scheduled, and then directly the assembly of products with colour 1. Since out of total of 10 pieces two products (2 and 7) are assembled with motor unit type 2, then for reasons of uniform assembly between jobs with type 2 four jobs with unit type 1 should be.

Table 14.2 Set of jobs in the line for scheduling

Job	1	2	3	4	5	6	7	8	9	10
Colour code	1	1	1	1	1	2	2	2	3	3
Engine unit type	1	2	1	1	1	1	2	1	1	1
Preferred shipping time point	–	3	–	–	–	–	–	–	3	–

Eventually a reasonable sequence corresponding to grouping and spacing algorithm is {10, 9, 2, 1, 3, 4, 5, 7, 6, 8}. For this sequence, grouping by colour occurs; joint shipment is possible after job 2 at a time point equal to 2.4 h; jobs with unit type 2 are evenly distributed along the assembly line.

Of course, the above example is just a special case of the possible diversity of real situations. However, grouping and spacing algorithm is quite simple and can be used satisfactorily for quick planning of job launching for assembly.

14.2.3 Scheduling for Mixed Assembly Lines

When using mixed-model line in Just-In-Time system, Toyota developed a method of job launch scheduling which is targeted at providing even consumption of resources (Monden 1983). Let us consider this method through the example.

Let us assume that three modifications of the product in quantity of $Q_1 = 3$, $Q_2 = 6$, $Q_3 = 2$ are produced on the line during one shift and so the total number of jobs in the shift target is $Q = 11$. Products of different types differ from each other by presence of various components of four kinds (Table 14.3).

In the ideal case of even consumption of components at launching of each new job the quantity of each consumed component must be equal to value N_j/Q, where N_j is the total requirement per shift. For example for the first component the total requirement

$$N_1 = q_{11}Q_1 + q_{21}Q_2 + q_{31}Q_3 = 1 \times 3 + 0 \times 6 + 1 \times 2 = 5,$$

and average consumption per job is $5/11 = 0.45$.

Since actually consumed amount of each component is determined in accordance with the configuration according to Table 14.3, then between this actual quantity and the average consumption some difference occurs. The idea of the described algorithm is to minimize standard deviation Δ_{ik} of the actual consumption of the components from the theoretical average value

$$\Delta_{ik} = \sqrt{\sum_{j=1}^{n} \left(\frac{kN_j}{Q} - X_{j,k-1} - q_{ij} \right)^2}, \tag{14.12}$$

Table 14.3 Configuration of manufactured products

	Per product q_{ij}			
Components	1	2	3	Total per shift N_j
1	1	0	1	5
2	0	1	0	6
3	1	1	0	9
4	1	0	1	5

where k is the sequential number of jobs sequence, $X_{j,k-1}$ is the summed consumed quantity of the j-th component when assembling previous $k-1$ jobs, q_{ij} is the quantity of the j-th component required for assembly of the i-th job, and n is the number of component types.

For example, at the beginning of planning $k=1$, and all values $X_{j,k-1}=0$. If we launch job type 1 as the first in sequence, then

$$\begin{aligned}\Delta_{1,1} &= \sqrt{\left(\tfrac{1\times5}{11}-0-1\right)^2+\left(\tfrac{1\times6}{11}-0-0\right)^2+\left(\tfrac{1\times9}{11}-0-1\right)^2+\left(\tfrac{1\times5}{11}-0-1\right)^2}\\ &= 0.96.\end{aligned}$$

Accordingly $\Delta_{2,1}=0.80$ and $\Delta_{3,1}=1.24$. Since the smallest value is given by value $\Delta_{2,1}$, then job type 2, which uses component 2 and 3, should be the first to go to assembling.

At the next step $k=2$, and values of already used components $X_{1,1}=0$; $X_{2,1}=1$; $X_{3,1}=1$; $X_{4,1}=0$. That is why

$$\begin{aligned}\Delta_{1,2} &= \sqrt{\left(\tfrac{2\times5}{11}-0-1\right)^2+\left(\tfrac{2\times6}{11}-1-0\right)^2+\left(\tfrac{2\times9}{11}-1-1\right)^2+\left(\tfrac{2\times5}{11}-0-1\right)^2}\\ &= 0.39.\end{aligned}$$

Values $\Delta_{2,2}$ and $\Delta_{3,2}$ equal 1.61 and 0.65 accordingly and so job 1 should be the second in the launching sequence.

By consistently performing the described process we obtain optimal sequence {2, 1, 2, 3, 2, 1, 2, 1, 2, 3, 2}. It can be seen that in this sequence jobs of different types are distributed evenly enough. Figure 14.10 shows diagram of consumption of component type 2 during the shift. You can see from Fig. 14.10 that at optimal sequence of start, the actual consumption of the component, shown by the solid line, fluctuates slightly against the average consumption level, which is described by the dashed line.

The described method of scheduling is one of the variants of application of so-called Shifted ideal method with the Euclidean metric described above in Sect. 4.4.1. Recall that the shifted ideal method was also used in Sect. 10.4.2 for selection of analogues of ordered products.

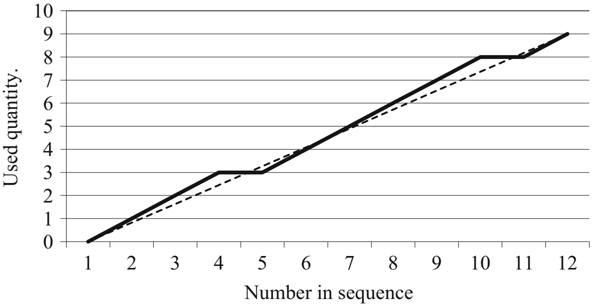


Fig. 14.10 Diagram of consumption of component type 2

14.3 Unsynchronized Flowshop Production

When you synchronize a serial discrete production the full processing time of jobs performed is divided into separate operations of short duration. This allows to choose the set of operations, having about the same duration for each machine in the line. Accordingly, the machine that performs these special operations usually should also be specialized. Therefore, the cost of the line is a significant value, and such lines can be used only in large-scale production.

If the scale of production is small, discrete products are manufactured on widespread general-purpose equipment providing a wide range of processing for such products. General-purpose equipment is usually combined into groups based on function, which facilitates its maintenance and contributes to its interchangeability. Herewith the performance significantly decreases because the absence of established material flow in the production leads to considerable transport costs, idling when awaiting processing, etc.

In many cases, however, it appears that the significant part of manufactured products can be divided into several groups having the same or similar processes. The manufacture of such products is advisable to do using group technology. Definition of this concept has changed significantly over time, since it included gradually not only design and technological properties of the products but also organizational indicators of production.

Currently, perhaps, the definition proposed in the paper (Burbidge 1991) is the most common. Here, the group technology is understood as a method to organize production of organizational units (groups), each of which contains its own set (family) of parts or other objects, and these sets should not intersect. Furthermore, it is very important that each such organizational unit should be composed of all or at least most of the required equipment.

Availability of such equipment in the organizational group united by the similar technology allows arranging this equipment in a sequence determined by the process, i.e. organize flow production. Of course, however, the duration of process steps in this flow is different, i.e. the flow will not be synchronized.

As the organizational group created this way is intended for manufacturing a particular set of different products (objects), it is naturally an area with cell specialization. If these products undergo full cycle of manufacturing within the processing in the area, then such organizational group is a cell manufacturing area.

In general, in a cell manufacturing area, except the serial production line for batch processing, other equipment may be used. However, the existence of such even elementary production line is the basis for creating a cell manufacturing. As the equipment located in the area is often underutilized, then the area staff must be qualified to service several units of available equipment.

Cell manufacturing is often considered in scheduling as separate production units (work centers), with own equipment and staff. In such cases, the planning parameters are set only for the entire work center, and the responsibility for their fulfilment rests with its staff. This approach often facilitates introducing methods of the so-called Lean Production.

Production lines for batch processing are usually organized on the basis of already operating general-purpose production. To divide manufactured products and process equipment into groups optimally the production flow analysis (PFA) is used. In the course of this analysis the extent of use of each piece of equipment is established for production of each of product and the groups similar in composition are selected. A detailed description of this method is given in the book (Burbidge 1997).

In Sects. 14.1 and 14.2 above, when the planning of synchronized production lines was discussed, it was indicated that the buffers are needed there for storing intermediate products in case synchronization is disturbed. In unsynchronized production lines, the situation with buffers is fundamentally different. Here buffers are used not for emergency situations and similar but are integral parts of the line providing its scheduled operation. The stock of intermediate product in a buffer of such line is called cycle stock and the definition of the latter is one of the important scheduling issues.

14.3.1 Modelling for Unsynchronized (Discontinuous) Flow Lines

For single-product discontinuous line, the rhythm of its operation can be determined, which is often called the period of line circulation. This period equals the time interval, after which the line repeats its state. The line circulation period can either be multiple of the shift duration or, vice versa, a shift can include several such periods. Quantity of products Q produced on each operation for the period of line circulation is the same and is called a cycle lot.

The value of interoperation cycle stock Z on different operations varies over time in different ways, with the largest interval of these changes in the range from zero to the value of cycle lot. The schedule of this change during the period of line circulation is called a stock epure. The largest value of the stock should ensure continued operation at the subsequent operation to perform one line circle; the value of the stock at the end of the circle must be equal to the stock at the beginning of this period.

The need to create a stock is defined not only by the difference in performance of the adjacent operations but also by possible shift of job start at these operations. For example, if two adjacent operations are sequentially serviced by one operator, the job at the second operation starts only after the operator goes from the first to the second workstation. In this case, the highest value of the stock equals a cycle lot. If the jobs at both adjacent operations begin at the same time, the greatest value of the stock is determined only by the difference in their throughput.

Typical stock epure is shown in Fig. 14.11. The epure in Fig. 14.11a is typical for the case when the jobs at adjacent operations start at the same time and the performance of the first operation is lower than the second one. In Fig. 14.11b the jobs also start simultaneously, but the performance is higher at the first operation. Figure 14.11c refers to the case of sequential operations by a single operator.

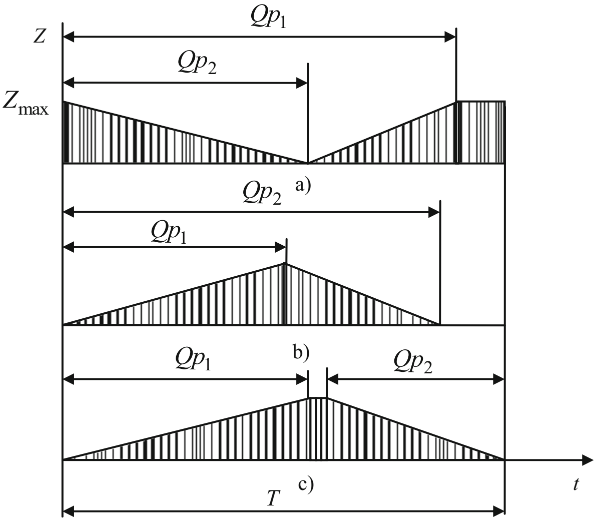


Fig. 14.11 Possible schedules at two adjacent operations

As can be seen from Fig. 14.11, during the period of line circulation T the stock epure consists of three areas: decreasing of the stock value, increasing of the stock value, and constant value. If processing time of the second operation p_2 is lower than processing time of the first operation p_1, then it is possible to start the second operation simultaneously with the first one only if there is a cycle stock (Fig. 14.11a). The quantity of this stock (Paramonov and Soldak 2009) is

$$Z_{max} = Q\left(1 - \frac{p_2}{p_1}\right). \tag{14.13}$$

On expiration of time Qp_2 stock value Z appears to be used up and operation 2 ends. At the same time, operation 1 continues until stock Z_{max} is replenished. If time $Qp_1 < T$, then the stock will not change before the start of new cycle.

If processing time $p_2 > p_1$, then the second operation can start simultaneously with the first one without any stock (Fig. 14.11b). In this case, the cycle stock will grow until at the first operation Q parts are processed and will become constant

$$Z_{max} = Q\left(1 - \frac{p_1}{p_2}\right). \tag{14.14}$$

Then, as the second operation is completed the stock will decrease to zero. At the end of period T the cycle will repeat.

In that case, when the adjacent operations are performed sequentially by a single operator (Fig. 14.11c) within the rhythm the cycle stock will grow first and then during transition of the operator from one operation to another $Z_{max} = Q$ and finally the stock decreases to zero during the second operation.

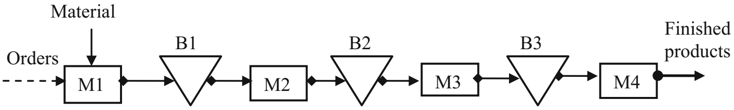


Fig. 14.12 Unsynchronized (discontinuous) production line

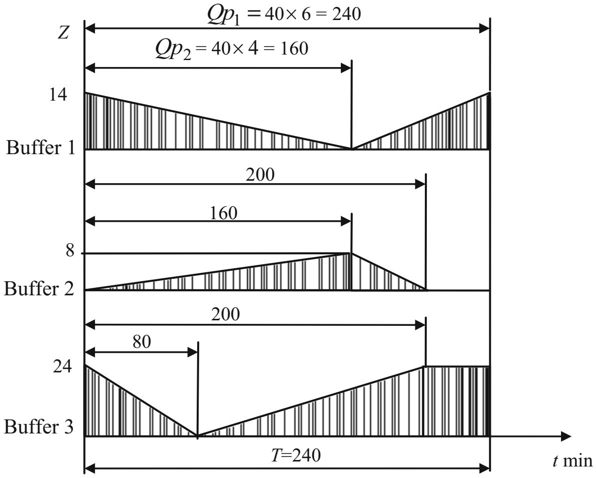


Fig. 14.13 Operating procedure of discontinuous production line

Now let us consider the line consisting of four machines and three buffers (Fig. 14.12).

Let us assume that the period of line circulation is $T = 4$ h, quantity of parts per cycle $Q = 40$ pcs., and processing time $p_1 = 6$ min, $p_2 = 4$ min, $p_3 = 5$ min, and $p_4 = 2$ min. Let us build epure of the stocks for this example, assuming that all operations can start simultaneously. The set of such epure for the line is called its operating procedure (Fig. 14.13).

The value of the largest stock in buffer 1 according to formulas (14.13) and (14.14) $Z_{\max} = 40\left(1 - \frac{4}{6}\right) \approx 14$; in the second buffer $Z_{\max} = 40\left(1 - \frac{4}{5}\right) = 8$; in the third $Z_{\max} = 24$. From Fig. 14.13, it can be seen that the line rhythm is set by machine 1. The largest stock should be created in buffer 3, as the operation on machine 4 has the shortest duration.

The sequence of operators' work can be changed giving them the duties of servicing several units of equipment. Considering various options of the resulting procedures, we can choose the most appropriate one. The simulation results for this option are sometimes executed in the form of so-called standard plan (Kozlovsky et al. 2003), which is the base of daily planning.

If processing on the discontinuous line occurs sequentially in alternating in time batches, which are changed on all operations at the same time or within a short period of time, such a line can be considered as a multi-model line (Eq. 14.2).

Production of each product on such a line is carried out in accordance with the special rhythm of the line for this product and with its own procedure.

In those cases where at various operations different product lots are processed at the same time, this is a mixed-model line (Eq. 14.2), which is most typical for batch processing. For such line, it is not possible to define the concept of rhythm and develop its operating procedure to develop by the method described above. At the same time, since operations in the line have a distinct technological sequence, it is possible to calculate the lead time (shifting) of the launching point of lot into processing.

The book (Paramonov and Soldak 2009) describes in detail various methods of such calculation, which is the base of the model of production group line. The quality of planning for such a line is characterized by the makespan, which essentially depends on the launch sequence of lots into processing. Optimization methods for this sequence are discussed below.

14.3.2 Optimization for Two-Machine Group Flow Lines

In the designations of classification (2.13), the problem of optimal sequence of jobs for the line with two machines is recorded as the structural formula

$$F2\ |prmu|C_{\max}, \tag{14.15}$$

where symbols $F2$ define the transfer production line of two machines, symbol *prmu* indicated the search of optimal sequence during processing, and objective function $C_{\max}$ is focused on achieving the minimum of makespan. The exact solution of this problem was obtained in the paper (Johnson 1954). Let us designate the processing time of the i-th job on the first machine as a_i, and on the second machine as b_i. Then to obtain the optimal sequence it is necessary to fulfil the below algorithm.

(A) To find the minimum among all values a_i and b_i.
(B) If this minimum is a_i, then the job with the corresponding operation will be put in the nearest possible position in the launch sequence. If this minimum is b_i, then the job with the corresponding operation will be put in the last possible position in the launch sequence.
(C) Job included into the sequence is excluded from the further consideration.
(D) If the list of jobs is used up, the algorithm is terminated; otherwise, step (A) will be repeated.

As an example, we make optimal launch sequence for the jobs given in Table 14.4.

It is convenient to execute the algorithm step by step using Table 14.5. At the first step, we find operation b_3 with the least processing time, equal to 1. Since this operation belongs to the second machine, relevant job 3 should be started last. At

Table 14.4 Processing time of the jobs for the line of two machines

Job	a_i	b_i
1	3	5
2	2	4
3	6	1
4	4	2
5	5	4

Table 14.5 Optimal sequencing

Step	List of jobs	$\min(a_i, b_i)$	Sequence
1	1, 2, 3, 4, 5	b_3	{x, x, x, x, 3}
2	1, 2, 4, 5	a_2, b_4	{2, x, x, 4, 3}
3	1, 5	a_1	{2, 1, x, 4, 3}
4	5	b_5	{2, 1, 5, 4, 3}

the second step, job 3 is excluded from the list, and for jobs 2 and 4, there are the same value of minimum processing time equal to 2. Since for job 2 this minimum refers to the first machine, then job 2 starts first. Accordingly, job 4 is placed in the optimal sequence before already entered job 3 as the last. At the third step, we find that job 1 should be started second in order, and finally, job 5 becomes the third in order. This algorithm is called Johnson's rule.

Let us use the record of the processing sequence in two entries for the first and second machines:

$$\begin{array}{llllllll}
\text{machine } 1: 0 & (2/2)\ 2 & (1/3)\ \ 5 & (5/5)\ 10 & (4/4)\ 14 & (3/6)\ 20; \\
\text{machine } 2: 0/2 & (2/4)\ 6/5 & (1/5)\ 11/10 & (5/4)\ 15/14 & (4/2)\ 17/20 & (3/1)\ 21.
\end{array}$$

The entry for machine 1 indicates the job start point, job number and (after slash) processing time, and then the job end point. In the entry for machine 2, the record structure is more complicated—here the job start point is defined as the largest value of the machine availability time point and the readiness time point for the operation on the second machine. It is also assumed that blocking of machine 1 by machine 2 cannot occur, i.e. the buffer capacity between these machines is unlimited.

For example, at first machine 2 is free, but readiness of job 2 to a corresponding operation occurs at a time equal to 2. On the contrary, for job 1 the machine availability time point, which is 6, comes later than the time point of this job readiness equal to 5. For job 5, its readiness comes at time point 10 and the machine is available at point 11, etc.

In the paper (Baker and Trietsch 2009), it is proved that launching of jobs in the sequence defined by Johnson's rule ensures minimal idling of machine 2. Yet this idling can become large if the processing time of an operation on the first machine is high. In such cases, it makes sense to switch from sequential processing of each lot to the parallel-serial processing on the time-consuming operation. Herewith the size of a transport lot and the transfer time point of the lot to the second machine should be determined. The corresponding detailed analysis is presented in the book (Paramonov and Soldak 2009).

For discontinuous production lines consisting of machines in the amount greater than two, the problem of optimal launch is considerably complicated. To solve it, a number of different heuristic algorithms were suggested among which two algorithms are the most common. They are discussed below.

14.3.3 Campbell, Dudek, and Smith Algorithm

Let us consider a discontinuous production line consisting of four machines, where it is necessary to perform five different jobs in sequence. Operating time standards p_{ij} for them are given in Table 14.6.

Campbell, Dudek, and Smith (CDS) algorithm (Campbell et al. 1970) allows for using Johnson's rule for the sequence of several so-called pseudoproblems. At the first step, it is suggested to confine oneself to considering only the first and last machines in the line. At that, it is obvious from the previous Sect. 14.3.2 that for this example $a_i = p_{i1}$ and $b_i = p_{i4}$. For illustrative purpose, these values in Table 14.6 coincide with the values in Table 14.4 and that is why Johnson's rule at the first step will give sequence $\{2, 1, 5, 4, 3\}$.

Let us calculate the overall duration of processing for this sequence on all four machines:

$$\begin{array}{lccccccccccc}
\text{machine 1}: 0 & (2/2) & 2 & (1/3) & 5 & (5/5) & 10 & (4/4) & 14 & (3/6) & 20; \\
\text{machine 2}: 0/2 & (2/6) & 8/5 & (1/4) & 12/10 & (5/2) & 14/14 & (4/3) & 17/20 & (3/1) & 21; \\
\text{machine 3}: 0/8 & (2/3) & 10/12 & (1/2) & 14/14 & (5/3) & 17/17 & (4/5) & 22/21 & (3/5) & 27; \\
\text{machine 4}: 0/10 & (2/4) & 14/14 & (1/5) & 19/17 & (5/4) & 23/22 & (4/2) & 25/27 & (3/1) & 28.
\end{array}$$

At the second and next steps of the algorithm, values a_i and b_i are understood as values

$$a_{ik} = \sum_{j=1}^{k} p_{ij} \quad \text{and} \quad b_{ik} = \sum_{j=m-k+1}^{m} p_{ij}, \tag{14.16}$$

where k is the step number, m is the quantity of machines in the line, and the quantity of steps is $m - 1$.

Table 14.6 Processing time of jobs for a line consisting of four machines

Job	p_{i1}	p_{i2}	p_{i3}	p_{i4}	$\sum p_i$
1	3	4	2	5	14
2	2	6	3	4	15
3	6	1	5	1	13
4	4	3	5	2	14
5	5	2	3	4	14

Table 14.7 Calculated launch sequences in CDS algorithm

Job	Step 1		Step 2		Step 3	
	$a_{i,1}$	$b_{i,1}$	$a_{i,2}$	$b_{i,2}$	$a_{i,3}$	$b_{i,3}$
1	3	5	7	7	9	11
2	2	4	8	7	11	12
3	6	1	7	6	12	8
4	4	2	7	7	12	10
5	5	4	7	7	10	9
Sequence	{2, 1, 5, 4, 3}		{1, 4, 5, 2, 3}		{1, 2, 4, 5, 3}	
Duration	28		29		30	

For example, with $k = 2\, a_{1,2} = p_{1,1} + p_{1,2} = 3 + 4 = 7; b_{1,2} = p_{1,3} + p_{1,4} = 2 + 5 = 7$.

Application of Johnson's rule at each step generates the corresponding launch sequence, the number of which in this case is $m - 1 = 3$. From these sequences, according to CDS algorithm, we should select the one which provides the shortest total processing time. The calculation results for this example are summarized in Table 14.7.

In this example the best result is provided by sequence {2, 1, 5, 4, 3}, obtained at the first step. CDS algorithm can also be used successfully to define the sequence in the case when the jobs arrive at the line at different time (Sule 2007).

14.3.4 Nawaz, Enscore, Ham Algorithm

This algorithm (Nawaz et al. 1983) is not associated with any exact solution like Johnson's rule but uses partial sorting of possible options for launch sequence. Performing the steps of the algorithm is associated with quite large scope of calculations; however, it usually leads to good results.

At the first step of the algorithm, the total processing time is determined for each job, and the jobs are sorted in descending order of the latter. In case of equality of processing time the sequence is remained in numerical order. After that, two sequences are compared—original and obtained from the original one by interchanging of the first and second jobs, and the one that gives the minimum of total duration is considered the best. The resulting order of the first two jobs is memorized and does not change in future.

Then we verify the options of the resulting sequence, where the position of the third job by processing time with respect to the first and second jobs changes—this job can have the first place in order, or to be between the first and second jobs. The best option is memorized and does not change subsequently. Similarly, the options for jobs next by processing time are calculated.

Let us consider the algorithm operation in terms of the data given in Table 14.6. The jobs in this list by descending of total processing time are arranged in sequence {2, 1, 4, 5, 3}.

Let us consider two possible sequences {2, 1, 4, 5 3} and {1, 2, 4, 5, 3}, different by execution order of the first two jobs with the highest processing time. The first of these have the total execution duration equal to 28, and the second one—30 time units. According Nawaz, Enscore, Ham (NEH) algorithm in this case job 2 should be executed in all other possible sequences before job 1.

At the next step, we consider three options of launch sequence {2, 1, 4, 5, 3}, {4, 2, 1, 5, 3}, and {2, 4, 1, 5, 3} different by the position of job 4. The duration values of these options are 28, 30, and 28 units, respectively. The second option is clearly worse than the others, while the third option does not improve the initial value and can be discarded as well.

Further calculation is performed for four possible options of job 5 position: {2, 1, 4, 5, 3}, {5, 2, 1, 4, 3}, {2, 5, 1, 4, 3}, {2, 1, 5, 4, 3}. The duration values of these options are 28, 30, 29, and 28 units, respectively. The permutation of job 5, as we see, does not reduce the processing time compared to the first option. Finally, we perform calculation for five possible positions of job 3: {2, 1, 4, 5, 3}, {3, 2, 1, 4, 5}, {2, 3, 1, 4, 5}, {2, 1, 3, 4, 5}, {2, 1, 4, 3, 5}, the duration of which is 28, 33, 30, 31, and 32, respectively.

Thus, the best option is {2, 1, 4, 5, 3} with duration of 28 units. Comparing this result with the calculation result of the optimal sequence {2, 1, 5, 4, 3}, obtained in the previous Sect. 14.3.3, we see that the duration of the optimal options in both algorithms is the same.

The results of applying NEH algorithm over the past 20 years, as indicated, for example, in the paper (Kalczynski and Kamburowski 2008), show that it is highly effective. Therefore, many researchers use this algorithm for any similar problems, attempting to reduce the number of required iterations as well.

14.4 Job-Shop Production

In job-shop production, the sequence of operations for each job is different. At the same time, the set of possible operations is limited by the machines at the disposal of the production unit. The task (usually daily) of operative planning is to produce optimal schedule of work in the department within the planned period. This period usually covers several upcoming days, so each subsequent schedule is built based on previously developed schedules and represents their adjustments and amendments.

The number of possible options of the plan is extremely high in general, but it is significantly reduced due to the factual constraints. First of all, the technology of processing rigidly defines the possibility of equipment varying to perform operations. Besides, for organizational and psychological reasons, as a rule, an operation previously initiated on the machine must not be interrupted. Finally, when scheduling it is necessary to take into account the technical condition of the equipment, availability of tools, and qualified personnel.

Therefore, purely mathematical techniques of schedule optimization cannot lead directly to a satisfactory result, but the schedules obtained this way can often serve as a basis for making specific scheduling decisions. The method described below is widespread and can be considered as the base in development of planning systems.

14.4.1 Shifting Bottleneck Algorithm

Let us consider the problem of execution of *n* jobs, each of which as going through the technological process can come one time to *m* machines or fewer machines. Let us assume that the exact due date of each job is not stipulated and the objective function is to minimize makespan $C_{\max}$.

In the designations of classification (2.13), the stated problem of optimal sequence of job execution is written as a structural formula

$$J\,|prec|C_{\max}, \tag{14.17}$$

where symbol *J* means job-shop production and symbol *prec* means presence of constraints of precedence during processing.

The Shifting Bottleneck Algorithm (Adams et al. 1988) is used in case of some additional constraints of the stated problem, listed below.

(A) All jobs are available at the beginning of the scheduling.
(B) Each work cannot be performed simultaneously on several machines.
(C) The duration of each operation for each job is known.
(D) Setup and transportation time is included in the job execution time.
(E) All jobs are of equal importance.
(F) All machines are available at the beginning of work.
(G) In the department, there is only one machine of each type, and it is ready for use.

There are several different options to use this algorithm. In this book, as in most recent studies, we will adhere to the methods described in the book (Baker and Trietsch 2009). For definiteness, we assume that there are three machines on which four jobs are executed. Above in Sect. 7.5 for a similar example, it was shown that the sequence of jobs execution is reasonable to display as a graph (Fig. 14.14).

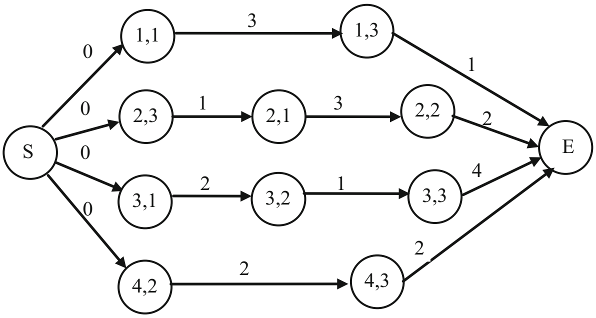


Fig. 14.14 Graph of processing four parts on three machines

Recall that the circles in Fig. 14.14 denote the operations performed on part i on machine j. For the convenience of the graph, dummy common starting point S and end point E of the schedule are introduced. The solid lines with arrows indicate the movement of the parts lot from one machine to another in the sequence provided by the process. These lines represent the precedence relationships between the operations of individual jobs and are called connecting (conjunctive) arcs. Above each arc (arrow) processing time p_{ij} on the relevant operation is indicated.

In the graph (Fig. 14.14), the possible sequence of operations of various jobs executed on a single machine can be shown using pairs of dashed lines similarly to Fig. 7.8. For example, for machine 1 such links can be made between nodes 1,1; 2,1; 3,1. The lines in these pairs are called disjunctive arcs. Solution of the stated problem as indicated in Sect. 7.5 is to select the best such sequence (a set of disjunctive arcs), providing the smallest value $C_{\max}$ in this case.

In the Shifting Bottleneck Algorithm, each machine in Fig. 14.14 for which the schedule has not yet been prepared is considered individually as a single machine. At that, the machine, because of which the maximum tardiness occurs, is regarded as a bottleneck and for it the schedule of operations is made in the first place.

Obviously, in the case where there are no delays in operations on any of the machines the total duration of execution of the entire job set is defined by the duration of the critical path for the graph in Fig. 14.14. Finding of the critical path and determining of its duration are described above in Sect. 12.3.1. In this example, execution of job 3 is critical with the duration of 7 units. We denote this duration as makespan in zero approximation $C_{\max}(0)$.

If you make a separate schedule for each of the machines in Fig. 14.14, it is necessary to take into account that the start point of the i-th job execution on the j-th machine r_{ij} is defined both by the arrival time of the job (as the previous operation ends) and the time point of the machine availability. The corresponding problem for a single machine was discussed above in Sect. 13.3.3, and in this case, the duration of delivery after any of these operations can be understood as a period of time ("tail") q_{ij} necessary to complete the entire set of jobs starting from the time point of operation completion. Note that in the early scheduling there are no disjunctive arcs and values r_{ij} and q_{ij} can be determined directly by the graph in Fig. 14.14.

At the first step of the algorithm, we summarize the calculated data for processing time p_{ij}, expected arrival time r_{ij}, and duration of "tail" q_{ij} into Table 14.8.

For example, in Fig. 14.14, expected arrival of job 2 to machine 1 is equal to the execution time point of this job on machine 3, i.e. $r_{2,1} = p_{2,3} = 1$. Accordingly, the duration of the rest of the operations is $q_{2,1} = p_{2,2} = 2$.

Let us find, for example, the optimal sequence of jobs for machine. We assume that at the initial moment of scheduling, machine 1 is free. At this moment either job 1 or job 3 can be started. To select one of these jobs according to the LT algorithm described above in Sect. 13.3.3, the first one should be the job with greater value q_{ij}. Since $q_{3,1} = 5 > q_{1,1} = 1$, then job 3 is started first.

Table 14.8 Calculated data at the first step of the algorithm

Machine	Job	p_{ij}	r_{ij}	q_{ij}	Optimal sequence	Duration
1	1	3	0	1	{3, 2, 1}	8
	2	3	1	2		
	3	2	0	5		
	4	0	–	–		
2	1	0	–	–	{4, 3, 2}	6
	2	2	4	0		
	3	1	2	4		
	4	2	0	2		
3	1	1	3	0	{ 2, 4, 1, 3}	9
	2	1	0	5		
	3	4	3	0		
	4	2	2	0		

At the moment of execution of job 3, which is 2 (Fig. 14.14), for execution on machine 1 jobs 1 and 2 (Table 14.8) are available. At that $q_{2,1} = 2$, and $q_{1,1} = 1$. So the processing sequence on machine 1 has the form {3, 2, 1} or in detail

$$0 \quad (3/2) \quad 2/1 \quad (2/3) \quad 5/0 \quad (1/3) \quad 8\,. \tag{14.18}$$

Recall that in sequence (14.18) in brackets the job numbers and their processing time are indicated and the figures without brackets are the completion time points on the machine and slash arrival time points of the next job. Similarly, for other machine we have:

$$\text{machine } 2: \quad 0 \quad (4/2) \quad 2/2 \quad (3/1) \quad 3/4 \quad (2/2) \quad 6\,, \tag{14.19}$$

$$\text{machine } 3: \quad 0 \quad (2/1) \quad 1/2 \quad (4/2) \quad 4/3 \quad (1/1) \quad 5/3 \quad (3/4) \quad 9\,. \tag{14.20}$$

Since the maximum duration of processing occurs for machine 3, we believe that it is a bottleneck and, therefore, the total duration of jobs in the first approximation $C_{\max}(1) = 9$. The sequence of jobs in the bottleneck is shown in Fig. 14.15 by bold lines (disjunctive arcs). As these lines represent the links, which during planning are replaced by the links with subsequent operations of the initial graph, shown by thin lines, then the processing times in these lines coincide. For example, the processing time on the new link between nodes (2,3) and (4,3) is equal to the processing time between nodes (2,3) and (2,1), i.e. 1.

At the second step of the algorithm we make Table 14.9 for machines 1 and 2, and for this purpose, we recalculate the arrivals of the jobs to these machines r_{ij} and "tail" durations q_{ij}, assuming that machine 3 is loaded in sequence {2, 4, 1, 3}.

The introduction of disjunction arcs in Fig. 14.15, in general case, changes values r_{ij} and q_{ij}. In this example, the introduction of these arcs, shown in Fig. 14.15, changes the start time points of processing on machine 3. For example,

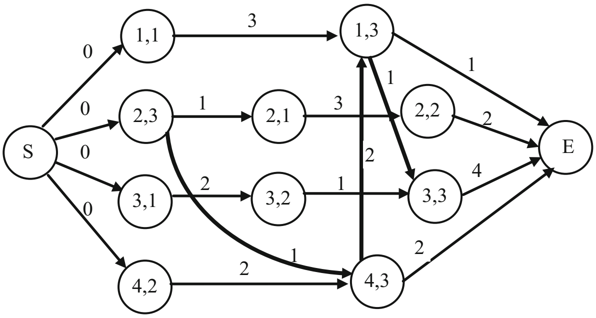


Fig. 14.15 Graph of processing after the first step of the algorithm

Table 14.9 Calculated data at second step of the algorithm

Machine	Job	p_{ij}	r_{ij}	q_{ij}	Optimal sequence	Duration
1	1	3	0	5	{3, 1, 2}	8
	2	3	1	2		
	3	2	0	9		
	4	0	–	–		
2	1	0	–	–	{4, 3, 2}	6
	2	2	4	0		
	3	1	2	9		
	4	2	0	7		

in node (3,3) the "head" duration, i.e. the longest path from the beginning of the graph to this node, $r_{3,3} = p_{4,2}+p_{4,3}+p_{1,3} = 2+2+1 = 5$. At the same time, drawing the disjunctive arcs does not affect the start time points on machines 1 and 2.

Values q_{ij} change for several nodes of processing on machines 1 and 2. For example, for node (1,1) the "tail" duration is determined by the longest path from the bottleneck to the end of the graph. This path now goes not only through node (1,3) but also through (3,3). In node (3,1) there are two ways from the beginning of the graph: through node (1,1) and through nodes (4,2) (4,3). Since the length of these paths are the same, $r_{1,3} = 2$ and $q_{1,1} = p_{1,3}+p_{3,3} = 1 + 4 = 5$.

Let us define the value of "tails" in nodes (3,1) and (3,2). For this we should take into account that the completion time point of machine 3 for job 3 as it was shown, $r_{3,3} = 5$, which is more than the total duration processing in nodes (3,1) and (3,2). So in these nodes, values q_{ij} are equal and defined by work of node (3,3), i.e. $q_{3,1} = q_{3,2} = r_{3,3}+p_{3,3} = 5+4 = 9$.

Using algorithm LT we obtain optimal sequences for machines 1 and 2 {3, 1, 2} and {4, 3, 2}, respectively. In detail they have the form:

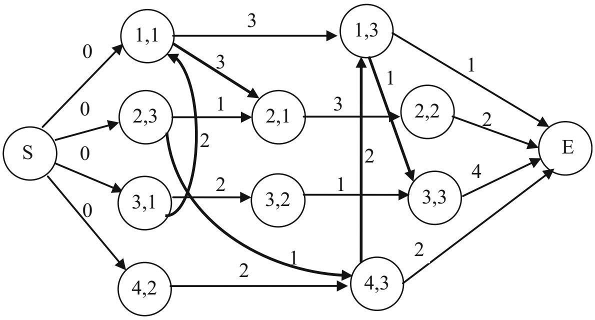


Fig. 14.16 Graph of processing after the second step of the algorithm

Table 14.10 Calculated data at the third step of the algorithm

Machine	Job	p_{ij}	r_{ij}	q_{ij}	Optimal sequence	Duration
2	1	0	–	–	{4, 3, 2}	6
	2	2	4	0		
	3	1	2	9		
	4	2	0	7		

$$\begin{array}{llllllll} \text{machine 1}: & 0 & (3/2) & 2/0 & (1/3) & 5/1 & (2/3) & 8; \\ \text{machine 2}: & 0 & (4/2) & 2/2 & (3/1) & 3/4 & (2/2) & 6. \end{array}$$

Thus, at the second step of the algorithm, the bottleneck shifts to machine 1 and the graph of processing displays new disjunctive arcs (Fig. 14.16).

Disjunctive arcs showing processing on machine 1, in general, alter the sequence of jobs on machine 3, scheduled at the previous step. In this case, due to the fact that the operation in node (1,1) will not be performed immediately, but after the operation in node (3,1), the arrival moment of job 1 to the machine becomes equal $r_{1,3} = p_{3,1} + p_{1,1} = 2 + 3 = 5$. That is why the sequence for machine 3 (14.20) will change and take the form 0 (2/1) 1/2 (4/2) 4/5 (1/1) 6/3 (3/4) 10, and total duration $C_{\max}(2) = 10$.

At the third step of the algorithm we find an optimal sequence for machine 2, for which we draw up Table 14.10.

As can be seen, the optimal sequence for machine 2 at the third step is maintained the same as at the previous step. The total duration at the last step does not change as well, i.e. $C_{\max}(3) = 10$.

The result of algorithm action can be presented in the form of Gantt chart (Fig. 14.17).

Using the Shifting Bottleneck Algorithm does not generally provide the optimal solution, even though, as a rule, the obtained result for the makespan $C_{\max}$ is close enough to the lowest possible value. As the number of jobs and machines grows, the computation scope required to obtain a solution by using this algorithm increases sharply, and also the probability of deviation of the obtained solution from the optimal one increases.

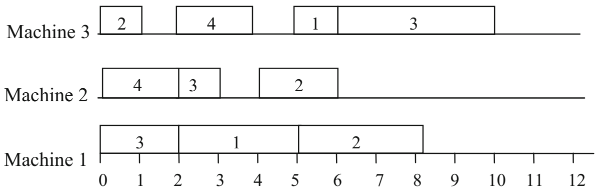


Fig. 14.17 Gantt chart for Shifting Bottleneck Algorithm

Therefore, a number of subsequent studies are dedicated both to improving the accuracy of the solution and to decreasing of the necessary calculations scope. Over the past 20 years, a considerable number of different heuristic methods, among which the so-called genetic method, the simulated annealing, and the method of tabu search, are the most common. The basics of using these methods for solving the problem described in this paragraph are given, for example, in the thesis (Yamada 2003). Practical use of the scheduling problem considered in this paragraph makes sense for the "make-to-stock" strategy, which, in turn, is used in the job-shop production with relatively small but stable demand.

14.4.2 Job-Shop Production Scheduling Using Dynamic List Algorithms

The vast majority of decisions in scheduling theory refer to static problems, such as in Sect. 14.4.1 above. In these cases, a number of specified activities for a particular technical structure are considered and the best option schedules in accordance with the set criterion are searched by exact or approximate method. In practice, however, the setting of jobs quantity for scheduling appears to be difficult.

This approach is only possible provided there is a quite stable demand. If the demand is constant and significant, it is really possible to divide the whole segment of the planning up to, for example, its monthly horizon into some time intervals, weeks for example. In this case, a schedule made for 1 week will be repeated next week, and obviously, it is very important that this schedule is as close as possible to the optimal one.

However, this situation in practice is quite rare, because in the context of market-oriented economy, the demand even for fast-moving consuming goods does not remain constant due to competition and other factors. Therefore, usually the plans vary from one period to another, and the plans of lower level change faster than the plans of the higher hierarchical level. Accordingly, the optimization accuracy of the lower level plan is significantly less important than when developing the high-level plan.

For the lower level (operational) plan, its exact optimality is not so much important as the adequacy of preparation of planning task to the actual state of the production process, simplicity, speed, and clarity of the solution algorithm, as well as the possibility of planning results modelling for different planning horizons.

As an analogy for dynamic scheduling we can present the problem of dynamic sizing of production lots (see Sect. 11.3). Section 11.3 summarizes various heuristic methods, in most of which a lot is formed by combining the demand for several time intervals. Dynamic scheduling is also advisable by consistently analysing the comparative values of evaluation criteria for horizons.

The simplest methods of this type are the so-called dynamic “list” algorithms (Timkovsky 1992). When planning with this algorithm the dynamic priorities of jobs execution are calculated as the machines become available and the jobs are assigned to the free machine in order of priority.

Dynamic priorities, as opposed to static, take into account the manufacturer’s reserve time available to fulfil the order. Section 2.3 describes several dynamic priority rules: Minimum Slack Time (MST) rule, Critical Ratio (CR) rule, and Apparent Tardiness Cost (ATC) rule. However, these criteria do not take into account the complex psychological relationship between people engaged in the production, which we believe to be a very important aspect when planning. Therefore, here we consider the list algorithm of scheduling, which is used as a criterion for the production intensity (see Sect. 2.4.1).

As an example, let us make a schedule for the area with three machines, on which the jobs with data specified in Table 14.11 are executed. The processing sequence on the machines is defined by the operation number. Let us assume that

Table 14.11 List of jobs for planning

Arrival time of the job list	No. of job *i*	Due time point *d*	Planned arrival time point *r*	Priority coefficient *w*	No. of operation *l*	Machine *j*	Processing time *p*
0	1	42	7	1	1	1	6
					2	3	14
	2	31	0	2	1	1	8
					2	2	5
					3	3	9
	3	4	0	1	1	1	4
					2	3	6
					3	2	7
	4	26	4	1	1	1	5
					2	2	7
					3	3	4
10	5	22	10	2	1	2	4
					2	1	4
					3	3	3
24	6	48	24	1	1	1	6
					2	3	14
	7	42	24	2	1	1	8
					2	2	5
					3	3	9

machine 1 is free from the previously planned job at time 0 and machines 2 and 3—at time 5 and 8, respectively.

Let us assume that the machines are serviceable, provided with personnel, tooling, and materials, and the actual processing times coincide with the established time standards. Such being the case, in principle, the planning must be carried out only at the moments of receipt of new lists of jobs. At the same time, the planning horizon, of course, is determined by the jobs included in the list and available at the time of planning.

We perform planning using Table 14.12. For each new planning, we create lines of this table corresponding to each completion time point of any of the machines. In column "possible jobs", all the jobs that are ready for processing at the time of machine availability are entered. If such jobs are not available, then this column has the job with the nearest point of readiness, or several of these jobs in the case of coincidence of their readiness time.

In the case, when several jobs are recorded in column "possible jobs", the priority belongs to that one, for which the intensity value is greater. The processing start point is set as the highest value between the time of the machine availability and the job readiness time point, and the end of processing is set by the relevant processing of the job from Table 14.11. The intensity value is calculated by formulas (2.28) in Sect. 2.4.1. In these calculations, we assume, as in Chap. 2, value $\alpha = 0.1$, and the planning period duration $G = 40$ units.

At time of planning 0, as shown in Table 14.11, the planning is possible for jobs 1÷4. At time 0 only machine 1 is free, for the processing on which jobs 2 and 3 are ready. Since the possible start time point of processing for both of these jobs is less than the required due time point, then to calculate the intensity we use of the first of formulas (2.28). For example, for job 3 on machine 1 at the first operation we have

$$\begin{aligned} H_3 &= \frac{w_3\left(p_{3,1} + p_{3,3} + p_{3,2}\right)}{G} \frac{1}{(d_3 - t)/\alpha G + 1} \\ &= \frac{1 \times (4 + 6 + 7)}{40} \frac{1}{(4 - 0)/(0.1 \times 40) + 1} = 0.21. \end{aligned}$$

Since the intensity value of job 3 is more than of job 2, job 3 is the first to be accepted for execution. Completion of job 3 on machine 1 is considered to be the time point of its availability. In this example, at time point 4 jobs 2 and 4 are possible to be executed on machine 1. Job 2 is selected in a similar manner.

According to the initial conditions machine 2 is free at time 5. At this point, there are no immediate jobs ready to be executed on machine 2, but nearest to this job is job 2, which will be ready by time 8. Therefore, it is possible to schedule this job for machine 2, starting with readiness time point 8.

At time 8, machines 1 and 3 are free simultaneously. For machine 1 execution of jobs 1 and 4 is possible, and for the latter the intensity has a greater value. The intensity of job 3 by time 8 becomes high, because the required due time, equal to 4, has already passed. In this case, the intensity at the second operation is defined by the second of formulas (2.28), namely

Table 14.12 Planning

Planning time point	Availability time point	Machine	Possible jobs	No. of operation	Readiness time point	Intensity	Start of processing	End of processing
0	0	1	2	1	0	0.125	–	–
			3	1	0	0.210	0	4
	4	1	2	1	0	0.141	4	8
			4	1	4	0.061	–	–
	5	2	2	2	8	0.093	8	13
	8	1	1	1	7	0.050	–	–
			4	1	4	0.072	8	13
		3	3	2	4	0.650	8	14
	13	1	1	1	7	0.060	13	19
		2	4	2	13	0.064	13	20
	14	3	2	3	13	0.085	14	23
	20	2	3	3	14	0.875	20	27
	23	3	1	2	19	0.060	–	–
			4	3	20	0.375	23	27
	27	3	1	2	19	0.073	27	41
10	13	1	1	1	7	0.060	13	19
		2	4	2	13	0.064	–	–
			5	1	10	0.169	13	17
	14	3	2	3	13	0.085	14	23
	17	2	3	3	14	0.743	17	24
	19	1	5	2	17	0.200	19	23
	23	3	1	2	19	0.060	–	–
			5	3	23	0.200	23	26
	24	2	4	2	13	0.183	24	31
	26	3	1	2	19	0.060	26	40
	40	3	4	3	31	0.450	40	44

(continued)

Table 14.12 (continued)

Planning time point	Availability time point	Machine	Possible jobs	No. of operation	Readiness time point	Intensity	Start of processing	End of processing
24	23	1	6	1	24	0.068	–	–
			7	1	24	0.191	24	32
	24	2	4	2	13	0.183	24	31
	26	3	1	2	19	0.072	26	40
	31	2	7	2	32	0.186	32	37
	32	1	6	1	24	0.100	32	38
	40	3	4	3	28	0.450	40	44
			6	2	38	0.116	–	–
			7	3	37	0.300	—	–
	44	3	6	2	38	0.175	–	–
		3	7	3	37	0.675	44	53
	53	3	6	2	38	0.780	53	67

$$H_3 = \frac{w_3(p_{3,3} + p_{3,2})}{G}((t - d_i)/\alpha G + 1) = \frac{1 \times (6 + 7)}{40}((8 - 4)/(0.1 \times 40) + 1)$$
$$= 0.65.$$

Similarly, we perform planning for the remaining operations of the jobs in the list, received at time 0. All such operations, according to the top part of Table 14.12, theoretically should be completed at time equal to 41.

However, at time 10, due to the receipt of a new list comprising urgent job 5, it is necessary to adjust the initial plan. To do this, the plan has to be revised for all machines that are available later than time 10 and add it with operations of job 5 according to their intensity. Adjustment of the plan leads to its extension until time point 44.

A similar situation occurs later when the following list arrives at time 24. In this case, it appears that for machine 1, which is free at time 23, the previous plan has not provided loading. While the preparation of new plan in this case has its initial point at time 23, the actual start of new jobs is only possible after the arrival time point, i.e. starting from 24. While preparing this plan it is necessary to take into account both the execution of received jobs 6 and 7 and jobs 1 and 4 not completed at the previous stages of planning.

Figure 14.18 shows the graphs of the executed jobs intensity variation, according to the plan in Table 14.12. In Fig. 14.19, thin lines show the graphs for each of the machines, and bold lines—the graphs of intensity for the entire area. Intensity values for each machine, as defined in Sect. 2.4.1, are calculated as the total of jobs intensity awaiting execution on the machine. Accordingly, the total intensity of the area is the sum of the intensity on all machines of the area.

The graphs in Fig. 14.18 show that the intensity of each job in process of its execution has a well expressed oscillatory form. The reason for these oscillations is

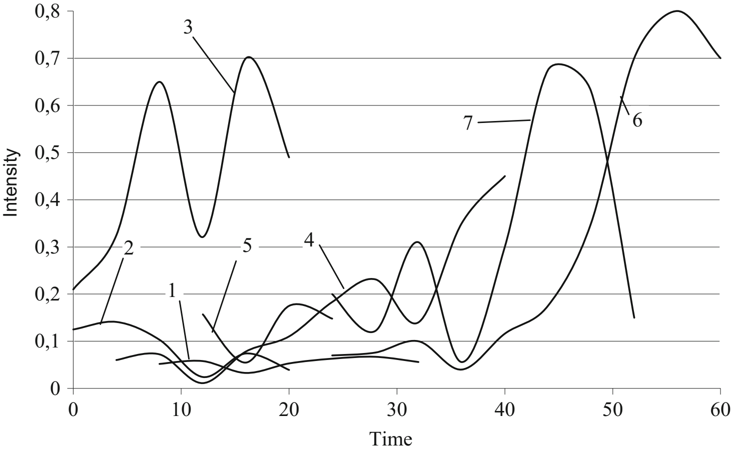


Fig. 14.18 Curves of intensity changing for seven jobs

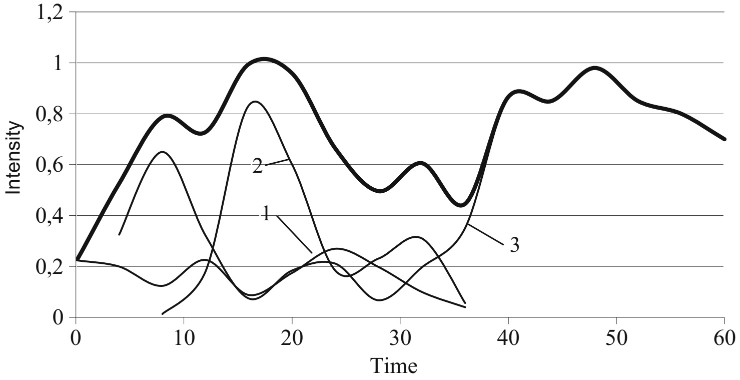


Fig. 14.19 Curves of intensity changing for three machines and the area

that interoperation idling leads to increase in intensity, and during processing, it decreases. The greatest increase in intensity occurs in cases where long operations for several jobs overlap in time on a single machine. For example, at time 40 when machine 3 is loaded (Table 14.12) jobs 4, 6, and 7 compete, which further leads to a sharp increase in intensity of jobs 6 and 7 (Fig. 14.18) and thus the intensity on machine 3 (Fig. 14.19).

Analysis of the graphs in Fig. 14.19 for a while allows defining the loading extent of each machine and its contribution to the total load of the processing area. If the plan calculation (Table 14.12) is automated enough, it is possible to simulate the load for different versions of the plan: with joining or dividing the lots, changing the order of machines load, changing of shifts of individual machines or in case of overtime, etc.

In this case, it is required to decide what index is the most important in planning. The described model works with the index of production intensity, which is subject to fluctuations, as can be seen from Fig. 14.19. The sharp increase in intensity is undoubtedly a negative factor, as it indicates weak performance of production tasks and causes deterioration of production relations between the executive person at the site and the management. At the same time, a significant decrease in intensity compared to some average value for the production area reduces the scope of planned jobs and, accordingly, to difficulties in the wages payment to operators. This implies that for each division there is some optimal value of average intensity, fluctuations around which should be limited as possible.

Efficient operation of this model is only possible if the source data for its operation are available in the database of the main transactional information system of the enterprise (Sect. 5.1.3). Often, this database operates under the ERP system employed at the enterprise. Of course, the algorithms used in the development of a specific information system are is much more complicated than the algorithm for developing the plan set out above in Table 14.12.

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15 Multi-criteria Scheduling

15.1 Just-in-Time Production Scheduling

For Just-in-Time (JiT) production, the schedule of starting different jobs should provide a regular supply of components to assembly exactly according to needs arisen. However, since the supply of components is possible only in lots, the sizes of which are often determined by the cost of setup and transportation, the dates necessary for lots to arrive to assembly are not always known. The values of these dates are determined by establishing a certain compromise between the possibility of delay and undesired long storing.

As a rule, in the studies on optimal JiT schedules the criteria are used based on the values of tardiness of deliveries T_i and waiting for release E_i. Two such algorithms for relatively simple cases are given below.

15.1.1 Starting Group of Jobs with Fixed Sequence

Let us consider the problem of scheduling for a single machine, which must process a group of jobs (product lots), for each of which desired delivery date d_i is established. It is also assumed that the sequence of these jobs is known and the objective functions are both minimization of total tardiness T and early production E of the entire group of jobs. The relevant classification formula of the problem has the form

$$1\left|d_i, seq\right|f(T, E), \tag{15.1}$$

where parameter *seq* is the symbol of the predefined sequence. In the simplest case, the objective functions has the form

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Table 15.1 Data on planned jobs

Job J_i	Processing time p_i	Required due time point d_i
1	2	4
2	4	7
3	3	12
4	5	14
5	3	16

$$f(T,E) = T + E = \sum_{i=1}^{n} |C_i - d_i|, \tag{15.2}$$

where C_i is the completion time point of each job.

We describe the algorithm for solving this problem proposed in Garey et al. (1988) in terms of the data in Table 15.1. Jobs in Table 15.1 are arranged in accordance with the mandatory sequence.

The main objective of the algorithm is that at the beginning of each i-th job 2 options may appear. In the first case

$$C_{i-1} + p_i < d_i, \tag{15.3}$$

and job J_i shall start at time point $d_i - p_i$. At that between completion time point C_{i-1} of job J_{i-1} and the start time point of job J_i a gap (machine idling) may occur. In the second case

$$C_{i-1} + p_i \geq d_i \tag{15.4}$$

and this kind of gap will not occur.

In this scheduling the execution diagram of the entire job group consists of several job blocks B_1, $B_2 \ldots$ B_k (Fig. 15.1), the time intervals between which correspond to the machine idling time.

When there is a gap between the blocks it makes sense to try to move the follow job blocks to the earlier point in time to obtain lower value of the objective function. Figure 15.2 shows the flowchart of the algorithm. When applying the algorithm, all jobs in the established order in Table 15.1 are consistently included in set S.

Let us use symbol r to designate the quantity of jobs in the last block B_k, for which $C_i > d_i$, i.e. the planned completion time point, is more than the required due time point. Then the condition of shifting job block (block 10 of the algorithm) consists in

$$r \geq \left(\left|B_k\right| - r\right), \tag{15.5}$$

where symbol $\left|B_k\right|$ means the total quantity of jobs in block B_k.

To recalculate the start time point of jobs in the shifted block, we determine start time point ω of the first job in block B_k, at completion time π of the last job in the previous block B_{k-1} and the parameter of this shifting

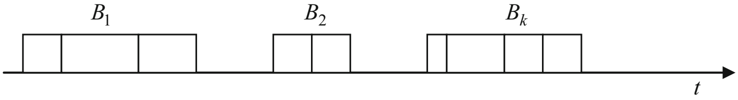

Fig. 15.1 Job blocks in scheduling

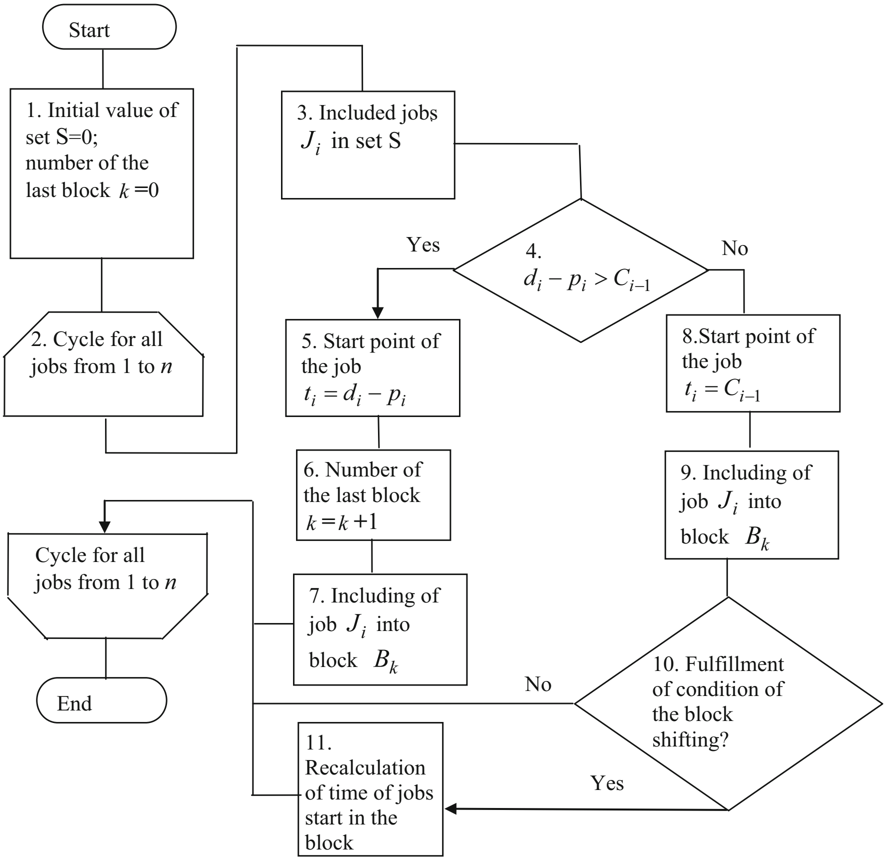
Start
1. Initial value of set S=0; number of the last block k=0
2. Cycle for all jobs from 1 to n
3. Included jobs J_i in set S
4. $d_i - p_i > C_{i-1}$
Yes
No
5. Start point of the job $t_i = d_i - p_i$
6. Number of the last block $k = k+1$
7. Including of job J_i into block B_k
8.Start point of the job $t_i = C_{i-1}$
9. Including of job J_i into block B_k
10. Fulfillment of condition of the block shifting?
No
Yes
11. Recalculation of time of jobs start in the block
Cycle for all jobs from 1 to n
End

Fig. 15.2 The flowchart of the algorithm of job launching with predefined sequence

$$\delta = \min_{J_i \in B_k / C_i > d_i} (C_i - d_i). \tag{15.6}$$

The minimum of values $C_i - d_i$ in expression (15.6) is taken for all the jobs in block B_k, for which $C_i > d_i$. After determining of values ω, π, and δ at each step of the algorithm for each i-th job included into the block, the start time point is recalculated as

$$t_i = t_i - \min(\delta; \omega - \pi). \tag{15.7}$$

In case when the difference $\omega - \pi$, i.e. the gap between blocks B_k and B_{k-1}, appears to be less than parameter δ, block B_k shifting to the left by the gap value leads to joining of blocks B_k and B_{k-1} in one. So the number of the last block k should be reduced by one.

Let us consider the algorithm application in terms of the data in Table 15.1. At the first step of the algorithm, we include job J_1 into schedule set S. As before J_1 there were no other jobs, then the end time point of job $C_{i-1} = C_0 = 0$. The condition in block 4 (Fig. 15.2) gives $d_1 - p_1 = 4 - 2 = 2 > C_0 = 0$ and we accept $t_1 = d_1 - p_1 = 2$. We create the first (and the last) schedule block with $k = 1$ and include job J_1 in it.

At the next step, $i = 2$, $S = \{J_{1,}J_2\}$. We find $C_{i-1} = C_1 = t_1 + p_1 = 2 + 2 = 4$, and $d_2 - p_2 = 7 - 4 = 3$. Since the condition in block 4 is not fulfilled then the start point of job 2 $t_2 = C_1 = 4$, and job J_2 and J_1 are included in the only block B_1. The total quantity of jobs in this block will be $|B_1| = 2$.

The completion time point of job J_2 in this case is $C_2 = t_2 + p_2 = 4 + 4 = 8$ and, accordingly, is more than set value $d_2 = 7$. Since job J_1 ends in time, then in block B_1 the number of jobs, for which $C_i > d_i$, $r = 1$. Verifying the condition (15.5) we have $|B_k| - r = 2 - 1 = 1$, the condition in block 10 (Fig. 15.2) is fulfilled, and accordingly, the block shifting should be performed.

Parameter δ equals value $C_i - d_i$ for job J_2 being the only in the block, which finishes later than the set period, i.e. $\delta = C_2 - d_2 = 8 - 7 = 1$. Start of the first job in block $\omega = t_1 = 2$. Since block B_1 is the first in the schedule, then the end time point of the jobs in the previous block can be considered to be equal to 0, i.e. $\pi = 0$. Using formula (15.7), we obtain $t_1 = t_1 - \min(1; 2 - 0) = t_1 - 1 = 2 - 1 = 1$. Similarly we have $t_2 = t_2 - 1 = 4 - 1 = 3$. After this shifting the completion time point of job J_2 becomes equal to $C_2 = 7$.

At the third step $i = 3$, $S = \{J_{1,}J_2,J_3\}$, $d_3 - p_3 = 12 - 3 = 9$. Since $C_2 = 7$, the condition in block 4 (Fig. 15.2) is fulfilled, then $t_3 = 9$ and it is necessary to create the second block of schedule with number $k = 2$, where job J_3 is included.

At step $i = 4$, $S = \{J_{1,}J_2,J_3,J_4\}$, $d_4 - p_4 = 14 - 5 = 9$. Since $C_3 = 12$, then the condition in block 4 is not fulfilled at $t_4 = 12$, and $C_4 = 17$. Job J_4 is included in block B_2, where now the number of jobs $|B_2| = 2$ and number of jobs not executed in time $r = 1$. As condition (15.5) is fulfilled then we shift the block B_2 to the left to B_1. Parameter $\delta = C_4 - d_4 = 17 - 14 = 3$; start of the first job in block B_2 $\omega = t_3 = 9$; the end of the job in the previous block $\pi = C_2 = 7$. Using formula (15.7), after shifting we obtain $t_3 = t_3 - \min(3; 9 - 7) = 9 - 2 = 7$ and accordingly $t_4 = 12 - 2 = 10$. The end time point of job J_4 after shifting becomes $C_4 = 15$.

In this case as a result of shifting block B_2 the gap between this block and block B_1 has become equal to zero, i.e. they have been joined. That is why the number of the last block is reduced by one, i.e. equals $k = 1$.

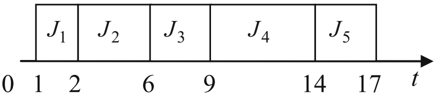

Fig. 15.3 Optimal schedule for the problem generated by the job sequence

At step $i = 5$, $S = \{J_1, J_2, J_3, J_4, J_5\}$, $d_5 - p_5 = 16 - 3 = 13$. Since $C_4 = 15$, then the condition in block 4 is not fulfilled and $t_5 = 15$, and $C_5 = 18$. Job J_5 is included into block B_1, where the total of jobs $|B_1| = 5$, and the quantity of jobs not executed in time $r = 1$. Verifying condition (15.5), we see that it is not fulfilled, since $r = 1 < |B_k| - r = 5 - 1 = 4$. That is why the block shifting is not performed.

The final schedule has the form shown in Fig. 15.3. For this schedule, objective function (15.2) has the value, equalling $\sum_{i=1}^{n} |C_i - d_i| = (4 - 2) + (7 - 6) + (12 - 9) + (14 - 14) + (17 - 16) = 7$.

15.1.2 Scheduling for Identical Parallel Machines with Common Shipment Date

Scheduling algorithms for problems with a given common due date for a single machine were shown above in Sect. 13.2. The paper (Sundararaghavan and Ahmed 1984) provides a solution of a similar problem for several identical parallel machines, provided that the specified date of shipment is not a constraint in scheduling. In fact, this condition means that the specified date is remote from the start of planning so that the optimal schedule start does not become negative. The corresponding classification formula problem has the form

$$P\left|d_i = d \;\; nonrestrictive, nmit\right| f(T, E). \tag{15.8}$$

In formula (15.8), symbol P indicates similar parallel machines; equality $d_i = d$ *non-restrictive* means that for all i-th jobs the date of shipping is the same and the non-negativity of the plan beginning; symbol *nmit* means that each following job in each machine starts as soon as this machine is free. The objective function in this case coincides with formula (15.2) in the previous paragraph.

Let us consider an example for two machines (Table 15.2), where the jobs are sorted in increasing order of processing time and set date $d = 30$.

Similar to the solution in Sect. 13.2, for each machine we make an optimal sequence of job execution, which contains two sets A and B, separated by the set date. Compiling of these two sequences is started with jobs 9 and 10, having the most processing time. Accordingly $A_1 = \{J_{10}\}$ и $A_2 = \{J_9\}$. Following jobs 8 and 7 are included into set $B_2 = \{J_8\}$ and $B_1 = \{J_7\}$. By proceeding with this process we obtain $A_1 = \{J_{10}, J_6, J_2\}$, $A_2 = \{J_9, J_5, J_1\}$, $B_2 = \{J_4, J_8\}$, $B_1 = \{J_3, J_7\}$, and the schedule given in Fig. 15.4.

Table 15.2 List of jobs with common shipment day

Job no.	1	2	3	4	5	6	7	8	9	10
Processing time	2	4	4	5	6	8	10	10	12	14

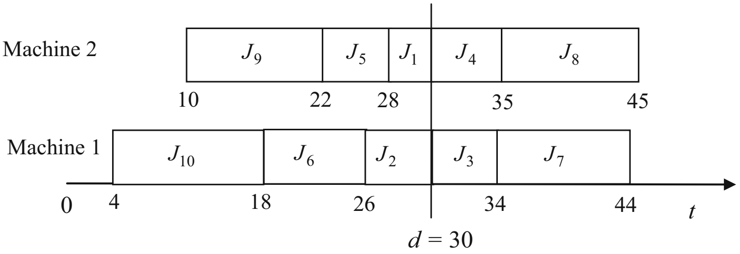

Fig. 15.4 Optimal schedule for two machines with given common date

In the problems described in this paragraph both optimization criteria—total tardiness criterion T and early production criterion E—can be reduced to one criteria using simple mathematical operations. Therefore, it allows further to use optimization methods of single-objective problems for their solution.

15.2 Multi-objective Algorithms for Some Simple Production Structures

In Sect. 4.3 the problem was considered of scheduling for a single machine with set due dates for job completions d_i and criteria that cannot be reduced to a single criterion. In this problem with classification formula $1\left|d_i\right|\overline{F}, T_{\max}$ optimization was carried out simultaneously by two criteria: minimum of average duration of each job $\overline{F}$ and minimal value of maximum tardiness $T_{\max}$.

The solution (VanWassenhoven and Gelders 1980), given in Sect. 4.3.2, contains a set of options forming a trade-off curve on the plane of criteria. This algorithm served as a basis for studies of many different problems of scheduling a single machine (T'Kindt and Billaut 2005). Most of these solutions, however, refer to the relatively narrow problems and the complexity of these solutions is usually too high. In the next paragraph, we present solutions of two multi-objective problems for a single machine based on the use of current utility functions. Here we will consider some problems for others but also simple production structures.

15.2.1 Scheduling for Two-Machine Flowshop Production

The paper (Sivrikaya-Serifoglu and Ulusoy 1998) suggests a simple solution of the problem, described by the following structural formula:

$$F2\left|prmu\right|f\left(C_{\max},\overline{C}\right), \tag{15.9}$$

where symbol $F2$ means the flow of two machines, *prmu*—possibility to find optimal sequence among the set of given jobs. The function of the target represents a combination $f\left(C_{\max},\overline{C}\right)$ from the total duration of all jobs $C_{\max}$ and average time of job completion $\overline{C}$. Since the start time point of all jobs is considered to be the same and equal to 0, then average time of completion $\overline{C}$ coincides with the average duration of production cycle $\overline{F}$.

At the first step of the solution (Sivrikaya-Serifoglu and Ulusoy 1998), a sequence is arranged that is though not optimal but can serve as a base for further quite fast optimization. For such an arrangement an algorithm is used which is close to SPT rule (Sect. 2.3.1) for the first machine and simultaneously on the second machine the time of idling is minimized. Let us demonstrate its work by the example (Table 15.3), where $p_{i,1}$ means processing time of the i-th job on machine 1 and $p_{i,2}$—the corresponding processing time on machine 2.

We will understand set T as the original set of jobs and set S as the set of schedules jobs. Let us denote the end time point of the last job of set S on the first machine as $C_{s,1}$ and the relevant time point on the second machine as $C_{s,2}$.

At the beginning of the process, we set job J_k, for which the processing time on the first machine has the smallest value. In this case it is obvious, $k = 9$. Let us exclude job 9 from the initial list T and include it into set S. It is obvious that at the first step of the algorithm time point $C_{s,1} = p_{k,1} = 2$, and time point $C_{s,2} = p_{k,1} + p_{k,2} = 2 + 8 = 10$.

If at each following step of the algorithm we define job J_k, which should be the next in order for processing on machine 1, then accordingly at this step the time point can be redefined using the dependencies

$$C_{s,1} = C_{s,1} + p_{k,1}\,\text{and}\,C_{s,2} = \max(C_{s,1}, C_{s,2}) + p_{k,2}. \tag{15.10}$$

As the start of the next job J_k on the second machine is only possible after J_k is completed on the first machine, and the second machine is free after the previous job, then in the second formula (15.10) it is necessary to take maximum from $C_{s,1}$ and $C_{s,2}$.

Further application of the algorithm is a cycle, in which jobs J_k are in sequence and are included in set S and removed from set T. On the each cycle iteration out of set T we select subset E with jobs J_i, for each of which the inequality is valid

$$p_{i,1} < (C_{s,2} - C_{s,1}). \tag{15.11}$$

Table 15.3 List of jobs for two machine flowshop

Job no.	1	2	3	4	5	6	7	8	9	10
$p_{i,1}$	5	4	6	3	10	8	12	10	2	4
$p_{i,2}$	6	3	7	2	2	9	2	5	8	7

If subset E appears to be empty, then as the next job J_k, we select that one, for which

$$p_{k,1} = \min_{J_i \in \mathrm{T}} (p_{i,1}); \qquad (15.12)$$

otherwise, the number of the following job k is defined as

$$p_{k,1} = \min_{J_i \in \mathrm{E}} (p_{i,1}). \qquad (15.13)$$

In that case, when it appears that in formula (15.12) or (15.13) there are several jobs with the same values $p_{i,1}$, the job with the least value $p_{i,2}$ is accepted as J_k. After defining J_k, this job is transferred from set T to set S; values $C_{s,1}$ and $C_{s,2}$ are recalculated by formulas (15.10). When set T becomes empty the cycle stops.

In this example at the first cycle iteration $(C_{s,2} - C_{s,1}) = 10 - 2 = 8$ and all the jobs, for which $p_{i,1} < 8$ are included into set E, i.e. $E = \{J_1, J_2, J_3, J_4, J_{10}\}$. According to formula (15.13) the next in the planned sequence job J_4 appears, for which we have the least $p_{4,1} = 3$. By recalculating values $C_{s,1}$ and $C_{s,2}$ by formulas (15.10), we have $C_{s,1} = 2 + 3 = 5, C_{s,2} = \max(5,\ 10) + 2 = 12$, and $\mathrm{S} = \{J_9, J_4\}$.

By continuing the operation of the cycle, at the second iteration we have $(C_{s,2} - C_{s,1}) = 7$ and set $E = \{J_1, J_2, J_3, J_{10}\}$. Since $p_{2,1} = p_{10,1} = 4$, so as the next job we accept job J_2 with value $p_{2,2} = 3 < p_{10,2} = 7$. The planned list after this iteration $\mathrm{S} = \{J_9, J_4, J_2\}$. We recalculate $C_{s,1} = 9$; $C_{s,2} = 15$.

At the third iteration $(C_{s,2} - C_{s,1}) = 6$; set $E = \{J_1, J_{10}\}$. Accordingly $k = 10$ and $\mathrm{S} = \{J_9, J_4, J_2, J_{10}\}$. We recalculate $C_{s,1} = 13$; $C_{s,2} = 22$.

At the following iteration, $(C_{s,2} - C_{s,1}) = 9$; set $E = \{J_1, J_3, J_6\}$; $k = 1$; $\mathrm{S} = \{J_9, J_4, J_2, J_{10}, J_1\}$; $C_{s,1} = 18$; $C_{s,2} = 28$.

Then $(C_{s,2} - C_{s,1}) = 8$; $\mathrm{E} = \{J_3\}$; $\mathrm{S} = \{J_9, J_4, J_2, J_{10}, J_1, J_3\}$; $C_{s,1} = 24$; $C_{s,2} = 35$.

Onward $(C_{s,2} - C_{s,1}) = 9$; $\mathrm{E} = \{J_6\}$; $\mathrm{S} = \{J_9, J_4, J_2, J_{10}, J_1, J_3, J_6\}$; $C_{s,1} = 32$; $C_{s,2} = 44$.

Similarly $(C_{s,2} - C_{s,1}) = 10$; set E is empty. According to expression (15.12) we have two possibilities—jobs J_5 and J_8. Since for the first of them value $p_{i,2}$ is less than for the second we obtain sequence $\mathrm{S} = \{J_9, J_4, J_2, J_{10}, J_1, J_3, J_6, J_5\}$, $C_{s,1} = 42$, $C_{s,2} = 46$.

At the following iteration $(C_{s,2} - C_{s,1}) = 4$, set E is empty; we add job J_8 to set S. At that $C_{s,1} = 52, C_{s,2} = \max(C_{s,1}, C_{s,2}) + p_{k,2} = \max(52,\ 46) + 5 = 57$ and in this case machine 2 will be idling.

The similar idling will be also before job J_7. Finally, we have the sequence $\mathrm{S} = \{J_9, J_4, J_2, J_{10}, J_1, J_3, J_6, J_5, J_8, J_7\}$, for which Gantt charts are valid, shown in Fig. 15.5.

From Fig. 15.5 it immediately follows that $C_{\max} = 66$. The average duration of a production cycle is defined as a sum of all end points of jobs on the second machine, referred to the total of jobs and equal to $\overline{F} = 33.5$.

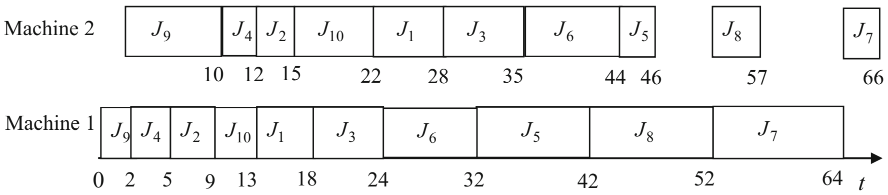

Fig. 15.5 Gant chart for flowshop production consisting of two machines

The described algorithm, generally speaking, is not aimed directly at minimizing criteria $C_{\max}$ and $\overline{F}$, and it does not set the weights of these criteria. For this optimization in the paper (Sivrikaya-Serifoglu and Ulusoy 1998), a number of different branch-and-bound algorithms are provided. In this algorithm, the lower value of (best) limit LB is described by the linear function

$$LB = w_1 LB_{C_{\max}} + w_2 LB_{\overline{F}}, \tag{15.14}$$

where $LB_{C_{\max}}$ is the possible lower limit only when using criterion $C_{\max}, LB_{\overline{F}}$—is the possible lower limit only when using criterion $\overline{F}$, and w_1 and w_2 are the relevant weighting factor.

At that it is obvious that limit $LB_{C_{\max}}$ can be set using Johnson's rule for two machines (Sect. 14.3.2). Lower limit $LB_{\overline{F}}$ can be calculated (Nagar et al. 1995) depending on the so-called load index

$$D_a = \sum_{i=1}^{n} p_{i,1} - \sum_{i=1}^{n} p_{i,2}, \tag{15.15}$$

showing which of the machines is loaded more. Obviously, if $D_a < 0$, then the second machine is loaded more; otherwise, the first machine has a larger load.

The results obtained by this algorithm of this type will depend essentially on weighting factors w_1 and w_2 established a priori, the values of which are difficult to define. In addition, the calculations performed in the paper (Sivrikaya-Serifoglu and Ulusoy 1998) show that the solutions confirmed by using the branch-and-bound approach does not differ much from the solutions described above as a first step. Therefore, generally, solving this problem we can be limited to this solution.

15.2.2 Schedule for Parallel Uniform Machines

In the above-discussed problems for parallel machines (Sect. 13.6), dividing jobs into separate parts for execution of these parts on different machines is not allowed. In case when such division is possible (symbol *pmtn*), then for the problem of the least makespan for uniform machines

$$Q|pmtn|C_{\max} \tag{15.16}$$

optimal planned sequence can be found by applying the rule of execution of the job with Longest Remaining Processing Time on Fastest Machine (LRPT-FM) first (T'Kindt and Billaut 2005). In case when the criterion is represented by the average process time $\overline{F}$, or as in this case it is the same, the average time of job completion $\overline{C}$, then for the corresponding problem

$$Q|pmtn|\overline{C} \tag{15.17}$$

we apply the rule of Shortest Remaining Processing Time on Fastest Machine (SRPT-FM) first (T'Kindt and Billaut 2005).

In the paper (McCormick and Pinedo 1995), the solution of a multi-objective problem was suggested

$$Q|pmtn|C_{\max}, \overline{C} \tag{15.18}$$

with simultaneous use of both criteria. At that the construction of ε-neighbourhood of the curve for the Pareto compromise is used to a certain extent similar to the method described above in Sect. 4.3 for a single machine.

In this case, on set S of possible sequences s some given value ε is set, for which

$$C_{\max}(s) \le \varepsilon, \tag{15.19}$$

and then in this set we find the sequence which gives the minimum of value $\overline{C}$. As ε decreases other possible solutions appear, the total of which forms trade-off curve E on plane $C_{\max}, \overline{C}$.

Let us assume that the numbering of the parallel machines increases with decreasing of their productivity factor γ_i, i.e. $\gamma_1 \ge \gamma_2 \ge \ldots \gamma_m$, and $\gamma_m = 1$. Let us position the available set of jobs in ascending order of their processing time p_i at $k_m = 1$, for example, like in Table 15.4.

Method (McCormick and Pinedo 1995) relies on the concept of the production capacity reserve for each machine. Since during planning with the use of rules LRPT-FM and SRPT-FM the preferability of machines use decreases as the performance reduces, then each following machine is involved when the capacity of already involved machines appears to be insufficient.

When planning for the most productive machine, its total operation time $C_{\max}$ according to expression (15.19) must not exceed the established value ε. As by time point $C_{\max}$ on this machine, the started job may be unfinished, then it makes sense to

Table 15.4 List of jobs with preemption

Job no.	1	2	3	4	5	6	7	8	9	10
Processing time	2	4	4	5	6	8	10	10	12	14

define what possibilities exist for involving other machines during execution of this job.

Production capacity reserve of the j-th machine at the time point of including new job in to the schedule at step r is understood as

$$V_{jr} = \sum_{l=0}^{m-j} (C_{l,r} - C_{l+1,r}) \times \gamma_{j+l}, \tag{15.20}$$

where $C_{l,r}$ is the time point of completion of the last scheduled job on the j-th machine, and $C_{0,r} = \varepsilon$.

Assume for example, $\varepsilon = 20$, the scheduling is done for two machines ($m = 2$) and $\gamma_1 = 4$; $\gamma_2 = 1$. At the first step (at the beginning) of scheduling $C_{1,1} = C_{2,1} = 0$. The capacity reserve for the first machine

$$V_{1,1} = \sum_{l=0}^{2-1} (C_{l,1} - C_{l+1,1}) \times \gamma_{j+l} = (C_{0,1} - C_{1,1}) \times \gamma_1 + (C_{1,1} - C_{2,1}) \times \gamma_2 = \\ = (20 - 0) \times 4 + (0 - 0) \times 1 = 80.$$

The capacity reserve for the second machine

$$V_{2,1} = (C_{0,1} - C_{1,1}) \times \gamma_2 = (20 - 0) \times 1 = 20.$$

When including each following job into the schedule the available capacity reserves are used gradually. The difference between the available reserve and the processing time of the job included into the schedule defines the so-called freedom or latency of planning

$$g_k = \sum_{j=1}^{\min(m,k)} V_{jr} - \sum_{i=n-k+1}^{n} p_i \quad \text{with } k = 1 \ldots n. \tag{15.21}$$

In the given example, at the beginning of planning (with $r = 1$)

$$g_1 = \sum_{j=1}^{\min(2,1)} V_{j,1} - \sum_{i=n-1+1}^{n} p_i = V_{1,1} - p_{10} = 80 - 14 = 66; \;\; g_2 \\ = \sum_{j=1}^{\min(2,2)} V_{j,1} - \sum_{i=n-2+1}^{n} p_i =$$

$$= V_{1,1} + V_{2,1} - p_{10} - p_9 = 80 + 20 - 14 - 12 = 74; \;\; g_3 = \sum_{j=1}^{\min(2,3)} V_{j,1} - \sum_{i=n-3+1}^{n} p_i =$$

$$= V_{1,1} + V_{2,1} - p_{10} - p_9 - p_8 = 64; \;\; g_4 = 54; \;\; g_5 = 46; \;\; g_6 = 40 \;\; g_7 = 35; \;\; g_8 = 31; \;\; g_9 = 27$$

and $g_{10} = 25$. The required condition of scheduling possibility is the condition

$$g_k \geq 0 \text{ with } \quad r = 1 \quad \text{for all}\, k = 1 \ldots \text{n}. \tag{15.22}$$

In the block diagram shown in Fig. 15.6, auxiliary set of jobs T is used that is initially equal to the original set sorted in ascending order of the processing time. Calculation of initial values $C_{l,1}$, $V_{j,1}$, and g_k is described above; if some of $g_k < 0$, then calculation is terminated.

At each step r of the cycle (block 4) there is a machine which is free first (block 5). After defining the minimal value g_k two options are possible. If $g > 0$, then rule SRPT-FM is used and the first out of jobs of set T (block 8) is included in the schedule of machine j for the time equalling x. In the other option, rule LRPT-FM (block 9) is used and the last job out of set T is included into the machine schedule.

Value x is defined by one of three methods. For the least productive machine with productivity coefficient $\gamma_m = 1$, value

$$x = \min\left(C_{j-1,r} - C_{j,r}, \frac{p_i}{\gamma_j}, g_{|\mathrm{T}|}\right) \tag{15.23}$$

where $\left|\mathrm{T}\right|$ is the quantity of items in set T. In this example, initial value $\left|\mathrm{T}\right| = 10$.

If $\gamma_j - \gamma_m = 1$, we use the formula

$$x = \min\left(C_{j-1,r} - C_{j,r}, \frac{p_i}{\gamma_j}, \min_{l=i+1\ldots\left|\mathrm{T}\right|\text{-}1} g_l\right), \tag{15.24}$$

and in other cases

$$x = \min\left(C_{j-1,r} - C_{j,r}, \frac{p_i}{\gamma_j}, \frac{g_{|\mathrm{T}|}}{\gamma_j - \gamma_m - 1}, \frac{\min_{l=i+1\ldots\left|\mathrm{T}\right|\text{-}1} g_l}{\gamma_j - \gamma_m}\right). \tag{15.25}$$

If the value obtained from formulas (15.23–15.25) $x < p_i/\gamma_j$ (block 12 in Fig. 15.6), then job J_i is performed on the j-th machine incompletely, and the remainder should be included in the schedule of other machine at the following iteration of the algorithm. After defining set T (block 13), it is necessary to recalculate (block 15) values $C_{l,r}$ as

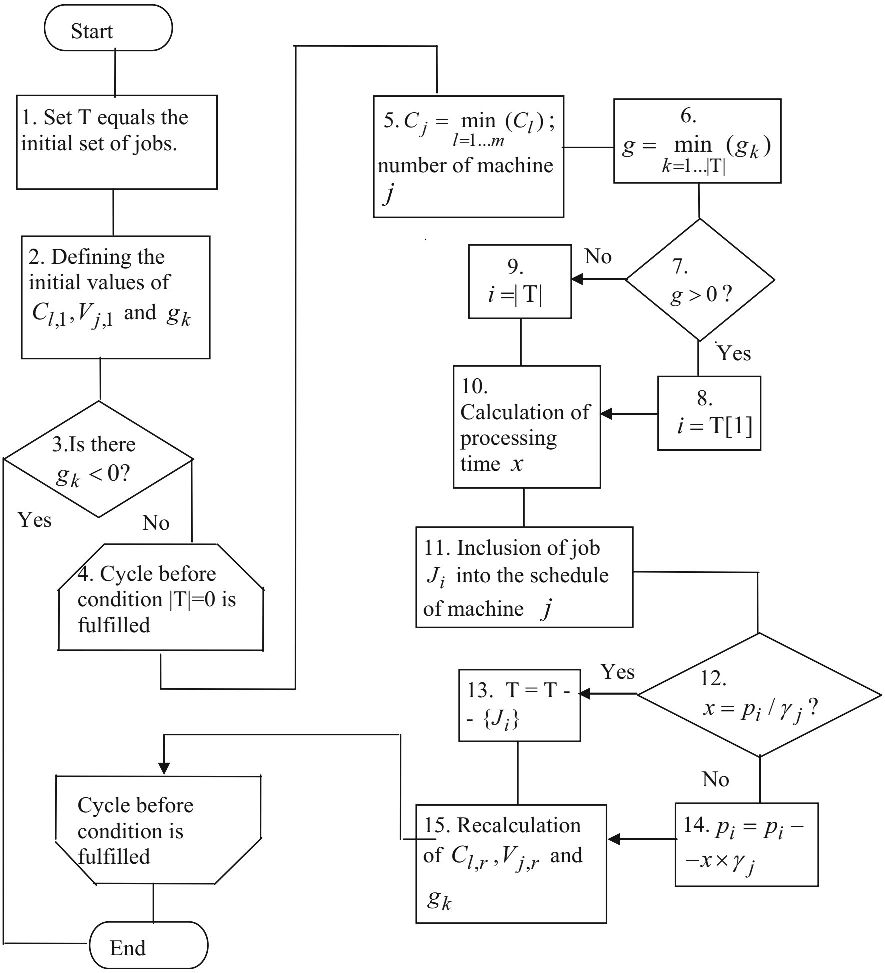
Start
1. Set T equals the initial set of jobs.
2. Defining the initial values of $C_{l,1}, V_{j,1}$ and g_k
3.Is there $g_k < 0$?
Yes
No
4. Cycle before condition |T|=0 is fulfilled
5. $C_j = \min_{l=1...m} (C_l)$; number of machine j
6. $g = \min_{k=1...|T|} (g_k)$
7. $g > 0$?
No
Yes
9. $i = |T|$
8. $i = T[1]$
10. Calculation of processing time x
11. Inclusion of job J_i into the schedule of machine j
12. $x = p_i / \gamma_j$?
Yes
No
13. T = T - - $\{J_i\}$
14. $p_i = p_i - -x \times \gamma_j$
15. Recalculation of $C_{l,r}, V_{j,r}$ and g_k
Cycle before condition is fulfilled
End

Fig. 15.6 Block diagram (algorithm 1) of problem (15.18) solution

$$C_{l,r+1} = C_{l,r} + x, \tag{15.26}$$

values $V_{j,r}$ and corresponding values g_k by formulas (15.20) and (15.21).

For example, by proceeding with step 1, according to block 5 we have $C_j = \min\limits_{l=1...m} (C_{l,r}) == \min(C_{1,1}, C_{2,1}) = \min(0, 0) = 0$. In case of equality the machine with larger capacity is selected, i.e. $j = 1$. According to block 6, we define minimal value $g = \min\limits_{k=1...|T|} g_k = g_{10} = 25$. As $g > 0$, then for further scheduling we use SRPT-FM rule and include job 1 into the schedule of the first machine (Table 15.4), i.e. $i = 1$.

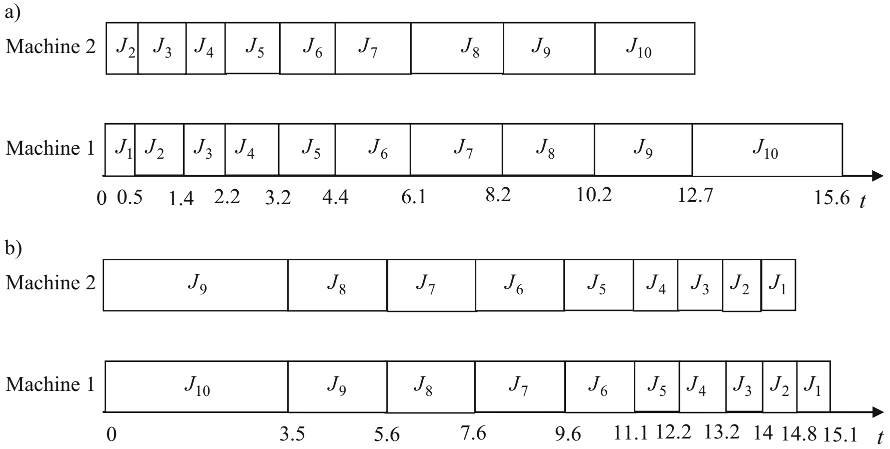

Fig. 15.7 Gantt charts for two parallel machines: (**a**) using SRPT-FM rule; (**b**) using LRPT-FM rule

Since $\gamma_1 = 4$, then for processing time x we use formula (15.25). In this case at the first step $C_{j-1,r} = C_{0,1} = \varepsilon = 20$; $C_{j,r} = C_{1,1} = 0$; $p_i = p_1 = 2$. The value of parameter $g_{|T|} = g_{10} = 25$; value $\min\limits_{l=1\ldots|T|\text{-}1}(g_l) = 27$. Hence we have $x = \min\left(20 - 0, \frac{2}{4}, \frac{25}{4-1-1}, \frac{27}{4-1}\right) = 0.5$. As $x = 0.5 = p_1/\gamma_1 = 2/4$, then job 1 on machine 1 is executed fully and can be excluded from set T.

Preparing step 2, using formulas (15.26), (15.20), and (15.21) values $C_{l,2}$, $V_{j,2}$, and g_k are recalculated. In this case $C_{1,2} = C_{1,1} + x = 0 + 0.5 = 0.5$; $C_{2,2} = 0$; $V_{1,2} = 78.5$; $V_{2,2} = 19.5$.

Consistent application of the algorithm in Fig. 15.6 in this example is actually equivalent to using SRPT-FM rule. The corresponding Gantt charts for two parallel machines are shown in Fig. 15.7a. As can be seen from Fig. 15.7a, an optimal schedule in this case provides for transferring the job from less productive machine 2 to machine 1 at the exact time of availability of the last one. Wherein $C_{\max} = 15.6$ and value $\overline{C} = (0.5 + 1.4 + 2.2 + 3.2 + 4.4 + 6.1 + 8.2 + 10.2 + 12.7 + 15.6)/10 = 6.45$.

If we reduce value ε-neighbourhood down to 15, then using formula (15.20), we obtain $V_{1,1} = 60$ and $V_{2,1} = 15$. Respectively, from expressions (15.21) $g_1 = 46$; $g_2 = 49$; $g_3 = 39$; $g_4 = 29$; $g_5 = 21$; $g_6 = 15$; $g_7 = 10$; $g_8 = 6$; $g_9 = 2$; $g_{10} = 0$. As minimal value $g = 0$, $i = |T| = 10$ is accepted as the first-in-order job (block 9 in Fig. 15.6), i.e. job J_{10} is with the highest processing time. Further application of the algorithm in Fig. 15.6 actually leads to consistent application of LRPT-FM rule, the result of which is shown in Fig. 15.7b. In this case $C_{\max} = 15.1$, $\overline{C} = 10.85$.

Comparing the obtained results in Fig. 15.7a, b, we see that the use of LRPT-FM rule leads to a slight decrease in the total processing time $C_{\max}$, but significantly increases the average processing time of a job $\overline{C}$.

The paper (McCormick and Pinedo 1995) describes a general algorithm that allows finding a few non-dominated options of scheduling, which use partly SRPT-FM rule and partly LRPT-FM rule. In this case, the algorithm in Fig. 15.6 is part of a more general algorithm. The complexity of the latter, however, is that its presentation in the scope of this book is not possible.

In practical terms, when solving the above problem it can be recommended first to find solutions according to SRPT-FM and LRPT-FM rules using the algorithm in Fig. 15.6. Then, if it is desirable to reduce the overall processing time $C_{\max}$, then some jobs with greater processing time should be moved to the beginning of the schedule drawn up by SRPT-FM rule.

15.2.3 Some Other Problems and Solving Challenges

Above in Sects. 4.3 and 15.2.2, three problems of multi-objective scheduling were considered: for a single machine, for two successive machines, and for uniform parallel machines. For these and similar to them industrial structures, a large number of studies were carried out with different set of criteria and constraints. Let us consider a few such examples, starting with the tasks for a single machine.

The paper (Lin 1983) considers the problem with classification formula

$$1\left|d_i\right|\overline{C},\overline{T}, \tag{15.27}$$

close to the described in Sect. 4.3 problem with formula $1\left|d_i\right|\overline{F},T_{\max}$. Virtually, the difference is in the fact that in the first case the criterion of average tardiness $\overline{T}$ is applied, while in the second case the criterion is the greatest tardiness $T_{\max}$. As for the criteria of the average job completion time $\overline{C}$ and the average processing time $\overline{F}$, then if all the jobs are available by the start time point of planning then these criteria are equal. The use of criterion $\overline{T}$ required to apply a quite complex algorithm based on dynamic programming method when solving the problem (Lin 1983) of finding Pareto solution.

The need for the multi-objective approach is expressed usually not for the problems with two temporal criteria as $\overline{F},T_{\max}$ or $\overline{C},\overline{T}$, but in the tasks, in which one of the criteria describes the time consumption, and the second criteria defines the cost values. The cost values for the problems of scheduling are mainly in the costs for changeover of the machine when transferring from one job to another.

In the paper (Bourgade et al. 1995) a typical task of this kind with structural formula is considered

$$1\left|d_i,s_{ik}\right|F(\overline{c},T_{\max}), \tag{15.28}$$

where s_{ik} means availability of time consumption for changeover from job i to job k. The objective function here is some function $F(\overline{c}, T_{\max})$ of average setup cost $\overline{c}$ and maximal tardiness $T_{\max}$. As this kind of function for n job it was suggested, for example, to use the expression of the form

$$F(\overline{c}, T_{\max}) = w_1\overline{c} + w_2\sum_{i=1}^{n} \exp\bigl(\max\bigl(0, T_i - T_{\max}^{*}\bigr)\bigr), \tag{15.29}$$

where $T_{\max}^{*}$ is understood as the problem solution with formula $1\big|d_i\big|T_{\max}$; w_1 and w_2—some weighting factors. This problem (Bourgade et al. 1995) was solved by branch-and-bound method.

Among the studies relating to scheduling for flowshop production, the problem outstands

$$F2\big|prmu,\, d_i\big|C_{\max}, T_{\max}. \tag{15.30}$$

In the framework of this problem for two sequential machines with possible permutations of jobs ($prmu$) and given due dates d_i in Daniels and Chambers (1990), a heuristic method for finding the optimal sequence was developed, which is close to the method of solving the problem, detailed above in Sect. 4.3. The developed method allows by use of special ε-neighbourhood of efficient points (Sect. 4.3.1) to find Pareto solutions and construct a trade-off curve for different schedule options. Moreover, in the paper (Daniels and Chambers 1990), it was also proposed to extend this method for the case of any number of serial arranged machines, i.e. for the problem

$$F\big|prmu,\, d_i\big|C_{\max}, T_{\max}. \tag{15.31}$$

For identical parallel machines, substantial results are obtained in the paper (Mohri et al. 1999), which allowed solving the problem

$$P3\big|pmtn, d_i\big|C_{\max}, L_{\max}. \tag{15.32}$$

In this paper, for three machines with specified due dates d_i and the possibility to interrupt jobs ($pmtn$), Pareto non-dominated solutions for two criteria are found—the makespan $C_{\max}$ and the greatest deviation $L_{\max}$ from the specified due date.

Returning to three problems described above in Sects. 4.2 and 15.2.2, one can notice that the complete solution including the trade-off curve construction between the criteria was given only for the first one, which is the most simple. For the other two tasks, due to the complexity of calculations, we had to be limited to only a partial their solution. Such a solution consists in scheduling for individual possible non-dominated options without constructing a complete Pareto solution.

Naturally, for the tasks described above in this paragraph, we had to develop even more complex algorithms than those given in Sect. 15.2.2. Also, while solving these problems the required scope of computation may depend heavily on the number of independent variables of planning (jobs and machines).

In some simple cases, such as in problems in Sect. 4.3 or Sect. 15.2.2, the scope of computation is a polynomial dependence on the number of variables. However, for so-called NP-hard problems, which are typical for the majority of planning cases, a much stronger growth in the scope of calculations is possible.

The concept of NP-hard is a quite difficult mathematical definition and it is usually believed that the NP-hard problems are those that have the scope of calculation exponentially depending on the number of variables. Not only complex problems but also simple ones can be NP-hard. For example, NP-hard problem is the problem for the flowshop production consisting of two machines, described in Sect. 15.2.1.

Due to the difficulty in computing of the majority of known algorithms, even with theoretical solutions, the practical use of multi-objective scheduling is limited to our time. In recent years, however, a large number of paper and studies are dedicated to development of new, as a rule, heuristic methods for finding optimal solutions. In these studies analogies to a variety of situations are used, for example, the behaviour of living things, etc.

The reason for such rapid growth of researches in this field is apparently initiated awareness of the fact that the optimization of operational planning in many cases cannot be achieved with a single criterion and requires the inclusion of at least two criteria, as it has been shown in Fig. 4.5.

A detailed overview of the major jobs in the field of multi-objective schedules for flowshop and job shop production over the past two decades is presented, for example, here (Parveen and Ullah 2010).

15.3 Scheduling Based on Cost and Average Orders Utility

Let us consider operational planning for a single machine in terms of serial production and "make-to-order" strategy. According to Table 2.5 it is advantageous here to use a schedule with two criteria—minimum costs K1 and timely order fulfilment C1. These criteria are contradictory, since, for example, to reduce K1 it is necessary to increase the sizes of production lots and conversely, to reduce the delay of order fulfilment C1 the lot sizes should be reduced. Thus, problems of this type have the so-called planning dilemma described previously in Sect. 3.2.

As shown above in Sect. 4.1.3, criterion C1 can be replaced by utility function of order processing V and criterion K1—by cost function U (Fig. 4.5). To solve the problem of planning with the help of these functions, it is necessary to construct the corresponding Pareto solution (trade-off curve) and then select a positioning point of this curve, i.e. specific solution that suits best the planner's ideas at the time of planning. Below there are two examples of such an approach for planning the operation of a single machine. Using this methodology for parallel machines, job shop production is described in Mauergauz (2013).

15.3.1 Sequenced Job Scheduling with Sequence-Dependent Setups

Assume (Mauergauz 2014a) that three kinds of jobs can be executed on the machine in any sequence, and the cost of setup depends on their order. Table 15.5 lists five planned jobs on the horizon of 7 working days, Table 15.6—matrix of setup cost C_{ik}. The number of the i-th column defines the type of previous product and the number of the k-th line—the type of subsequent product. Suppose that the planning period $G = 10$, the average cost of 1 day of job $c = 8$, "psychological" coefficient $\alpha = 0.3$. We also assume that by the beginning of the first day of the planned interval the machine is set to the product of type 2.

In Table 15.5, date g_i required for fulfilling the shipments plan is defined as

$$g_{i=}d_i - p_i + 1, \tag{15.33}$$

as the due dates are defined by completion of the working day, and the start dates—by their start.

To solve this problem first it is necessary to determine the quality criteria of the solutions. A similar problem with one criterion of maximum average utility of orders $\overline{V}$ was solved in Sect. 2.6.2. In this case, obviously, it makes sense to use also criterion $\overline{V}$ to be determined by the expression (2.40), but other than that, it is necessary to introduce a criterion reflecting the costs associated with setups.

Generally speaking, similarly to the criterion of average orders utility $\overline{V}$ it is possible to use the criterion of average costs $\bar{U}$ as a second criterion. Its value should be determined by the ratio of total costs to the time for which these costs are incurred. However, numerical calculations show that the best planning results can be obtained if not the average value of the costs but their current value U determined by the formula (4.4) is used as the costs criterion. In this case, the structural formula of the considered problem has the form

Table 15.5 Job characteristics

Job	1	2	3	4	5
Product type	1	2	1	3	1
Processing time in days p_i	1	2	1	2	1
Required due date d_i	−1	2	3	3	6
Possible date of release r_i	−4	0	1	1	2
Required date of release g_i	−1	1	3	2	6

Table 15.6 Matrix of setup cost

Product type no.	1	2	**3**
1	0	1	2
2	2	0	3
3	3	2	0

$$1\left|r_i, d_i, C_{ik}\right|U, \overline{V}. \tag{15.34}$$

To some extent, similar problem (15.28) was studied in the paper (Bourgade et al. 1995), but compared to (15.34), this problem was much easier. Furthermore, in the initial conditions of example (Table 15.5), one of the required shipping dates at the time of planning has expired, which does not allow using any static approach.

To solve the problem, first of all, we note that the value of the average utility function $\overline{V}$ is directly related to the time interval between the actual start time point of the job and the necessary release time g_i.

Indeed, according to (2.40) and (2.41) value $\overline{V}$ depends on the intensity integral for the execution time of each job p_k. For this job, if the conditions are fulfilled

$$d_k - C_l - p_k \geq 0, \tag{15.35}$$

i.e. the required due date is more than the total of the completion date of the previous job C_l and processing time p_k, intensity integral (2.44) has the form

$$\int\limits_{C_l}^{C_l+p_k} H_k dt = \alpha w_k \left[p_k - (d_k - C_l - p_k + \alpha G)\ln\left(\frac{(d_k - C_l)/\alpha G + 1}{(d_k - C_l - p_k)/\alpha G + 1}\right)\right].$$

Using expression (15.33), we have

$$\int\limits_{C_l}^{C_l+p_k} H_k dt = \alpha w_k \left[p_k - (g_k - C_l + 1 + \alpha G)\ln\left(\frac{(d_k - C_l)/\alpha G + 1}{(g_k - C_l + 1)/\alpha G + 1}\right)\right]. \tag{15.36}$$

Let us introduce value $x = g_k - C_l$, representing the time integral between the required release time point and the completion time point of the previous job. If condition (15.35) is fulfilled as expected, then $x > 0$. Expression (15.36) takes the form

$$\int\limits_{C_l}^{C_l+p_k} H_k dt = \alpha w_k \left[p_k - (x + 1 + \alpha G)\ln\left(\frac{(d_k - C_l)/\alpha G + 1}{(x+1)/\alpha G + 1}\right)\right]. \tag{15.37}$$

Let us assume that value $(x+1)/\alpha G$ much more than 1, we denote $a = 1 + \alpha G$ and $b = (d_k - C_l)/\alpha G + 1$. Then

$$\int\limits_{C_l}^{C_l+p_k} H_k dt = \alpha w_k [p_k - (x + a)\ln(b)]. \tag{15.38}$$

Expression (15.38) shows that the intensity integral decreases linearly with the increase of x. From here it follows that the earlier the actual launching towards the value of the required release time point g_i is, the higher value $\overline{V}$ is in accordance with (2.41). A similar result can be obtained for the cases when $d_k - C_l - p_k < 0$.

This result shows that to enhance utility $\overline{V}$ of each job it is necessary to tend to reduce the time interval between the required release time point and its actual possible value. For the cases where the priority of the jobs is the same, it is logical to assume that the best opportunity to achieve minimization of these intervals is to start jobs in ascending order of the required release time point g_i.

To solve problem (15.34) we use the branch-and-bound method similarly to the solution set out in Sect. 2.6.2 above, but taking into account cost criterion U.

At time point C_l the job finishes in the node of the tree, which is located at level l. If the k-th job starts at time t_k, then the average utility of all available set of jobs for all time from the start of jobs until the end of the k-th job at time $t_k + p_k$ in the node at level $l + 1$ is determined by formula (13.22) as

$$\overline{V}_{l+1,k} = \frac{1}{t_k + p_k} \int\limits_0^{t_k+p_k} V dt = \frac{1}{t_k + p_k} \left(\overline{V}_l \times C_l + \int\limits_{C_l}^{t_k+p_k} V_k dt \right).$$

When executing the k-th job in the node at level $l + 1$, the value of the cost utility function (loss function) for the unit of time is determined by the total of costs by the relevant sequence of the tree from $l + 1$. Respectively, dimensionless function of costs

$$U_{l+1,k} = \frac{1}{c} \sum_{i=1}^{l+1} c_i = U_l + \frac{1}{c} c_k. \tag{15.39}$$

Let us assume that the initial function of costs $U_0 = 0$.

According to (4.5), and accepting that the weighting factors $w_i = 1$, we obtain

$$V_0 = \frac{1}{G} \sum_{i=1}^{n} w_i p_i - \sum_{i=1}^{n} H_i = \frac{1}{10} \times \sum_{i=1}^{5} p_i - \sum_{i=1}^{5} H_i.$$

The intensity value for job 1 is defined by the second out of formulas (2.28), since the shipping date at time $t = 0$ is already expired. That's why

$$H_1 = \frac{w_1 p_1}{G} ((t - d_1)/\alpha G + 1) = 0.1 \times 1 \times ((0 + 1)/(0.3{*}10) + 1) = 0.133.$$

For job 2 and following jobs the intensity is defined by the first formula (2.28), for example,

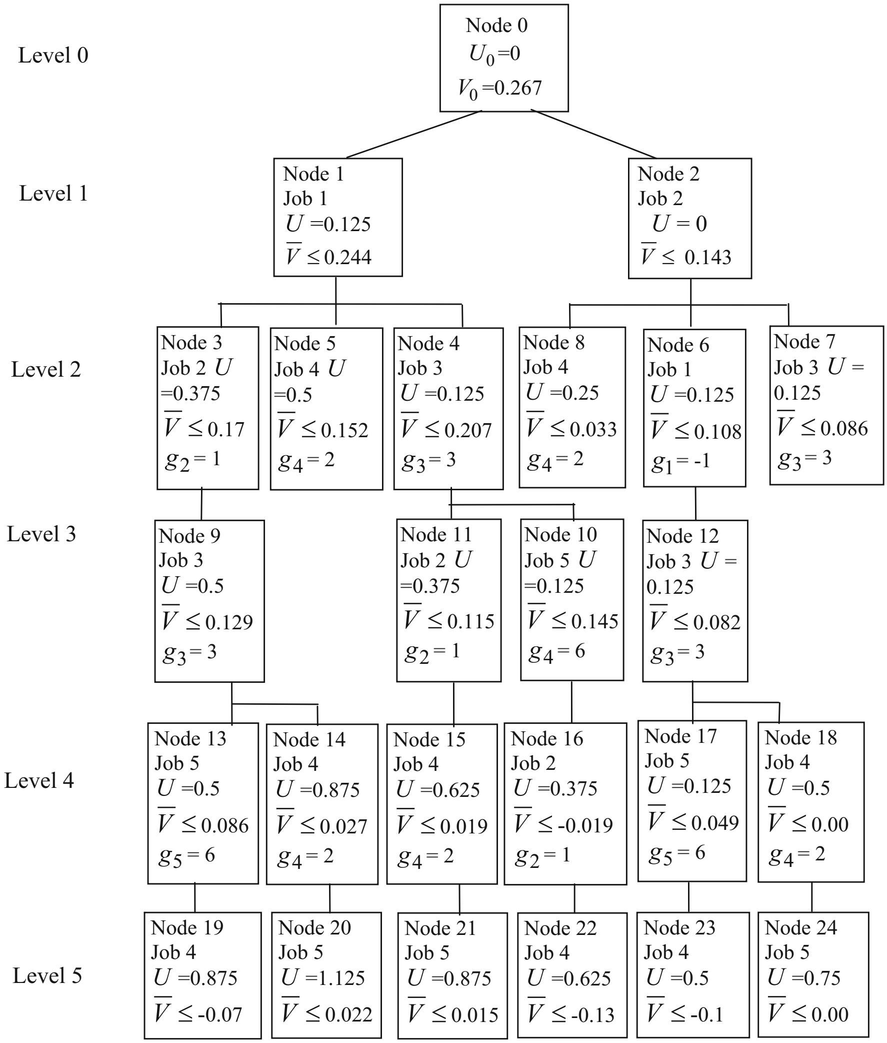
Level 0
Node 0 U_0=0 V_0=0.267
Level 1
Node 1 Job 1 U=0.125 $\overline{V}\le 0.244$
Node 2 Job 2 U= 0 $\overline{V}\le$ 0.143
Level 2
Node 3 Job 2 U=0.375 $\overline{V}\le 0.17$ g_2= 1
Node 5 Job 4 U=0.5 $\overline{V}\le 0.152$ g_4= 2
Node 4 Job 3 U=0.125 $\overline{V}\le 0.207$ g_3= 3
Node 8 Job 4 U=0.25 $\overline{V}\le 0.033$ g_4= 2
Node 6 Job 1 U=0.125 $\overline{V}\le 0.108$ g_1= -1
Node 7 Job 3 U= 0.125 $\overline{V}\le 0.086$ g_3= 3
Level 3
Node 9 Job 3 U=0.5 $\overline{V}\le 0.129$ g_3= 3
Node 11 Job 2 U=0.375 $\overline{V}\le 0.115$ g_2= 1
Node 10 Job 5 U=0.125 $\overline{V}\le 0.145$ g_4= 6
Node 12 Job 3 U= 0.125 $\overline{V}\le 0.082$ g_3= 3
Level 4
Node 13 Job 5 U=0.5 $\overline{V}\le 0.086$ g_5= 6
Node 14 Job 4 U=0.875 $\overline{V}\le 0.027$ g_4= 2
Node 15 Job 4 U=0.625 $\overline{V}\le 0.019$ g_4= 2
Node 16 Job 2 U=0.375 $\overline{V}\le$ -0.019 g_2= 1
Node 17 Job 5 U=0.125 $\overline{V}\le 0.049$ g_5= 6
Node 18 Job 4 U=0.5 $\overline{V}\le 0.00$ g_4= 2
Level 5
Node 19 Job 4 U=0.875 $\overline{V}\le$ -0.07
Node 20 Job 5 U=1.125 $\overline{V}\le 0.022$
Node 21 Job 5 U=0.875 $\overline{V}\le 0.015$
Node 22 Job 4 U=0.625 $\overline{V}\le$ -0.13
Node 23 Job 4 U=0.5 $\overline{V}\le$ -0.1
Node 24 Job 5 U=0.75 $\overline{V}\le 0.00$

Fig. 15.8 Tree of decisions search

$$H_2 = \frac{w_2 p_2}{G}\frac{1}{(d_2 - t)/\alpha G + 1} = 0.1 \times 2/(2-0)/((0.3*10)+1) = 0.12$$

and the function of the current utility $V_0 = 0.1 \times (1+2+1+2+1) - (0.133 + 0.12 + 0.05 + 0.1 + 0.03) = 0.267$. The values of initial U_0 and V_0 are given as a node of level 0 in Fig. 15.8.

Average utility value $\overline{V}_{l+1,q}$ at level $l+1$ in node q, where job J_k is executed, is defined using Table 15.7, similar to Table 13.18. Like in Sect. 13.5.3 parameter a_k^i may have one of three values: 0—for the i-th jobs already completed by start time t_k

Table 15.7 Calculation of average utility for the nodes of search tree

Level $l+1$; set I_l	Node q; job J_k in node q; beginning; p_k	Job i	a_k^i	γ_k^i	$d_i - t_k - p_k$	$d_i - t_k$	$d_i - C_l$	Formula no. for H_k^i	H_k^i	$\overline{V}_k^i$
$l = 0$ $I_0 = \{\}$ $C_0 = 0$ $\overline{V}_0 = 0$	$q = 1$ J_1 $t_1 = 0$ $p_1 = 1$	1	0.5	0.05	−2	−1	−1	7	0.072	−0.022
		2	1	0.2	1	2	2	1	0.134	0.066
		3	1	0.1	2	3	3	1	0.055	0.045
		4	1	0.2	2	3	3	1	0.109	0.091
		5	1	0.1	5	6	6	1	0.035	0.065
		Average utility in node 1 at level 1 $\overline{V}_{1,1} = 0.244$								0.244
	$q = 2$ J_2 $t_2 = 0$ $p_2 = 2$	1	1	0.2	−2	4	6	5	0.333	−0.067
		2	0.5	0.2	0	6	8	2	0.14	0.03
		3	1	0.2	1	3	3	1	0.122	0.039
		4	1	0.4	1	3	3	1	0.243	0.078
		5	1	0.2	4	6	6	1	0.075	0.062
		Average utility in node 2 at level 1 $\overline{V}_{1,2} = 0.143$								0.143
$l = 1$ $I_1 = \{1\}$ $C_1 = 1$ $\overline{V}_1 = 0.244$	$q = 3$ J_2 $t_2 = 1$ $p_2 = 2$	1	0	0	–	–	–	–	0	0
		2	0.5	0.2	−1	1	1	4	0.183	0.006
		…								
		5	1	0.2	3	5	5	1	0.086	0.038
		Average utility in node 3 at level 2 $\overline{V}_{2,3} = 0.17$								0.09
	$q = 4$ J_3 $t_3 = 1$ $p_3 = 1$	1	0	0	–	–	–	–	0	0
		2	1	0.2	0	1	1	1	0.173	0.014
		3	0.5	0.05	1	2	2	2	0.032	0.009
		…								
		5	1	0.1	4	5	5	1	0.04	0.03
		Average utility in node 4 at level 2 $\overline{V}_{2,4} = 0.207$								0.086
	…									

$l = 1$ $I_1 = \{2\}$ $C_1 = 2$ $\overline{V}_1 = 0.143$	$q = 6$ J_1 $t_1 = 2$ $p_1 = 1$	1	0.5	0.05	−4	−3	−3	7	0.106	−0.019
		2	0	0	–	–	–	–	0	0
		. . .								
		5	1	0.1	3	4	4	1	0.046	0.018
		Average utility in node 6 at level 2 $\overline{V}_{2,6} = 0.108$								0.013
	. . .									
. . .										
$l = 4$ $I_4 = \{1, 2, 3, 4\}$ $C_4 = 6$ $\overline{V}_5 = 0.0$	$q = 23$ J_5 $t_5 = 6$ $p_5 = 1$	1	0	0	–	–	–	–	0	0
		2	0	0	–	–	–	–	0	0
		3	0	0	–	–	–	–	0	0
		4	0	0	–	–	–	–	0	0
		5	0.5	0.05	0	0	0	4	0.055	0.0
		Average utility in node 23 at level 5 $\overline{V}_{5,23} = 0.037$								0.0

of job J_k, 1—for the jobs not yet executed and not scheduled in node q, and 0.5—for not completed yet and planned in node. Values of γ_k^i are defined by formulas similar to (13.25) and (13.26):

$$\gamma_k^i = p_i \frac{p_k + t_k - C_l}{G} \text{ for } i \neq k, \tag{15.40}$$

$$\gamma_k^i = p_k \frac{p_k/2 + t_k - C_l}{G} \text{ for } i{=}k. \tag{15.41}$$

For example, at the initial time point of the tree level of solution search $l = 0$; $C_l = C_0 = 0$; planned set of jobs is empty, i.e. $I_0 = \{\}$; $\overline{V}_0 = 0$. At this time jobs 1 and 2 are available (Table 15.5).

Let us calculate the average utility in node 1, in which job J_1 is executed and accordingly $t_1 = 0$. In this case $a_1^1 = 0.5$; $\gamma_1^1 = 1 \times (1/2 + 0 - 0)/10 = 0.05$; parameter $d_i - t_k - p_k = d_1 - t_1 - p_1 = -1 - 0 - 1 = -2$; parameter $d_i - t_k = d_1 - t_1 = -1 - 0 = -1$; parameter $d_i - C_l = d_1 - C_0 = -1 - 0 = -1$; and for the intensity formula 7 should be used (Appendix D), according to which intensity $H_1^1 = 0.072$. Utility $\overline{V}_1^1 = \frac{\gamma_1^1 - H_1^1}{t_1 + p_1} = (0.05 - 0.072)/(0 + 1) = -0.022$.

Job 2 in node 1 is not executed. That is why parameter value $a_1^2 = 1$; $\gamma_1^2 = 2 \times (1 + 0 - 0)/10 = 0.2$; $d_i - t_k - p_k = d_2 - t_1 - p_1 = 2 - 0 - 1 = 1$; parameter $d_i - t_k = d_2 - t_1 = 2 - 0 = 2$; parameter $d_i - C_l = d_2 - C_0 = 2 - 0 = 2$; and for intensity formula 1 is used. At that the intensity $H_1^2 = 0.134$, utility $\overline{V}_1^2 = 0.066$. The calculation results for other jobs in the node are given in the following lines. The utility of all jobs in node 1 located at level 1 $\overline{V}_{1,1} = \frac{\overline{V}_l C_l}{t_k + p_k} + \sum\limits_i \overline{V}_k^i = 0 + 0.244 = 0.244$.

Since job 1 is executed for the product of type 1 and initially the machine is set up for product 2 then in node, the setup from product 2 to product 1 should be performed with the cost equalling 1 (Table 15.6). The value of cost functions in node 1, according to expression (15.39), is

$$U_{1,1} = \frac{1}{c} \sum_{i=1}^{1} c_i = \frac{1}{8} \times 1 = 0.125.$$

In node 2 job J_2 is executed and, respectively, $t_2 = 0$ and $a_2^2 = 0.5$, and for the rest of the jobs values $a_2^i = 1$. By calculating the utilities of jobs in node 2 at level 1 similar to the calculation of node 1, we have $\overline{V}_{1,2} = 0.143$. When executing job 2 at the initial time point of planning there is no need for changeover, since job 2 is executed for product 2 (Table 15.5), and the initial setup of the machine, as was specified, is performed for this product and that is why $U_{1,2} = 0$. In Fig. 15.8 nodes 1 and 2 are shown as nodes of level 1.

Let us now perform branching from node 1 of level $l+1=1$. At the end time point of job 1, time $C_l = C_1 = p_1 = 1$. By this time, jobs 2, 3, and 4 (Table 15.5) are available for execution; they are shown as nodes 3, 4, and 5 at level 2 in Fig. 15.8. In node 3 at level 2 job 2 is performed and in this case $t_2 = 1$, $a_2^1 = 0$, $a_2^2 = 0.5$, other $a_2^i = 1$. After performing calculation of value $\overline{V}_2^i$, we define the average utility $\overline{V}_{2,3} = \frac{\overline{V}_1 C_1}{t_2 + p_2} + \sum_i \overline{V}_2^i = \frac{0.244 \times 1}{1+2} + 0.09 = 0.17$. Similarly for node 4 at level 2, in which job 3 is executed, we obtain $\overline{V}_{2,4} = 0.207$, and for node 5 with executed job 4 we have $\overline{V}_{2,5} = 0.152$.

Values of loss functions according to formula (15.39) give in node 3 at level 2 with executed job 2 $U_{2,3} = \frac{1}{c} \times (c_1 + c_2) = \frac{1}{8} \times (1+2) = 0.375$. Similarly in node 4 $U_{2,4} = 0.125$ and in node 5 $U_{2,5} = 0.5$.

Using dependence (15.38) it was shown that utility $\overline{V}$ of each i-th job increases at job launching in ascending order of the required release time g_i. Let us assume that for domination at level $l+1$ of the q-th node of the tree of search with the i-th job over the r-th node with the k-th job is enough to observe inequality.

$$U_{l+1,q} \le U_{l+1,r},\ \overline{V}_{l+1,q} \ge \overline{V}_{l+1,r} \text{ and } g_i \le g_k, \tag{15.42}$$

at that at least one of the inequalities was strict.

Let us compare the values of criteria $U, \overline{V}$ in node 3 and node 4 (Fig. 15.8). Value $U_{2,3} = 0.375 > U_{2,4} = 1.25$ and $\overline{V}_{2,3} = 0.207 > \overline{V}_{2,4} = 0.17$, i.e. in node 3 compared with node 4 the first criterion is worse, and the second criterion is better. That’s why both these criteria should be used for the further branching.

When comparing node 3 with job 2 and node 5 with job 4 (Fig. 15.8) $U_{2,3} < U_{2,5}$, $\overline{V}_{2,3} > \overline{V}_{2,5}$. Besides, $g_2 < g_4$, from here it follows that it is possible to omit the consideration of the further branching from node 5.

Let us consider now branching from node 2 at level 1. At the end time of job 2, time $C_l = C_1 = p_2 = 2$. By now jobs 1, 3, 4, and 5 (Table 15.5) are available for execution. Figure 15.8 shows nodes 6, 7, 8 for jobs 1, 3, and 4. Comparison of the criteria using the inequalities (15.42) shows that node 6 with job 1 dominants over nodes 7 and 8. Similarly, node 6 dominates also over omitted node with job 5.

We transfer to branching from non-dominated nodes 3, 4, and 6 at level 2. At level 3 only non-dominated nodes $9 \div 12$ are shown; at level 4 the number of non-dominated nodes is six. When considering the dominance of the nodes at the last level 5, it is possible to take into account only the first two terms of inequality (15.42), because here branching of nodes is stopped. In this case, node 19 is dominated by nodes 21 and 24, and node 22 is dominated by node 23.

According to the results of the constructed search tree in Fig. 15.8, we draw up a table of criteria values for all non-dominated options (Table 15.8). The values of the criteria in Table 15.8 correspond to horizons, in which jobs are completed. For horizons in which job is in progress, the criteria values can be obtained by interpolation.

Table 15.8 Criteria values on various horizons for sequenced job scheduling

Sequence Option no.	Criteria	Horizon						
		1	2	3	4	5	6	7
1, 2, 3, 4, 5 1	U	0.125		0.375	0.5		0.875	1.125
	$\overline{V}$	0.244		0.17	0.129		0.027	0.022
1, 3, 2, 4, 5 2	U	0.125	0.125		0.375		0.625	0.875
	$\overline{V}$	0.244	0.207		0.115		0.019	0.015
2, 1, 3, 4, 5 3	U		0	0.125	0.125		0.5	0.75
	$\overline{V}$		0.143	0.108	0.082		0.0	0.0
2, 1, 3, 5, 4 4	U		0	0.125	0.125	0.125		0.5
	$\overline{V}$		0.143	0.108	0.082	0.049		−0.1

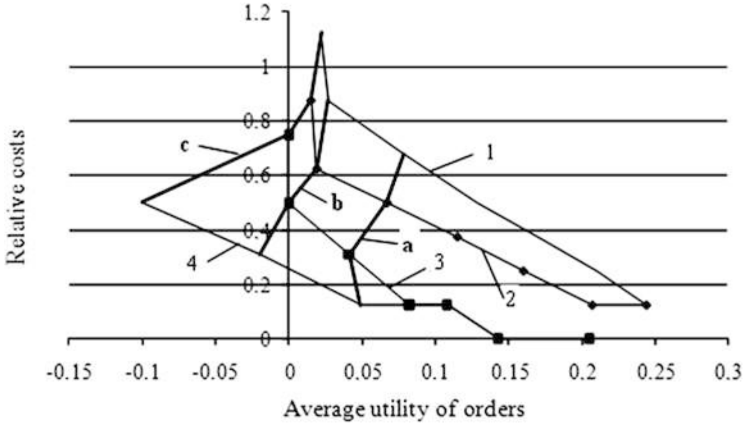

Fig. 15.9 Utility graphs of schedule options 1÷4 (Table 15.8); (**a**) horizon 5; (**b**) horizon 6; (**c**) horizon 7

Each option of 1÷4 sequence has its own trajectory on the plane and the average utility of orders $\overline{V}$ and costs U (Fig. 15.9). By connecting the points of different options for a single horizon, it is possible to get a set of Pareto solutions. In the example in Fig. 15.9, such trade-off curves **a, b, c** are plotted for horizons 5, 6, 7, respectively.

The study of the curves in Fig. 15.9 allows drawing some conclusions about the properties of the solutions on the plane $U\,\overline{V}$: the average utility of orders $\overline{V}$ on each next horizon is clearly reduced and the relative costs U somewhat increase; dispersion of point of the options on the plane increases with the horizon.

Note that on trade-off curve a there is an section between options 3 and 4, on which cost reduction U leads to increase in utility $\overline{V}$. This section corresponds to nodes 17 and 18 in Fig. 15.8. Node 17, referring to option 4 {2, 1, 3, 5, 4}, dominates against node 18 by function values U and $\overline{V}$. However, in node 17 the required release time point of job 5 $g_5 = 6$ is more than the corresponding time of release for job 4 $g_4 = 2$ in node 18, and therefore the branch of option 3 {2, 1, 3, 4, 5} should not be excluded from consideration.

Indeed, the subsequent consideration of horizons on curves **b**, **c** shows that further branching of option 3 leads to non-dominated values on plane $U\,\overline{V}$.

The existence of several trade-off curves for different horizons allows the user while scheduling to select the horizon, for which the data are known with high probability. On the trade-off curve for this horizon, it is obvious that a point should be selected (sequence option) using any of the methods of decision-making (Sect. 4.4). More details on this are set out below in Sect. 15.4.1.

15.3.2 Group Scheduling for Parallel Batches Based on Maximum Average Utility and Minimum Setup Costs

Let us assume (Mauergauz 2014a) that on the machine several jobs belonging to one of the two possible job types can be executed simultaneously and the cost of the machine setup depends on their sequence. For example, let us consider the input data for scheduling shown in Table 13.17, which we rewrite as Table 15.9 for convenience. Table 15.10 presents data on the setup cost c_{ik}. The required release time point g_i is defined by formula (15.33).

In this case the classification formula has the form

$$1\,(\text{batch}, C_{batch})|g = 2, c_{ik}|\,U, \overline{V}, \tag{15.43}$$

where, like in Sect. 13.5.2, parameter "batch" means that the schedule should be made by using load batches consisting of several jobs; parameter C_{batch} indicates

Table 15.9 Input data for scheduling

Job no.	Process group	Processing time p_i	Occupied volume, l	Expected arrival time r_i	Required due time point d_i	Required release time point g_i
1	1	4	30	0	6	3
2	2	6	50	2	8	3
3	2	6	40	3	9	4
4	1	4	30	5	9	6
5	1	4	60	8	12	9
6	1	4	30	12	16	13
7	2	6	50	13	20	15
8	1	4	30	18	22	19
9	2	6	40	18	24	19

Table 15.10 Matrix of setup cost

Job type no.	1	2
1	0	1
2	2	0

the parallel execution of all loaded jobs; parameter g indicates the quantity of job groups requiring a different process setup.

Assuming that, like in Sect. 13.5.3, duration of planning period $G = 24$, usable volume of the machine in litres $b = 100$, psychological coefficient $\alpha = 0.1$. Let us also assume that the average cost of one working $c = 8$, and at initial time $t = 0$, the machine is set up for execution of job type 2.

According to Table 15.9 it is possible to define the required release time in node q. This time is defined by job m from set of jobs J_k included into the k-th batch belonging to this node

$$g_m = \min_{i \in J_k} (g_i), \tag{15.44}$$

For example, in node 3, set of jobs included into the first batch of processing $J_1 = \{1, 4\}$. In this case we have $g_m = \min(g_1, g_4) = g_1 = 3$.

At level 0, the value of costs function $U = 0$, and utility function $V = 1.52$ (Fig. 15.10).

The possible nodes at the first level of the searching tree of optimal solution in the considered problem are the same as in Sect. 13.5.3; the calculation of the average utility in nodes $\overline{V}$ is performed using Table 13.18. Besides, it is necessary here to define costs values U. For example, the value of costs in node 1 according to expression (15.39) and Tables 15.9 and 15.10 is

$$U_{1,1} = \frac{1}{c} \sum_{i=1}^{1} c_i = \frac{1}{8} \times 1 = 0.125.$$

The values of costs for nodes 3 and 5 equal the costs in node 1 (Fig. 15.10); for nodes 2 and 4, the values of costs are 0.

The highest utility $\overline{V}$ takes place in node 1, and the lowest costs function U in nodes 2 and 4. That is why these nodes are taken as initial nodes for branching at level 2.

Just as in Sect. 13.5.3, we assume that if for some k-th branch worse results are observed consistently on two levels l and $l + 1$, than on other branches, then further branching from corresponding nodes is not advisable. This means that if out of the set of branches J the i-th branch exists, the node of which dominates over the node of k-th branch at level l, and the s-th branch, the node of which dominates over the node of k-th branch at level $l + 1$, i.e.

$$\begin{gathered} \overline{V}_{l,k} \le \overline{V}_{l,i}, \;\; U_{l,k} \ge U_{l,i} \\ \text{and} \\ \overline{V}_{l+1,k} \le \overline{V}_{l+1,s}, \;\; U_{l+1,k} \ge U_{l+1,s}, \end{gathered} \tag{15.45}$$

and in each pair of the inequalities at least one is fulfilled strictly, then further branching of the k-th branch is not advisable.

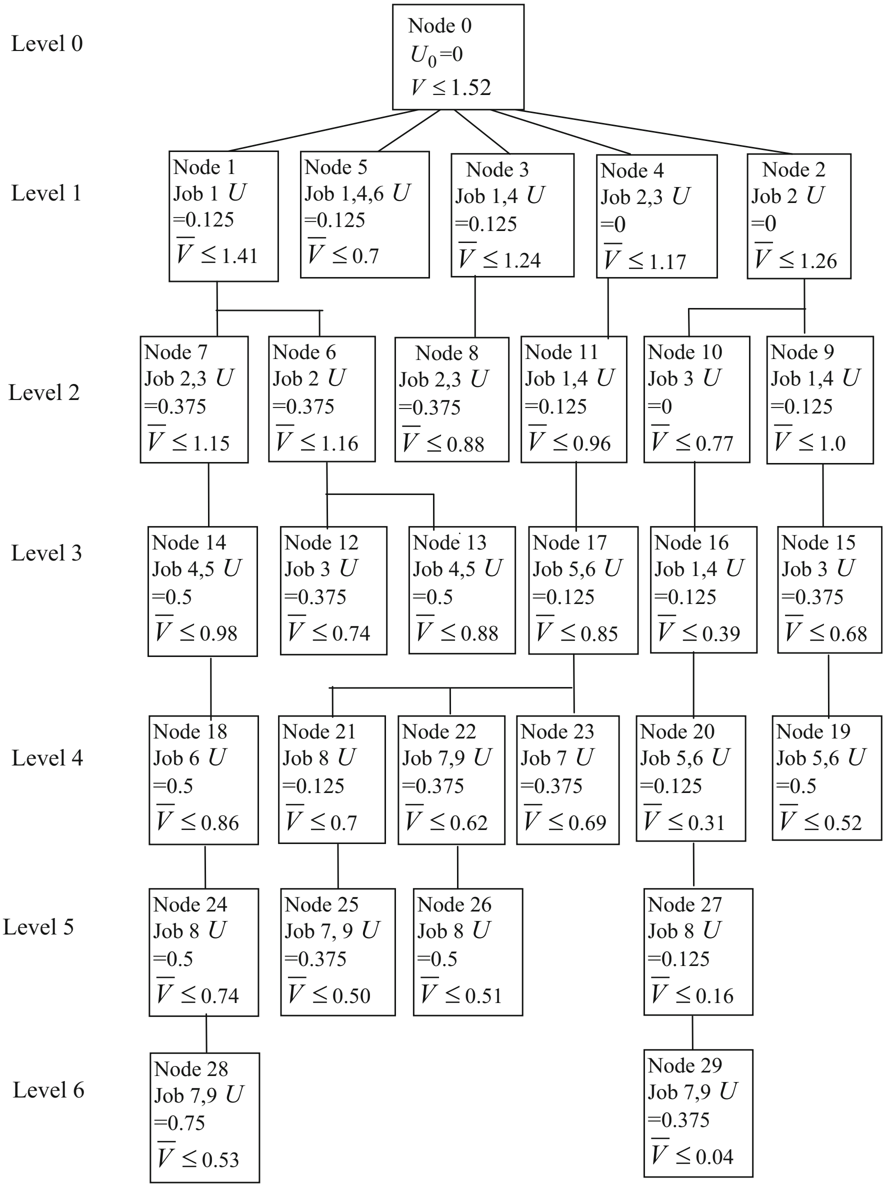
Level 0
Level 1
Level 2
Level 3
Level 4
Level 5
Level 6
Node 0 U_0=0 $V \le 1.52$
Node 1 Job 1 U =0.125 $\overline{V} \le 1.41$
Node 5 Job 1,4,6 U =0.125 $\overline{V} \le 0.7$
Node 3 Job 1,4 U =0.125 $\overline{V} \le 1.24$
Node 4 Job 2,3 U =0 $\overline{V} \le 1.17$
Node 2 Job 2 U =0 $\overline{V} \le 1.26$
Node 7 Job 2,3 U =0.375 $\overline{V} \le 1.15$
Node 6 Job 2 U =0.375 $\overline{V} \le 1.16$
Node 8 Job 2,3 U =0.375 $\overline{V} \le 0.88$
Node 11 Job 1,4 U =0.125 $\overline{V} \le 0.96$
Node 10 Job 3 U =0 $\overline{V} \le 0.77$
Node 9 Job 1,4 U =0.125 $\overline{V} \le 1.0$
Node 14 Job 4,5 U =0.5 $\overline{V} \le 0.98$
Node 12 Job 3 U =0.375 $\overline{V} \le 0.74$
Node 13 Job 4,5 U =0.5 $\overline{V} \le 0.88$
Node 17 Job 5,6 U =0.125 $\overline{V} \le 0.85$
Node 16 Job 1,4 U =0.125 $\overline{V} \le 0.39$
Node 15 Job 3 U =0.375 $\overline{V} \le 0.68$
Node 18 Job 6 U =0.5 $\overline{V} \le 0.86$
Node 21 Job 8 U =0.125 $\overline{V} \le 0.7$
Node 22 Job 7,9 U =0.375 $\overline{V} \le 0.62$
Node 23 Job 7 U =0.375 $\overline{V} \le 0.69$
Node 20 Job 5,6 U =0.125 $\overline{V} \le 0.31$
Node 19 Job 5,6 U =0.5 $\overline{V} \le 0.52$
Node 24 Job 8 U =0.5 $\overline{V} \le 0.74$
Node 25 Job 7, 9 U =0.375 $\overline{V} \le 0.50$
Node 26 Job 8 U =0.5 $\overline{V} \le 0.51$
Node 27 Job 8 U =0.125 $\overline{V} \le 0.16$
Node 28 Job 7,9 U =0.75 $\overline{V} \le 0.53$
Node 29 Job 7,9 U =0.375 $\overline{V} \le 0.04$

Fig. 15.10 Tree of search for criteria of costs U and average utility $\overline{V}$

When branching from node 1, the load batches consisting of jobs {2}, {2, 3}, {4}, {4, 5} are possible and herewith the best option of utility $\overline{V}$ is provided by node 6 at the second level with utility 1.16. Besides, as it was shown above in Sect. 13.5.3, it is necessary to take into account node 7 with jobs {2, 3}, since it give better values $\overline{V}$ at the next level.

Values of criterion U in the nodes of the first level 1, 3, 5 are the same. That is why for finding the need for branching from nodes 3 and 5 after branching from node 1, we compare only values $\overline{V}$ on nodes 6 and 7 at the second level with the corresponding values $\overline{V}$ in nodes 3 and 5. Obviously, that only branching from node 3 makes sense and the best results of branching occur for node 8.

As a result of this branching, it appears that at level 1 node 3 of branch {0, 3, 8} is dominated by node 1 of branch {0, 1} and at level 2 is dominated by node 9 of branch {0, 2, 9} and by node 11 of branch {0, 4, 11}. Hence, it appears that according to (15.45) further branching from node 8 is not advisable.

Going to branching of nodes 2 and 4, at level 2 we obtain nodes 9, 10, 11. The rest possible nodes are not shown, as they are further dominated by nodes 9, 10, 11.

At the third level we have six non-dominated nodes. With further branching, the criteria values in nodes originating from 12 and 13 are dominated by the criteria values in nodes 17 or 14, and so are not considered any more. Six non-dominated nodes also exist at the fourth level and from nodes 14, 15, and 16 each of non-dominated nodes 18, 19, and 20 originate, respectively, and from node 17—three non-dominated nodes 21, 22, 23.

From branches {0, 4, 11, 17, 21, 25} and {0, 4, 11, 17, 22, 26}, the planning is ended at level 5, i.e. to execute all scheduled jobs (Table 15.9) in this case it is enough to consistently load five batches while branches {0, 1, 7, 14, 18, 24, 2} and {0, 2, 10, 16, 20, 27, 29} provide six load batches.

To demonstrate the dependence of the criteria values from the sequence of jobs, we compare, for example, the criteria values in nodes 15 and 16. Upon execution of jobs in these nodes, it appears that same jobs 1, 2, 3, and 4 are executed; however, the criteria values are very different.

According to the results of construction of the searching tree in Fig. 15.10, we draw up a table of criteria values on each horizon for all non-dominated options (Table 15.11).

The execution time points of the jobs in the nodes of the searching tree do not always coincide with the values of the horizons in Table 15.11. In such cases, it is possible to refer the execution time point to the larger or smaller horizon using the rule:

$$\begin{aligned} &\text{with}\quad h_{i+1} > C_l \geq h_i + \frac{h_i + h_{i+1}}{2} \quad \text{refer execution to} \quad h = h_{i+1};\\ &\text{with}\quad h_{i+1} > C_l < h_i + \frac{h_i + h_{i+1}}{2} \quad \text{refer execution to} \quad h = h_i. \end{aligned} \tag{15.46}$$

For example for node 7 $C_l = 10$ and assume that horizon $h = 12$.

Table 15.11 Criteria values on various horizons for parallel batches

Sequence Option no.	Criteria	Horizons							
		4	8	12	16	20	24	28	32
{1}, {2,3}, {4,5}, {6}, {8}, {7,9} 1	U	0.125		0.375	0.5	0.5	0.5		0.75
	$\overline{V}$	1.41		1.12	0.98	0.86	0.74		0.73
{2,3}, {1,4}, {5,6}, {8}, {7,9} 2	U		0	0.125	0.125		0.125	0.375	
	$\overline{V}$		1.2	0.98	0.85		0.7	0.50	
{2,3}, {1,4}, {5,6}, {7,9}, {8} 3	U		0	0.125	0.125		0.375	0.5	
	$\overline{V}$		1.2	0.98	0.85		0.62	0.51	
{2},{3}, {1,4}, {5,6}, {8}, {7,9} 4	U		0		0	0.125	0.125	0.125	0.375
	$\overline{V}$		1.26		0.65	0.39	0.31	0.16	0.04

15.4 Application of Decision Theory Methods

The decision to choose the most appropriate scheduling option in the current operating environment should be made on the basis of common methods of the decision theory, the foundations of which are set out above in Sect. 4.4. According to the theory, the right solution can be obtained, if, first of all, the quality criteria of this decision are selected correctly. Selection of the criteria for different production structures in various situations is described in detail above in Sect. 2.2.

As stated in Sect. 4.2, there are two ways of multi-objective optimization. In the first of them, the solution of the optimization problem with one objective function is searched, which is a combination of several criteria, and in the second, a trade-off curve is constructed directly and the solution is selected on this curve.

When scheduling, they usually confine themselves to linear combination of two or three criteria, each of which comes with its own weight, such as in the model set out in Sect. 7.5.2.

The criteria significance for the same structure may vary depending on the current situation in production, and in such cases, it is advisable to use the so-called calculated priority method (Frolov 2010). This method takes into account such process parameters as intensity, setup time, processing time, the readiness percentage of items in a lot, etc., by the values of which the priority of any given order processing on each work center is calculated.

Yet the use of such models in practice faces great difficulties in determining the weight of each criterion. Therefore, the majority of current researches are aimed at direct construction of Pareto solutions in order to offer the planner the choice from a relatively small set of possible sustainable options. Such options are represented by trade-off curves constructed for multiple planning horizons, as for example the curves in Fig. 15.9.

If these diagrams are constructed the selection of the optimal decision consists of two problems:

- selection of the estimated planning horizon;
- selection of a point mostly suitable for the current operating situation on the trade-off curve of this horizon.

The selection of the estimated planning horizon depends on the reliability of information about future receipts of jobs to be executed. Obviously, it makes no sense to prepare an optimal schedule for a long time, if timely receipt of jobs is at issue. The specific value of the estimated planning horizon must be apparently set in accordance with statistical data on the targets fulfilment by the relevant departments.

When selecting the best option for the estimated planning horizon, it is possible to use one of the theoretical methods of decision-making set out in Sect. 4.5 above.

15.4.1 Application of Savage Principle for Decision-Making

As noted above in Sect. 4.5.2, to apply Savage Principle first of all on the basis of the existing utility values the loss functions are determined. To this end, for each *j*-th criterion and the *i*-th set of alternatives the best values of utility functions are determined, and then possible loss values are calculated as deviations from these best values.

Let us consider the application of Savage Principle for the problem described in Sect. 15.3.1 on the seventh planning horizon. Since Savage Principle uses monotonically increasing and normalized criteria that may vary from zero to one, we replace initial criteria U and $\overline{V}$ by criteria

$$f_1 = 1 - \frac{U - U_{\min}}{U_{\max} - U_{\min}} \text{ and } f_2 = \frac{\overline{V} - \overline{V}_{\min}}{\overline{V}_{\max} - \overline{V}_{\min}}. \tag{15.47}$$

For example, for option 3, using Table 15.8, we have

$$f_{1,3} = 1 - \frac{0.75 - 0.5}{1.125 - 0.5} = 0.6; f_{2,3} = \frac{0.0 - (-0.1)}{0.022 - (-0.1)} = 0.82.$$

In this case for the set of non-dominated options on the seventh horizon we have the table of utilities (Table 15.12).

According to expression (4.40), the regret by the first criteria is $f_{1,\max} - f_1$ and by the second criteria $f_{2,\max} - f_2$ (Table 15.13).

Thus, the application of Savage Principle leads to the conclusion that when using the seventh planning horizon as estimated the best alternative is option 3 with the

Table 15.12 Criteria utilities for various alternatives and sequenced job scheduling

Alternative (option)	f_1	f_2
1	0	1
2	0.4	0.943
3	0.6	0.82
4	1	0
$f_{\max,j} = \max_i f_{ij}$	1	1

Table 15.13 Determination of the best option by Savage Principle

Alternative	Regrets by criteria		Maximal regret
	f_1	f_2	
1	1	0	1
2	0.6	0.057	0.6
3	0.4	0.18	0.4
4	0	1	1
Regret min–max			0.4

Table 15.14 Criteria utilities for various alternatives and parallel batches

Alternative (option)	Regrets by criteria		Maximal regret
	f_1	f_2	
1	1	0	1
2	0	0.33	0.33
3	0.33	0.32	0.33
4	0	1	1
$f_{\max,j} = \max\limits_i f_{ij}$	1	1	0.33

smallest maximal regret by both criteria. Returning to the problem in Sect. 15.3.1, we see that option 3 provides sequence of job {2, 1, 3, 4, 5}. In this case, calculations show that option 3 remains the best by Savage Principle for estimated horizons with the value less than 7 as well.

Let us also consider the application of Savage Principle for the problem in Sect. 15.3.2. In this case, using Table 15.11 it is also possible to calculate the table of utilities for each horizon. For example, for horizon 32 the criteria values and the maximal regrets are given in Table 15.14.

Defining the best options by Savage Principle on several estimated planning horizons we obtain the data given in Table 15.15.

From consideration of Table 15.15 it follows that options 2 and 3 have absolute advantage over options 1 and 4 on all horizons. Options 2 and 3 up to horizon 16 coincide (Table 15.11) and show respectively the same regret value. In this case, the best alternative according to Savage is option 2 with sequence {2, 3}, {1, 4}, {5, 6}, {8}, {7, 9}.

Table 15.15 Maximal regrets and best options

	Horizons			
Options	20	24	28	32
1	1	1	1	1
2	0.19	0.09	0.51	0.33
3	0.33	0.67	0.76	0.33
4	1	1	1	1
Best options	2	2	2	2.3

15.4.2 Application of Hurwitz Principle for Decision-Making

Hurwitz Principle as was specified above in Sect. 4.5.1 is some combination of the Guaranteed Result Principle (Sect. 4.4.3) and Optimism Principle (Sect. 4.4.4) namely (4.37):

$$f^* = \gamma f_1^* + (1 - \gamma) f_2^*,$$

where γ is the importance factor of Guaranteed Result Principle and $1 - \gamma$ is the importance factor of Optimism Principle.

We construct the solution to the problem that is described in Sect. 15.3.1 on the seventh planning horizon using the original Table 15.12. If we set factor $\gamma = 0.5$, we obtain Table 15.16.

As shown in Table 15.16, resulting from the use of Hurwitz Principle in this case, the best option is option 2, which differs from the result of applying Savage Principle in Sect. 15.4.1. Option 3 appears to be the best by Hurwitz Principle only if we use parameter γ with a value greater than 0.7, which corresponds to substantially greater influence of the guaranteed result principle. Thus, the use of different methods of decision theory in this case leads to the two most efficient sequence options for machine loading and the final selection between them should be performed by the planner.

Returning to Fig. 15.9, we can compare options 2 and 3, the paths of which are marked. As can be seen from these graphs, the points corresponding to these two solutions on the seventh horizon are very close. At the same time, on the previous horizons the difference between the options is much more sufficient, and the costs of option 3 are clearly lower than the costs of option 2. Since the accuracy of the planning falls with increasing of the horizon, in this case it seems to make sense of using option 3 recommended by of Savage Principle.

For the problem from Sect. 15.3.2, the similar application of Hurwitz Principle gives (Table 15.17):

Optimal variant 2, obtained by Hurwitz Principle, coincides with the option obtained by Savage Principle. The reason for this coincidence consists in application of the value of factor $\gamma = 0.5$. As it was shown if Sect. 4.5.1, changing in factor value γ may change the selection of optimal option only in case of sufficient deviation of this value from value 0.5 (Table 4.12).

Table 15.16 Determination of the best result by Hurwitz Principle for sequenced job scheduling

Alternative (option)	f_1	f_2	$\min(f_j)$	$\max(f_j)$	$\gamma \min\left(f_j\right) + (1-\gamma)\max\left(f_j\right)$
1	1	0	0	1	0.5
2	0.6	0.057	0.057	0.6	0.325
3	0.4	0.18	0.18	0.4	0.29
4	0	1	1	0	0.5
$f^* = \max\limits_i \left[\gamma \min\limits_j \left(f_{ij}\right) + (1-\gamma) \max\limits_j \left(f_{ij}\right)\right]$					0.325

Table 15.17 Determination of the best result by Hurwitz Principle for parallel batches

Alternative (option)	f_1	f_2	$\min(f_j)$	$\max(f_j)$	$\gamma \min\left(f_j\right) + (1-\gamma)\max\left(f_j\right)$
1	0	1	0	1	0.5
2	1	0.7	0.7	1	0.85
3	0.67	0.7	0.67	0.7	0.68
4	1	0	0	1	0.5
$f^* = \max\limits_i \left[\gamma \min\limits_j \left(f_{ij}\right) + (1-\gamma) \max\limits_j \left(f_{ij}\right)\right]$					0.85

15.5 Decision-Support Systems

To support decision-making various special systems were developed and one of them is presented in Sect. 5.1.4 above in terms of application of the knowledge base. Below several examples of other support systems for decision-making are presented.

15.5.1 Decision-Support System for Hybrid Flow Lines

The structure of hybrid flow lines, for which explicitly the decision-support system is developed (Riane et al. 2001), is described above in Sect. 14.1.3. The results obtained in this paper, however, can be used more extensively as they contain the basic provisions characterizing the structure, function, and operation of any modern decision-support system in production planning. The system layout suggested in the paper is shown in Fig. 15.11.

The database of the information system in Fig. 15.11 contains information about the events of business processes, as well as planning and modelling results. So here transactional and analytical bases are combined (Sects. 5.1.3 and 5.1.4). Knowledge base (Sect. 5.1.5) is pointed out and contains previously used or developed rules for planning.

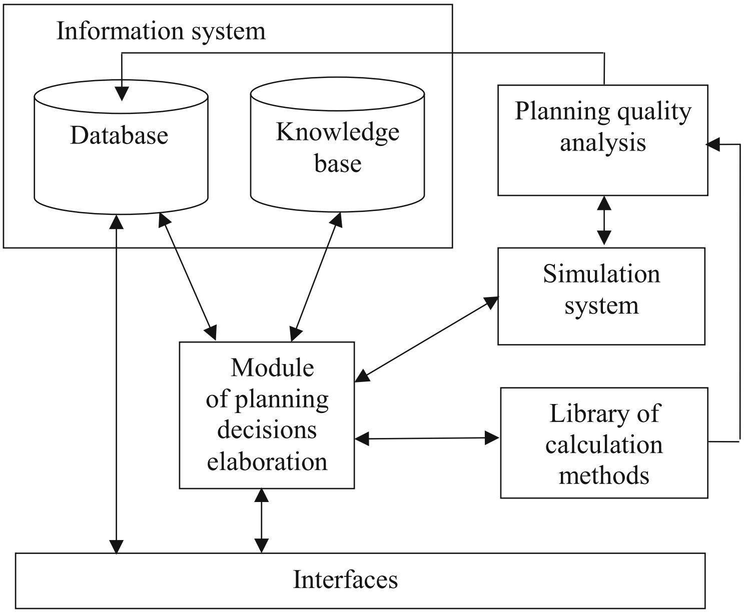
Information system
Database
Knowledge base
Planning quality analysis
Simulation system
Module of planning decisions elaboration
Library of calculation methods
Interfaces

Fig. 15.11 Architecture of decision-support system [based on Riane et al. (2001)]

The module of planning, scheduling, and decision-making relies on both bases of the information system, on the library of known calculation methods, and on the simulation system. Special module analyses the quality of plans by using the results of calculations and simulation, and the results of this analysis are recorded in the analytical part of the database. The system is controlled by a set of interfaces.

During planning, it is assumed that the value of demand is known for several discrete periods with horizon h. For each product, it is required to define the total output, quantity of lots for each planning period, and shop floor level schedule in each period.

Planning is done in three stages, the first of which is determined by the production volume on each work center (machine) for each period, optimal in terms of minimal costs for production and storage of products. Here with the capacities of work centers, availability of stocks, demand rate, duration of manufacturing operations, etc., are taken into account.

At the second stage, the quantity of lots of each product output for the planning period is calculated. The lot sizes of each product are considered to be constant, i.e. the so-called model with a fixed size of the order take place (Sect. 8.2.1). This approach is typical for process production, in which the lot size is defined by the capacity of machines.

At the third stage, schedules for all machines by the planning period are developed, while using different heuristics methods available in the library. The resulting schedules are loaded then into the simulation system, the main purpose of which is to assess the quality of planning. In addition, the simulation should allow introducing various additional constraints for consideration that were not included in original planning and determining their impact.

The system uses the so-called resource-actions-operations (RAO) simulation. In this model, the knowledge base is composed of actions and operations performed by resources by certain production rules. In the general case, each such rule has the form:

IF (condition) THEN (action 1) WAITING (time interval) NEXT (action 2).

As an example, we present the rule:

IF job *i* is planned on machine *j* and machine *j* is free THEN at time *t* the setup of machine *j* for job *i* begins WAITING duration *S* depends on the types of job *i* and previous job *k* NEXT machine *j* starts processing job *i*.

Using the rules of this type, the simulation system reproduces the production process and calculates the time of its performance and the quality of planning respectively.

After calculating the schedule, the user can analyse it in numerical and graphical form, control various parameters, and compare with the simulation results. The user is able to do various changes in the original data, to calculate new schedule options reflecting these changes, to control as follow-up, etc. It is possible to redistribute jobs among the machines, adjust lot sizes for better matching the given volume of production, introduce time and cost for transportation of products, etc.

15.5.2 Some Other Decision-Support Systems

Existing decision-support systems can be conditionally divided into two groups. The systems of the first group are intended for use in several though relative industries. Highly specialized systems can be referred to the second group.

A demonstrative example of the first group of systems is the Finnish system VTT_GESIM, described in the paper (Heilala et al. 2010). This system is designed for discrete manufacturing that is quick to respond to user requests (Customer Driven Manufacturing). The specific distinctions of this type of manufacturing are compulsory timely fulfilment of orders, fast reconfiguration according to changes in demand, acceleration of delivery, coordination in supply chains, etc.

The basis of the system (Heilala et al. 2010) is the so-called Discrete Event Simulation (DES). Such simulation techniques are possible for all kinds of discrete manufacturing including assembly and project manufacturing as well as supply chains. Simulation is possible at all stages of manufacturing starting from the creation of production lines and cell areas to development of operational plans.

The feature of the system, which allows its use in various industries, is application of special “parametric” files that influence the database (Fig. 15.11). Due to the parameter values that are set during setup the system can work with databases of different content.

Visualizing results allows developing scenarios of different situations with analysis of answers to queries such as “what if”. At that, the simulation time interval can vary from days to months, or even years. The system provides for user intervention in the process and simulation results. This paper (Heilala et al. 2010) presents a number of examples of the system application at mechanical engineering and woodworking enterprises.

System developers indicate greater flexibility of the system adaptation features to the requirements of various industries. For this purpose, a special technique was developed, which includes a number of steps to describe business processes and prepare a database, refine the model and interfaces, organize work with ERP and MES systems, etc. Certain opportunities here are found through the use of general-purpose interfaces, which are configured then using XML programming tools.

As an example of a highly specialized system we present the decision-support system (Kargin and Mironenko 2008) for production planning (metal cutting) for procurement workshops of Mariupol Heavy Machine Plant (Ukraine), the diagram of which is shown above in Fig. 5.2.

The solution to this problem is possible only if several optimality criteria are used. A detailed study of the problem (Kargin and Mironenko 2009) showed that these include the duration of production cycle to manufacture a part, equipment downtime, work-in-progress volumes, costs, delaying of parts to the specified date and time of their submission for further processing or assembly, the total processing costs of given number of parts, etc. In addition, to account the dynamic nature of planning process the system also uses the criterion of production intensity (Sect. 2.4.1), which is calculated for every detail of each equipment unit and in general for a production department.

Based on this research, the decision-support system includes a simulation model containing a static database and dynamic database, the so-called simulation driver including the event files, model hours, and simulation knowledge.

Once the order situation is identified, it is transferred to the planner who accepts it as the initial state of the system. The objective of scheduling in the system is to find the optimal path leading from the initial state to the target one. The transition from one state to the other is performed by using information from the knowledge database, simulation, and based on the experience and knowledge of experts. At the output, the scheduler generates an action plan due to which a “Schedule of cutting” is created or a way out of the situation is suggested.

The structure of intellectual system includes a subsystem of adjustments and learning, providing an opportunity to review and adapt the obtained solution, if necessary, and to save the newly adopted decision as a part of new precedent.

Here is another example of a decision-support system developed for manufacturing of fibres in the textile industry. The main issue in the planning of this manufacturing is the optimization of operation in the spinning machines area, which is the bottleneck (Silva and Magalhaes 2006). Since the area has several machines of various types, the scheduling for them relates to the problems for one purpose parallel machines with different design (Sect. 13.6.2).

Schedule is usually made on the basis of monthly or weekly targets, which set production volumes of each type of fibre in accordance with the orders for finished goods (textiles, ropes, etc.). The complexity of scheduling is caused by the need to consider two mutually contradictory criteria: cost of setups and just-in-time fulfilment of targets, i.e. the problem under consideration in this aspect is similar to the problem described above in Sect. 15.3.1.

However, in reality, the real problem stated in the paper (Silva and Magalhaes 2006) is much more complicated, because its solution is imposed with a number of additional constraints. In particular, in addition to the need to consider the relation of the setup cost to the sequence of jobs, it is also necessary to take into account the wearing degree of the jets through which the yarns are pulled, as well as the possibility to use the machines only for certain types of yarns, different capacities of the machines, etc.

In this paper on the basis of the Discrete Lot sizing and Scheduling Problem (DLSP) described above in Sect. 11.1.4, a special scheduling algorithm for multiple parallel different machines is developed. However, the use of this algorithm to develop a satisfactory schedule was insufficient, and further the optimization algorithm was used as a part of the decision-support system. This system also included a database and a user interface. Further development of this system (Silva 2009) based on specialized software tool enables the planner to use background experience (knowledge base) in order to adjust the result of the calculations and verify own decisions on the model.

In some cases, it is advisable to use relatively simple systems that can be called decision-support tools. In the paper of Buehlmann et al. (2000), a simple decision-support system for the production of wood panels is given. The system includes MS Excel forms, by which the user in the workshop can optimize the schedule in case of changes in delivery time and prices of materials produced. For the job shop manufacturing, such decision-support tool using MS Excel and the techniques described above in Sect. 15.3 is described in the paper of Mauergauz (2014b).

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Appendix A: Symbols

C	planned time point of job completion
$C_{\max}$	makespan
c	cost of labour per shift
c_o	costs for order arrangements
c_h	costs of storage per product measure unit
c_p	product cost
D	amount of consumed product per time unit (demand)
d	required time point of job completion (due time)
E	mean value of random variable (mathematical expectation)
$\overline{F}$	average duration of job from the set of jobs (production cycle)
$F_{\max}$	max duration of the job from the set of jobs
G	average quantity of working hours or days in the planning period
H	production intensity
h	planning horizon
$\overline{K}$	equipment load average coefficient
k	wave propagation coefficient in the supply chain
$L_{\max}$	max deviation of the job from the due time out of the set of jobs
$\overline{L}$	average deviation of the job from the due time out of the set of jobs
L	duration of order delivery (lead time)
$\overline{N}$	average quantity of orders present in the system
$\overline{P}$	average production rate
P	machine production rate
$P_{\max}$	the greatest possible machine working time
P	probability of order receipt within the time interval
p	job processing time
$\overline{p}$	average processing time of the jobs out of the set
Q	quantity of order (shipment batch)
Q^*	optimal quantity of order (batch)
$\overline{R}$	average processing time of orders set, launched into production
r	expected date of batch arrival for processing
S	schedule time point of the job start

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S_L	service level
$\dot{S}$	upper level of the stock
s	setup time
$\dot{s}$	quantity of stock at the reorder point
$T_{\max}$	max job tardiness out of the job set
$\overline{T}$	average job tardiness out of the job set
T	cycle (period) time of shipment
U	function of negative utility of costs (loss function)
V	function of current order utility
$\overline{V}$	function of average orders utility
$\overline{W}$	average production workload
$\overline{w}$	average queuing time of order
w	job weighting factor
Y	alternative version
Z	current stock
Z_c	safety stock

α	psychological coefficient
δ	binary variable
ε	effective point neighbourhood (distance from the trade-off curve)
σ	standard deviation of a random variable from its mean value
κ	safety factor (factor of accounting the spread in values of random function)
ƛ	Lagrange multiplier
λ	mean rate of incoming orders in the system (demand)
χ	correction factor for the optimal batch size
η	readiness percentage of operation
v	variation coefficient of a random variable
Π	expected profit
τ	processing time of one part
ω	frequency of a product in one schedule cycle
Ω	orders frequency

Appendix B: Abbreviations

ABC	Activity-based costing
ALBP	Assembly line balancing problem
ANSI	American National Standards Institute
AP&S	Advanced planning and scheduling
APICS	American Production and Inventory Control Society
APS	Advanced planning system
ARIS	Architecture of integrated information systems
ATO	Assemble-to-order
ATP	Available-to-promise
BOM	Bill of materials
BPwin	Business process windows
BSC	Balanced scorecard
CCR	Capacity constraint resource
CDS	Algorithm of Campbell, Dudek, and Smith for intermittent flowshop line
CLSP	Capacitated multi-item lot-sizing problem
CR	Critical ratio
CSLP	Continuous setup lot-sizing problem
CSV	Comma separated values
DBMS	Database management system
DBR	Drum–buffer–rope
DES	Discrete event simulation
DLSP	Discrete lot-sizing and scheduling problem
DRP	Distribution resource planning
EDD	Earliest due date
ELSP	Economic lot scheduling problem
EOQ	Economic order quantity
EPC	Electronic product code
EPQ	Economic production quantity
ERP	Enterprise resource planning
ESB	Enterprise service bus
FFS	Frequency fixing and sequencing

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FIFO	First in first out
FIR	Fast innovation reinforcement
IDEF0	Integration definition for function modelling
IEC	International Electric Commission
ISA	Instrumentation, Systems and Automation Society
JIT	Just-in-time
KPI	Key performance indicators
LIFO	Last in first out
LIMS	Laboratory information management systems
LPT	Longest processing time
MAUT	Multi-attribute utility theory
MES	Manufacturing execution system
MESA	Manufacturing Enterprise Solutions Association
MRP2	Manufacturing resource planning
MSLSP	Multi-stage lot-sizing problem
MST	Minimum slack time
MTO	Make-to-order
MTS	Make-to-stock
NEX	Nawaz, Enscore, and Ham algorithm
OLAP	Online analytical processing
OLTP	Online transaction process
OPC	OLE for process control
OLE	Object linking and embedding
OPT	Optimized production technology
PFA	Production flow analysis
POQ	Period order quantity
PPB	Part period balancing
PPM	Production process model
PI	Planning item
SADT	Structured analysis and design technique
SCADA	Supervisory control and data acquisition
SCC	Supply chain council
SCM	Supply chain management
SCOR	Supply chain operation reference
SKU	Stock-keeping unit
SOA	Service-oriented architecture
SPT	Shortest processing time
SQL	Structured query language
STN	State-task network
TOC	Theory of constraints
UBSC	Utility-based balanced scorecard
VMI	Vendor-managed inventory
WIP	Work-in-process

Appendix C: Classification Parameters of Schedules

Classification (2.13) of form α | β | γ is of great importance in ordering the scheduling problems. The classification has fields with three directions: type of production (type of machines used), type of jobs performed and various constraints, and type of objective function.

C.1 Parameters in Field α

These parameters have the structure of type $\alpha_1\alpha_2(\alpha_3)$, i.e. often consist of two and sometime three elements. The list of possible values α_1 is presented below.

empty value in field α_1 means that a single machine is under consideration;

$\alpha_1 = P$	parallel identical machines
$\alpha_1 = Q$	parallel uniform machines
$\alpha_1 = R$	parallel unrelated machines
$\alpha_1 = F$	flowshop production
$\alpha_1 = J$	jobshop production
$\alpha_1 = O$	openshop production.

Symbol α_2 serves to describe the quantity of machines. If $\alpha_2 = m$, it means that the quantity of machines is fixed and equal to m.

Symbol α_3 is used for description of batch characteristics and can have several elements. Let us provide some such possibilities.

$\alpha_3 = C_{job}$	after processing each object leaves the machine separately
$\alpha_3 = C_{batch}$	the objects in the batch are processed sequentially but leave the machine after completion of processing in the joint batch
$\alpha_3 = C_{batch}, p_{\max}$	the objects in the batch are processed simultaneously and leave the machine after processing with the longest duration

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$\alpha_3 = C_{batch}, p_{\max}, b = a$	the objects in the batch are processed simultaneously in the quantity not exceeding a and leave the machine after processing with the longest duration.

C.2 Parameters in Field β

Field β can contain up to eight elements, the most common values of which are given below.

$prec$	operations are executed in specific sequence
r_i	jobs arrive at different time
d_i	the required due time point is set for each job
$d_i = d$	the jobs must be fulfilled by one fixed date
d_i $unknown$	the jobs must be fulfilled by one date, which has no fixed value
$split$	it is allowed to divide a batch into parts
$pmtn$	it is allowed to interrupt the batch processing and transfer a part of it to another machine
s_{nd}	before processing the time for setup is required. It depends on the job sequence
S_i	the desired start time point is set for each job
$p_i \in \left[\underline{p}_i; \overline{p}_i\right]$	duration of job is not fixed and can be in the range from $\underline{p}_i$ to $\overline{p}_i$.
$no - wait$	when performing sequential operations there should not be any breaks in between
$nmit$	there should not be any breaks in the machine operation
$prmu$	there should be a sequence of jobs in the flowshop production

The order of these elements in field β is usually arbitrary.

C.3 Parameters in Field γ

In this field, the criteria of the scheduling problem are usually recorded. Their possible values are as follows:

$C_{\max} = \max\limits_{i=1,\ldots n}(C_i)$	common time point of processing completion
$T_{\max} = \max\limits_{i=1,\ldots n}(T_i)$	maximal tardiness
$L_{\max} = \max\limits_{i=1,\ldots n}(L_i)$	maximal time deviation
$F_{\max} = \max\limits_{i=1,\ldots n}(F_i)$	maximal duration of job
$f_{\max} = \max\limits_{i=1,\ldots n}(f_i)$	maximal cost of one job

$\overline{C}^w = \sum_{i=1}^{n} w_i C_i$ weighted average total of completion time

$\overline{T}^w = \sum_{i=1}^{n} w_i T_i$ weighted average total of tardiness

$\overline{I}^w = \sum_{i=1}^{n} w_i I_i$ weighted average number of delayed orders

$H = \sum_{i=1}^{n} H_i$ total intensity of jobs

$U = \sum_{i=1}^{n} U_i$ function of relative setup loss

$\overline{V} = \sum_{i=1}^{n} \overline{V}_i$ total average utility of orders.

When compiling multicriteria schedules, a combination of criteria can be used, such as a linear combination of the average setup cost $\overline{c}$ and maximum tardiness $T_{\max}$ $F_l(\overline{c}, T_{\max})$. If the criteria in multicriteria schedule are and their effect is independent, then in field γ all such criteria are recorded. For example, $\gamma = U, \overline{V}$.

Appendix D: Production Intensity Integral Calculations

Let us assume that at some time point C_l, l of jobs are already completed and after that at time t_k the k-th job is started. The average utility of the entire available set of jobs for all time $t_k + p_k$ from the beginning of jobs and to the k-th job is determined by the recurrent formula (13.23):

$$\overline{V}_{l+1,k} = \frac{1}{t_k + p_k} \int\limits_{0}^{t_k+p_k} V dt = \frac{1}{t_k + p_k} \left(\overline{V}_l \times C_l + \int\limits_{C_l}^{t_k+p_k} V_k dt \right).$$

To find the value of integral $\int\limits_{C_l}^{t_k+p_k} V_k dt$, we use the expression of the orders current utility function by formula (2.32):

$$V = \sum_{i=1}^{N} V_i = \frac{1}{G} \sum_{i=1}^{N} w_i p_i - \sum_{i=1}^{N} H_i,$$

where N is the number of jobs, G is the duration planning period, w_i, p_i, and H_i are weight, processing time, and current intensity of the i-th job, respectively.

The intensity values are defined by formulas (2.28):

$$H_i = \frac{w_i p_i}{G} \frac{1}{(d_i - t)/\alpha G + 1} \quad \text{at} \quad d_i - t \geq 0$$

and

$$H_i = \frac{w_i p_i}{G} ((t - d_i)/\alpha G + 1) \quad at \quad d_i - t \leq 0,$$

where α is the setup coefficient for a specific enterprise reflecting the level of complacency with the available time reserves or nervousness when failing to meet the production due date; d_i—required due time point of job execution.

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Thus, to determine the value of average utility $\overline{V}_{l+1,k,}$ it is necessary to calculate the values of certain integrals $\int\limits_{C_l}^{t_k+p_k} H_i dt$. These values are defined as formulas of different types depending on the mutual arrangement of parameters C_l, t_k, $d_{i,}$ and t_k+p_k. Figure D.1 shows seven of such possible variants.

In variant 1, the intensity integral is calculated for the job, which is non-executable at time t_k and the due time point d_i of which is behind point t_k+p_k of the k-th job execution. Variant 2 differs from variant 1 by the fact that the intensity is defined for the job, the execution of which starts at time t_k.

Variant 3 exists for the non-executable job in that case when its required due time point d_i is between points C_l of previous job completion and point t_k+p_k of the k-th job execution. In variant 4, the intensity integral is defined for the job the execution of which starts at time t_k, and the required die time point is between points C_l of previous job completion and point t_k+p_k.

In variant 5, for the non-executable job, its required due time point d_i is earlier than point C_l. In case of variant 6, for the job, the execution of which starts at point t_k, its required due time point is between points C_l of previous job completion and point t_k. In the last seventh variant, required due time point of the job started at time t_k is earlier than point C_l.

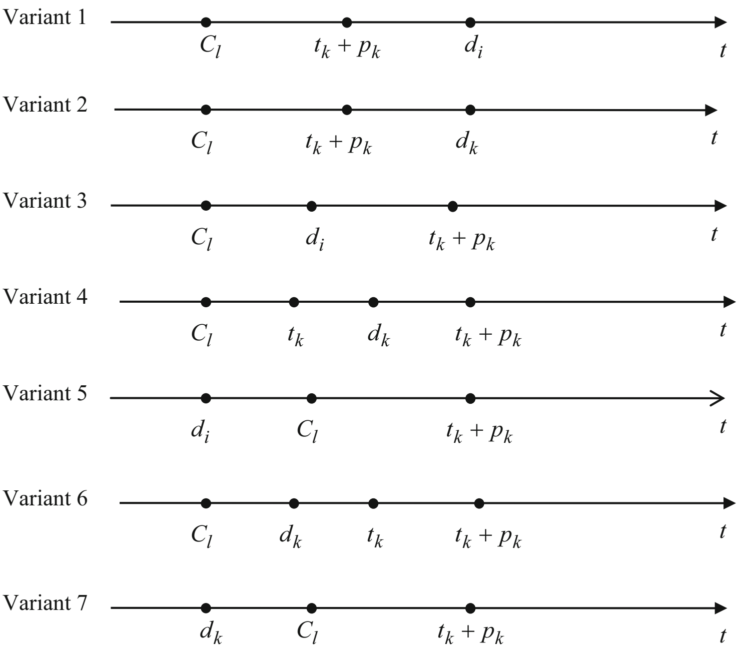

Fig. D.1 Variants of parameters arrangement for defining $\overline{V}_{l+1,k}$

Table D.1 The rules of selection of correct variant of intensity integral formulas

a_k^i	$d_i - t_k - p_k$	$d_i - t_k$	$d_i - C_l$	Variant number for H_k^i
1	≥ 0	–	–	1
0.5	≥ 0	–	–	2
1	<0	<0	>0	3
0.5	<0	≥ 0	–	4
1	<0	<0	<0	5
0.5	<0	<0	≥ 0	6
0.5	<0	<0	<0	7

The rules of selection of correct variant are provided in Table D.1. Parameter a_k^i can have one of three values:

0—for the jobs already performed by time point t_k;
1—for the jobs not performed yet and not included in the k-th batch;
0.5—for the jobs not performed yet and included in the k-th batch.

If $a_k^i = 0$, the intensity integral is calculated.

The values of defined intensity integrals are as follows:

Variant 1:
$$\int_{C_l}^{t_k+p_k} H_i dt = \alpha w_i p_i \ln\left(\frac{(d_i - C_l)/\alpha G + 1}{(d_i - t_k - p_k)/\alpha G + 1}\right);$$

Variant 2:
$$\int_{C_l}^{t_k+p_k} H_i dt = \alpha w_k \left[p_k - (d_k - t_k - p_k + \alpha G)\ln\left(\frac{(d_k - t_k)/\alpha G + 1}{(d_k - t_k - p_k)/\alpha G + 1}\right)\right] +$$
$$+\alpha w_k p_k \ln\left(\frac{(d_k - C_l)/\alpha G + 1}{(d_k - t_k)/\alpha G + 1}\right)$$

Variant 3:
$$\int_{C_l}^{t_k+p_k} H_i dt = \alpha w_i p_i \ln\left(\frac{d_i - C_l}{\alpha G} + 1\right) + \frac{\alpha w_i p_i}{2}\left[\left(\frac{t_k + p_k - d_i}{\alpha G} + 1\right)^2 - 1\right];$$

Variant 4:
$$\int_{C_l}^{t_k+p_k} H_i dt = \alpha w_k \left[d_k - t_k + (t_k + p_k - d_k - \alpha G)\ln\left(\frac{d_k - t_k}{\alpha G} + 1\right)\right] +$$
$$+\alpha w_k p_k \ln\left(\frac{(d_k - C_l)/\alpha G + 1}{(d_k - t_k)/\alpha G + 1}\right) + \frac{\alpha w_k (t_k + p_k)}{2}\left[\left(\frac{t_k + p_k - d_k}{\alpha G} + 1\right)^2 - 1\right] -$$
$$-\frac{w_k}{2G}\left(1 - \frac{d_k}{\alpha G}\right)\left[(t_k + p_k)^2 - d_k^2\right] - \frac{w_k}{3\alpha G^2}\left[(t_k + p_k)^3 - d_k^3\right]$$

Variant 5:

$$\int\limits_{C_l}^{t_k+p_k} H_i dt = \tfrac{\alpha w_i p_i}{2}\left[\left(\tfrac{t_k+p_k-d_i}{\alpha G}+1\right)^2-\left(\tfrac{C_l-d_i}{\alpha G}+1\right)^2\right]$$

Variant 6:

$$\int\limits_{C_l}^{t_k+p_k} H_i dt = \alpha w_k p_k \ln\left(\frac{d_k-C_l}{\alpha G}+1\right)+\frac{\alpha w_k p_k}{2}\left[\left(\frac{t_k-d_k}{\alpha G}+1\right)^2-1\right]+$$

$$+\frac{\alpha w_k(t_k+p_k)}{2}\left[\left(\frac{t_k+p_k-d_k}{\alpha G}+1\right)^2-\left(\frac{t_k-d_k}{\alpha G}+1\right)^2\right]-$$

$$-\frac{w_k}{2G}\left(1-\frac{d_k}{\alpha G}\right)\left[(t_k+p_k)^2-t_k^2\right]-\frac{w_k}{3\alpha G^2}\left[(t_k+p_k)^3-t_k^3\right]$$

Variant 7:

$$\int\limits_{C_l}^{t_k+p_k} H_i dt = \frac{\alpha w_k p_k}{2}\left[\left(\frac{t_k-d_k}{\alpha G}+1\right)^2-\left(\frac{C_l-d_k}{\alpha G}+1\right)^2\right]+$$

$$+\frac{\alpha w_k(t_k+p_k)}{2}\left[\left(\frac{t_k+p_k-d_k}{\alpha G}+1\right)^2-\left(\frac{t_k-d_k}{\alpha G}+1\right)^2\right]- \cdot$$

$$-\frac{w_k}{2G}\left(1-\frac{d_k}{\alpha G}\right)\left[(t_k+p_k)^2-t_k^2\right]-\frac{w_k}{3\alpha G^2}\left[(t_k+p_k)^3-t_k^3\right]$$

Appendix E: Scheduling Software Based on Order Utility Functions

E.1 General

On the Springer website in electronic books File1.xls to File8.xls, there are eight planning programs using average order utility functions as criteria. Each book contains several sheets with tasks for planning as well as a software module and user forms. The programs are macros in VBA for MS Excel. To work with the e-books, they must be copied to the user's computer. At the beginning of work, the macros must be enabled. References in the text are made to the relevant paragraphs of this book.

Below you will find information required for initial familiarization with the programs. For more details, please contact the author at prizasu@yandex.ru.

E.2 Description of Work with File1.xls

Please see http://extras.springer.com/2016/978-3-319-27522-2

File1.xls is intended for scheduling of a single-machine operation with sequential execution of each job. The book contains two task sheets, a software module, and three user forms.

E.2.1 Worksheet

Sheet 1 table $B$4:$F$8 contains a simple task (Fig. E.1) consisting of five jobs of three different types. Each row (number of job) describes the job parameters: processing time in hours, required due time (hours), expected time (hours) of arrival, job type, and weighting coefficient (importance) of a job. It is advisable to enter the jobs in ascending order of the required due time points. A negative value of the due time indicates lateness in execution.

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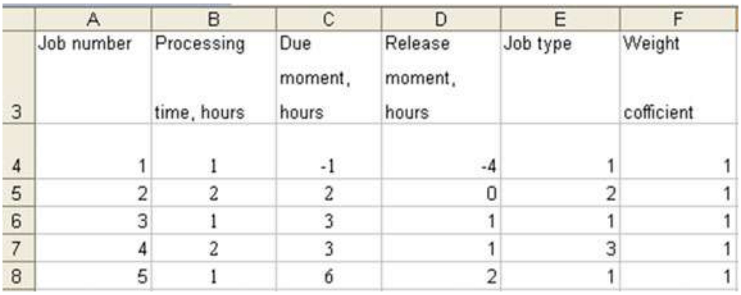


	A	B	C	D	E	F
3	Job number	Processing time, hours	Due moment, hours	Release moment, hours	Job type	Weight cofficient
4	1	1	-1	-4	1	1
5	2	2	2	0	2	1
6	3	1	3	1	1	1
7	4	2	3	1	3	1
8	5	1	6	2	1	1

Fig. E.1 Simplest job set for a single machine operation

On the same sheet, table $K$5:$M$7 shows the time standards in hours on machine setups from one type of job to another. It is assumed that the number of the previous job type is set by the table column and the number of the following job type is set by the table row.

The main parameters of the planning process are entered in the second row of the sheet; the data processing parameters and the planning results records are entered in the first row.

The data in the table of tasks and the time standards can be changed before planning by entering their values directly in the worksheet cells. Parameter values should be managed in the planning process using the forms.

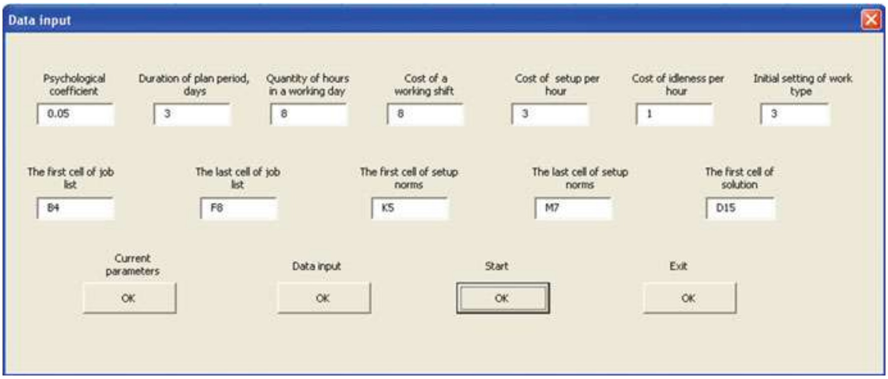
The program operation is based entirely on the planning method by costs criteria *U* and average orders utility *V* set out in Sect. 15.3.1. The planning results are recorded in the worksheet in several possible versions.

In each of these versions, the collection of values *U* and *V* is not dominant with respect to the collection of these values for any other version. This means that if, for example, value *U* is better (less) for the first version than for the second one, then at the same time value *V* for the first version is necessarily worse (less) than for the comparable second version.

Sheet 2 shows an example of the task for 20 jobs of seven different types.

E.2.2 How to Use the Program

To use the planning program in MS Excel, set a relevant worksheet, for example, sheet 1. Then enable macro Utility1, which opens a “Data input” form (Fig. E.2).



Data input

Psychological coefficient: 0.05
Duration of plan period, days: 3
Quantity of hours in a working day: 8
Cost of a working shift: 8
Cost of setup per hour: 3
Cost of idleness per hour: 1
Initial setting of work type: 3

The first cell of job list: B4
The last cell of job list: F8
The first cell of setup norms: K5
The last cell of setup norms: M7
The first cell of solution: D15

Fig. E.2 Data input form

In the top row of windows, the basic process parameters are set. Psychological coefficient α is a setting coefficient for a particular enterprise reflecting the degree of "complacency" with existing float time or "nervousness" in case of failure to meet the production time (Sect. 2.4.1). Recommended values of this coefficient are 0.03–0.1.

The plan period duration should be set with consideration of the number of working hours per day, so that the total number of working hours during the plan period is comparable to the total processing time of all jobs in the task. For example, with an 8-h working day for the task on sheet, it is advisable to install a 3-day plan period. The cost of the working shift, setup per hour, and idleness per hour should be set in the current monetary terms.

The result of the planning depends significantly on the job type for which the machine is set up at the initial time of task execution. This value must be entered at the time of planning.

In the second row of windows, cell names are entered that describe the table of the task and the table of the time standards. The names are entered in the form of a Latin letter (column) and a number (row) without $, such as L5. The upper left cell is considered to be the first and the lower right cell is the last. Here the cell with job numbers in the task table and cells with numbers of job types in the table of setup time standards are not considered. The last window of the second row has the name of the cell into which the output of the planning is entered. It is advisable to select a cell located slightly below the task cells on the left of the sheet for this purpose.

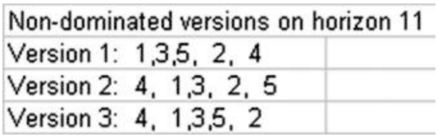
At first initiation of the "Data input" form, the windows can be empty. To open the data from the worksheet use the "Current parameters" button. The modified data should be recorded in the sheet using the "Data input" button.

When you click "Start" the program runs. The working time essentially depends on the number of jobs in the task. With five jobs in the task, the execution time is a few seconds. The typical number of jobs in the planned task is 20–30, and the execution time in this case is 1–2 min.

At the end of the calculation of possible non-dominated versions, these versions are displayed on the screen (Fig. E.3). In each version, the jobs are grouped by type and the groups are separated by spaces. Within the group, the sequence of jobs does not necessarily coincide with the order of their numbers, as jobs in the same group may have different processing times.

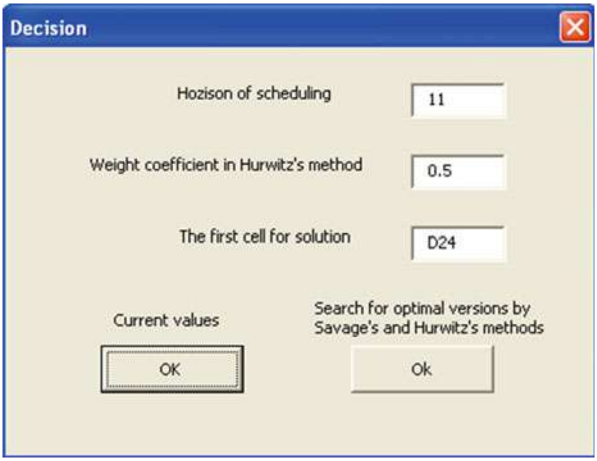
Then a “Decision” form (Fig. E.4) appears on the screen. In this form, the horizon (time in hours) for which the most acceptable decision will be calculated must be specified. This horizon is usually taken to be somewhat less than the total length of all the scheduled jobs. When the accuracy of the information about upcoming jobs is low, it makes sense to reduce the estimated horizon. When defining an estimated horizon greater than the total duration of jobs in the task, the total duration is used as the estimated horizon.

To determine the recommended solutions, the decision-making methods of Savage and Hurwitz are used in the program (Sect. 15.4). To calculate the latter, it is required to specify the weighting factor of Hurwitz’s method (Sect. 4.5.1), and usually, this factor is in the range 0.3–0.7. In the last window of the form, the cell name is recorded where the start of the results of the Savage and Hurwitz calculation are output.



Non-dominated versions on horizon 11	
Version 1: 1,3,5, 2, 4	
Version 2: 4, 1,3, 2, 5	
Version 3: 4, 1,3,5, 2	

Fig. E.3 Versions of decisions



Decision

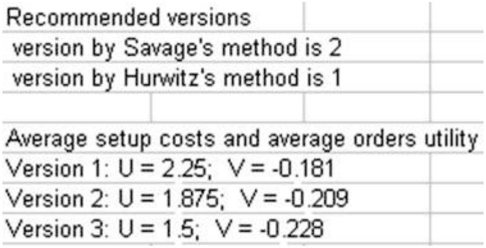
Hozison of scheduling: 11

Weight coefficient in Hurwitz's method: 0.5

The first cell for solution: D24

Current values
OK
Search for optimal versions by Savage's and Hurwitz's methods
Ok

Fig. E.4 Form of decision-making



Recommended versions
version by Savage's method is 2
version by Hurwitz's method is 1

Average setup costs and average orders utility
Version 1: U = 2.25; V = -0.181
Version 2: U = 1.875; V = -0.209
Version 3: U = 1.5; V = -0.228

Fig. E.5 Recommended versions of calculation

Upon initial opening of the “Decision” form the windows may be empty. To display the data from the sheet on the screen, use the “Current values” button. The “Search for optimal versions. . .” button is used to start the relevant program.

After calculation, the numbers of versions are displayed that are recommended in accordance with the methods of Savage and Hurwitz (Fig. E.5). In addition, for all non-dominated versions the data on values *U* and *V* are displayed for the specified estimated horizon.

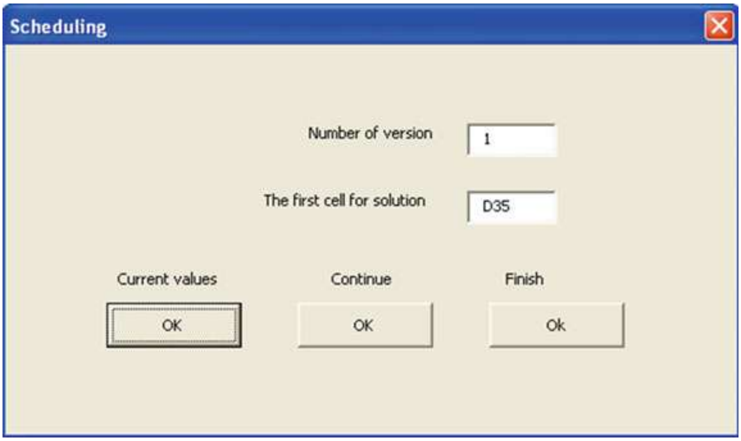
Negative values of the function of average orders utility *V* indicate probable overdue completion because of machine overloading.

E.2.3 Planning Result Analysis

The need for such analysis may occur, first of all, in cases where after calculations by Savage’s and Hurwitz’s methods different versions of the plan are suggested. In addition, the calculations by these methods may not produce results that satisfy the user.

The analysis can be done by studying the derived data on values *U* and *V* for the established estimated horizon. If the calculation finds that there is a version where value *V* is not too different from the greatest possible value and thus value *U* is much less than the maximum, the decision is reasonable.

To calculate the plan by the analysed version, the system offers a “Scheduling” form (Fig. E.6). Enter the number of the calculated version in the “Number of version” window. In “The first cell for solution” window, enter the name of the cell in which it is advisable to place scheduling for further study.

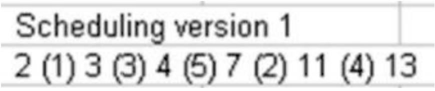


Scheduling

Number of version: 1

The first cell for solution: D35

Fig. E.6 Form of scheduling versions



Scheduling version 1

2 (1) 3 (3) 4 (5) 7 (2) 11 (4) 13

Fig. E.7 Job sequence

When scheduling (Fig. E.7) the time point of job arrival, the duration and time standards for changeovers are considered. The figure without brackets corresponds to the job start time point, which then is entered in the brackets.

The actual sequence of job execution in the group created by the plan does not necessarily match the sequence shown in the plan. In fact, each of these jobs may be executed in parts—for example, when cutting various parts from a single sheet, and these parts belong to various jobs.

The system allows us to calculate consistently several schedule versions by placing the results in different cells. To finish the calculation, use the “Finish” button.

Upon initial starting of “Scheduling”, the windows may be empty. To display the data from the sheet on the screen, use the “Current values” button.

A new task should be set up in a separate sheet, a new sheet or corrected sheet. One of the available sheets may be used. When using a new sheet, it is advisable first to copy the first two rows of any previously used sheet, which significantly reduces the probability of errors. To enter data on the scheduled jobs time standards for setups, it is advisable to use the data available on the previous sheets as much as possible.

When running the program, be careful about checking the compliance of cell names in the “Data input” form determining the position of the tables of jobs and standards with their actual position in the sheet.

E.2.4 Errors During Program Run

Some possible errors are checked by the program which generated the relevant message. Other errors are detected by the compiler, which indicates a system error. If this message appears, the user must indicate the end of work (command "End") and study the data entered.

The most frequent errors occur due to mismatching cell names in the "Data input" form and the tables in the current worksheet. With a large number of jobs, overflow of the memory array dedicated to calculation may occur. In this case, the planned task should be reduced.

E.3 Description of Work with File2.xls

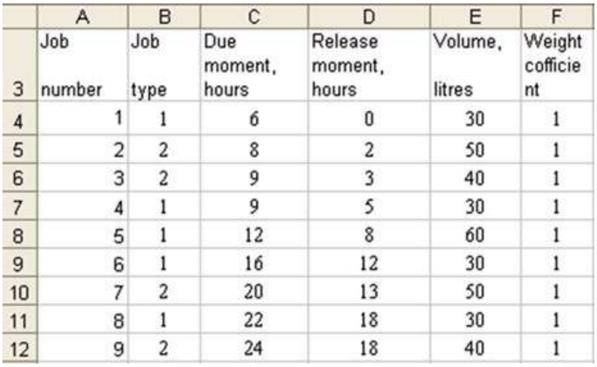
Please see http://extras.springer.com/2016/978-3-319-27522-2

File2.xls is intended for scheduling of a single-machine operation (furnace, bath, etc.) with parallel execution of tasks. The book contains two task sheets, a software module, and three user forms.

E.3.1 Worksheet

Sheet 1 table $B$4:$F$12 contains a task consisting of nine jobs of two different types (Fig. E.8). Each row (number of job) of the task describes the job parameters: job type, required due time (hour), expected time (hour) of arrival, volume used by the job, and weighting coefficient (importance) of the job. It is advisable to enter the jobs in ascending order of their required due dates.

Table $L$5:$N$6 shows the duration of the job of each type and the time standards in hours for processing and machine setups from one type of job to



	A	B	C	D	E	F
3	Job number	Job type	Due moment, hours	Release moment, hours	Volume, litres	Weight cofficie nt
4	1	1	6	0	30	1
5	2	2	8	2	50	1
6	3	2	9	3	40	1
7	4	1	9	5	30	1
8	5	1	12	8	60	1
9	6	1	16	12	30	1
10	7	2	20	13	50	1
11	8	1	22	18	30	1
12	9	2	24	18	40	1

Fig. E.8 Work task

another. It is assumed that the number of the previous job type is set by the table column and the number of the following job type is set by the table row.

The main parameters of the planning process, as in File1.xls, are entered in the second row of the sheet; the data processing parameters and the planning results records are entered in the first row. The program operation is based on the planning method by costs criteria *U* and average order utility *V* set out in Sect. 15.3.2. The planning results are recorded in the worksheet in several possible versions.

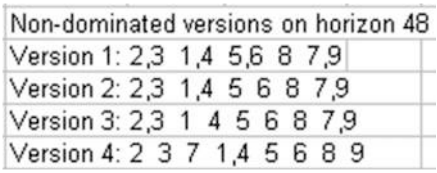
Sheet 2 shows an example of the task for 25 jobs of four different types.

E.3.2 How to Use the Program

When enabling macro Utility2, the data input form opens. This form is similar to the form in Fig. E.2, but the available physical volume of the machine in litres must be specified. At first initiation of the "Data input" form, the windows can be empty. To open the data from the worksheet, similar to Sect. E.2, use the "Current parameters" button. The modified data should be recorded in the sheet using the "Data input" button.

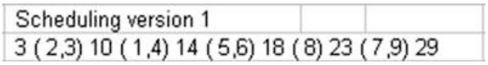
When you click "Start", the program runs. The working time essentially depends on the number of jobs in the task. With nine jobs in the task, the execution time is a few seconds, with 13 jobs the execution time may reach 1 min. At the end of calculation of possible non-dominated versions, these versions are displayed on the screen (Fig. E.9). In each version, the executed jobs are grouped by type and the groups are separated by spaces.

Further work with the program is entirely similar to the work with the macro in File1. To calculate the plan by the analysed version, the system offers a "Scheduling" form, where the number of the selected version must be entered. When scheduling (Fig. E.10) the time point of job arrival, their duration and time standards for changeovers are considered. The figure without brackets corresponds to the start time point of the job group, which then is entered in the brackets.



Non-dominated versions on horizon 48
Version 1: 2,3 1,4 5,6 8 7,9
Version 2: 2,3 1,4 5 6 8 7,9
Version 3: 2,3 1 4 5 6 8 7,9
Version 4: 2 3 7 1,4 5 6 8 9

Fig. E.9 Versions of decisions



Scheduling version 1
3 (2,3) 10 (1,4) 14 (5,6) 18 (8) 23 (7,9) 29

Fig. E.10 Job sequence

E.4 Description of Work with File3.xls

Please see http://extras.springer.com/2016/978-3-319-27522-2

File3.xls is intended for scheduling the operation of a production site consisting of several parallel machines of the same purpose and using a "make-to-order" strategy. The book contains two task sheets, a software module, and four user forms.

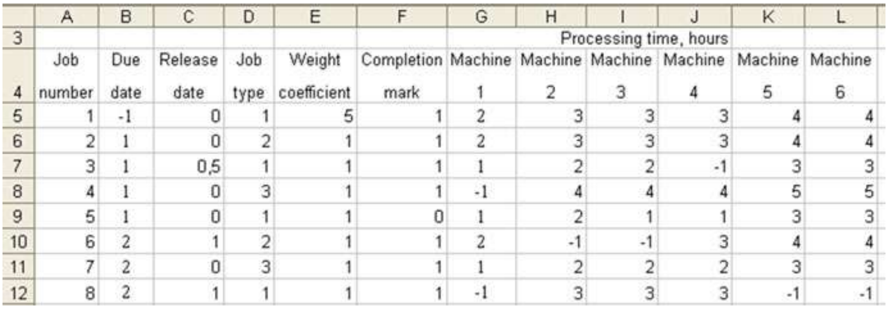
E.4.1 Worksheet

Sheet 1 table $A$5:$L$64 contains a task consisting of 60 jobs of six different types (Fig. E.11). Each row of the task describes the job parameters: job number, required due calendar day after start of schedule execution, expected calendar day of arrival after start, job type, weighting coefficient (importance) of the job, and processing time in hours on each machine. It is advisable to enter the jobs in ascending order of their required due time points.

The main parameters of the planning process are entered in the second row of the sheet; the data processing parameters and the planning results records are entered in the first row.

Table $R$6:$BA$11 shows the time standards in hours for machine setups from one type of job to another. It is assumed that the number of the previous job type is set by the table column and the number of the following job type is set by the table row. Region $S$13:$X$19 contains data on the machine condition at the start of planning: cost of machine operation per hour, cost of idleness per hour, mark of machine activation, and setup for the job type. Region $S$21:$AC$23 shows the working calendar for 11 days starting from the first day of planning. For the working day and machine utilization records, values 1 or 0 are used, and the operation value is 1.

The program operation, as in the previous paragraphs, is based on the planning method by costs criteria *U* and average order fulfilment utility *V*. The planning results are recorded in the worksheet in several possible versions.



	A	B	C	D	E	F	G	H	I	J	K	L
3								Processing time, hours				
4	Job number	Due date	Release date	Job type	Weight coefficient	Completion mark	Machine 1	Machine 2	Machine 3	Machine 4	Machine 5	Machine 6
5	1	-1	0	1	5	1	2	3	3	3	4	4
6	2	1	0	2	1	1	2	3	3	3	4	4
7	3	1	0,5	1	1	1	1	2	2	-1	3	3
8	4	1	0	3	1	1	-1	4	4	4	5	5
9	5	1	0	1	1	0	1	2	1	1	3	3
10	6	2	1	2	1	1	2	-1	-1	3	4	4
11	7	2	0	3	1	1	1	2	2	2	3	3
12	8	2	1	1	1	1	-1	3	3	3	-1	-1

Fig. E.11 Work task fragment

E.4.2 How to Use the Program

After enabling of the macro, the program offers the user two options: go to preparation planning for a new date or perform planning. This paragraph considers the second option. Upon selection of the latter, the “Data input” form opens.

At first initiation of the “Data input” form, the windows can be empty. To open the data from the worksheet, use the “Current parameters” button. The modified data should be recorded in the sheet using the “Data input” button.

When you click “Start”, the program starts operation. Scheduling can be performed for all the jobs in the task or for part of this task starting with its first row. It is assumed that the schedule execution starts from the set estimated hour corresponding to the start of the first working day as on the date of planning. In particular, if the date of planning is the working day, then the start of jobs can occur at 8 o’clock on this day.

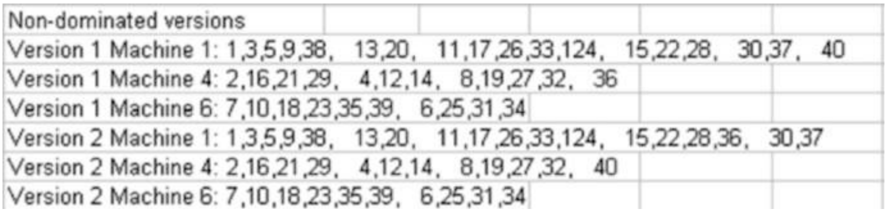
Values of the required due moments and expected arrival are set as the end time of the relevant calendar days. Within the program, the required due moments and expected arrival in accordance with the working calendar are automatically converted to the number of working hours after the start of schedule execution.

The working time essentially depends on the number of jobs in the task and the number of machines involved as well as various parameters of the system. For example, with 50 jobs scheduled for execution for six machines, the estimated time does not exceed 2 min. At the end of calculation of possible non-dominated versions, these versions are entered in the worksheet (Fig. E.12). In each version, the jobs are grouped by type and the groups are separated by spaces. Within the group, the jobs are given in numerical order.

In the example shown, three machines are involved: the first, the fourth, and the sixth. As can be seen from Fig. E.12, to reduce the setups the jobs scheduled for each machine are grouped by type.

Then a “Decision” form appears on the screen. In the form, the horizon (time in hours) for which the most acceptable decision will be calculated must be specified. This horizon is usually taken to be somewhat less than the total length of all the scheduled jobs. In the case where the accuracy of the information about the upcoming jobs is low, it makes sense to reduce the estimated horizon.

To determine the recommended solutions, the decision-making methods of Savage and Hurwitz are used in the program similarly to the previous paragraphs.



Non-dominated versions
Version 1 Machine 1: 1,3,5,9,38, 13,20, 11,17,26,33,124, 15,22,28, 30,37, 40
Version 1 Machine 4: 2,16,21,29, 4,12,14, 8,19,27,32, 36
Version 1 Machine 6: 7,10,18,23,35,39, 6,25,31,34
Version 2 Machine 1: 1,3,5,9,38, 13,20, 11,17,26,33,124, 15,22,28,36, 30,37
Version 2 Machine 4: 2,16,21,29, 4,12,14, 8,19,27,32, 40
Version 2 Machine 6: 7,10,18,23,35,39, 6,25,31,34

Fig. E.12 Fragment of calculation of two versions

At the end of the calculation, a set of possible versions of the plan can be derived in one of two ways. In the first case, all sustainable versions are derived from the version with the greatest utility to the version recommended by the methods of Savage or Hurwitz. In the second case, we can derive a minimum set of versions between the version with the greatest utility and the version of the least duration. Usually, it is recommended to use the first version.

For all non-dominated versions, the data on values *U* and *V* are displayed for the specified estimated horizon. Also, the value of the load coefficient and the number of the version with the least duration of total fulfilment of the task are recorded.

E.4.3 Planning Results Analysis

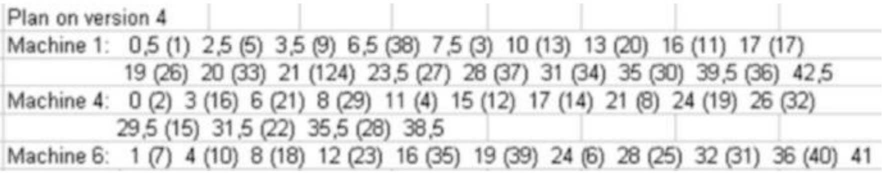
To calculate the plan by the analysed version, the system offers a “Scheduling” form. Enter the number of the calculated version in the “Number of version” window. The version with the least duration is used as the recommended one. In “The first cell for solution” window, the name of the cell is entered in which it is advisable to place scheduling for further study. When scheduling the time point of job arrival, their duration and time standards for changeovers are considered (Fig. E.13). The figure without brackets corresponds to the job start time point, which then is entered in the brackets.

The quality of scheduling for parallel machines is significantly defined by the uniformity of loading of all active machines. To evaluate this, at the end of the schedule row for each machine the end time point of task execution is specified.

E.4.4 New Task

A new task is set up in a separate sheet, using a new sheet or a corrected sheet. One of the available sheets may be used. It makes sense to copy the sheet with the last plan task, rename this worksheet by the number in the file, and delete the planning results belonging to the previous task from it.

In preparation for the new plan, put the completion mark (number 1) in the rows of the last task. In the region describing the state of the machines, specify the date for which the new plan is performed and put the marks of machines activation. Then indicate the job type for which each machine will be set up by the beginning of the



```
Plan on version 4
Machine 1:  0,5 (1)  2,5 (5)  3,5 (9)  6,5 (38)  7,5 (3)  10 (13)  13 (20)  16 (11)  17 (17)
            19 (26)  20 (33)  21 (124)  23,5 (27)  28 (37)  31 (34)  35 (30)  39,5 (36)  42,5
Machine 4:  0 (2)  3 (16)  6 (21)  8 (29)  11 (4)  15 (12)  17 (14)  21 (8)  24 (19)  26 (32)
           29,5 (15)  31,5 (22)  35,5 (28)  38,5
Machine 6:  1 (7)  4 (10)  8 (18)  12 (23)  16 (35)  19 (39)  24 (6)  28 (25)  32 (31)  36 (40)  41
```

Fig. E.13 Job sequence on parallel machines

task start and the expected time of machine availability. In the region of the working calendar, specify the date of the new task and put the marks of the working days after the task start date.

An example of a new task is shown on sheet 2 in File3.xls. With the command "Recalculate", the system automatically processes the remains of the old task, deleting the executed rows and updating the required due dates to the new time of planning indicated in the "Preparation for planning" form.

E.5 Description of Work with File4.xls

Please see http://extras.springer.com/2016/978-3-319-27522-2

File4.xls is intended for scheduling the operation of a production site consisting of several parallel machines of the same purpose and using a "make-to-stock" strategy. The book contains two task sheets, a software module, and four user forms.

E.5.1 Worksheet

Sheet 1 table $B$6:$AK$13 shows the characteristics of technical batches and time standards in hours for each machine setup from one job to another. The technical batch is understood as the minimal volume of production, due to the technical capabilities of production. This amount can be determined by the physical volume of the machine, the number of products in a single package, the size of the minimum quantity shipped, etc. The technical batch size in the corresponding physical unit and the duration of its production in hours may depend on the type of product and the machine on which the product is produced.

Region $F$16:$K$22 contains data on the states of each machine at the planning start time, the marks of machine activation, setup for the job type, and expected time of availability in working hours. In region $Q$16:$V$23, there are data on the stock of each manufactured product: designation, current stock and arrears in deliveries and expected consumption per working day, safety stock, and the expected day of raw material readiness for production. Region $F$26:$P$28 shows the working calendar for 11 days starting from the first day of planning. For the working day and machine utilization records, values 1 or 0 are used, and the operation value is 1.

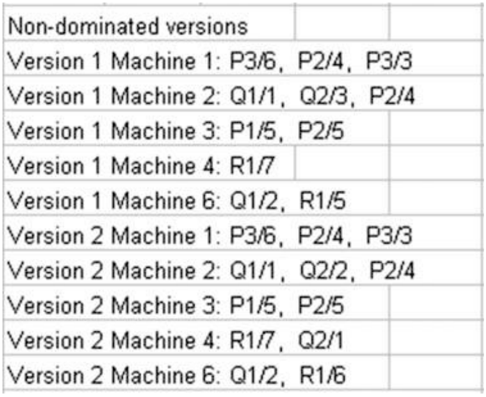
E.5.2 How to Use the Program

At the beginning, the program offers the user two options: go to preparation planning for a new date or perform planning. This paragraph considers the second option. Upon selection of the latter, a "Data input" form opens similarly to the previous paragraph.

When using a "make-to-stock" strategy, the value of the plan horizon is essential for planning. The greater the plan horizon is, the more technical batches are combined into one group. The value of the average orders utility falls with growth of the horizon, because the backlog in deliveries can increase.

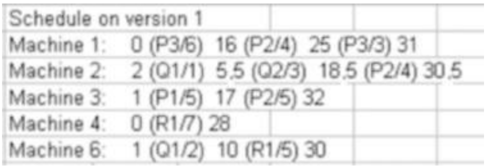
When you click "Start", the program starts operation. The working time essentially depends on the number of jobs in the task and the number of machines involved as well as various parameters of the system. For example, with ten products scheduled for execution on six machines, the estimated time does not exceed 2 min. At the end of the calculation of possible non-dominated versions, these versions are entered in the worksheet (Fig. E.14). In each version, the technical batches of a product of one type are grouped, and the quantity of batches per group is specified after a "/" symbol. For example, in version 1 on machine 2, it is planned to produce one technical batch of product Q1, then three batches of product Q2, and four batches of product P2.

To determine the recommended solutions, the decision-making methods of Savage and Hurwitz are used in the program similarly to the previous paragraphs. As a recommended version, the version with the least total duration is often used. In the obtained schedule (Fig. E.15), the figure without brackets corresponds to the planned start time point of the technical batch group on each machine in working hours.



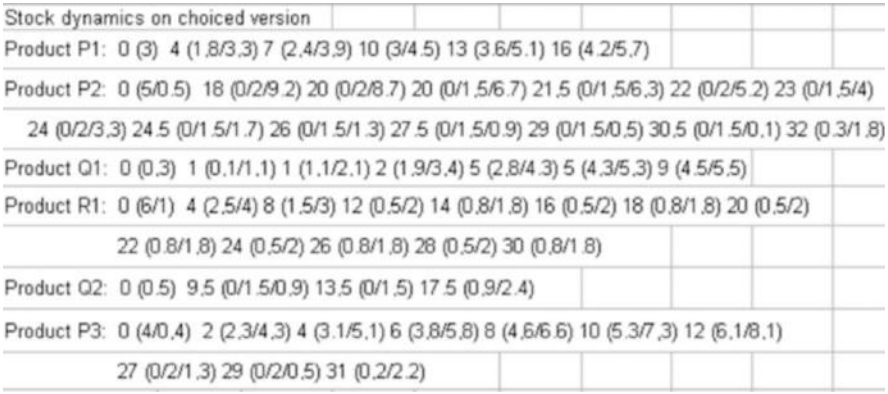
Non-dominated versions
Version 1 Machine 1: P3/6, P2/4, P3/3
Version 1 Machine 2: Q1/1, Q2/3, P2/4
Version 1 Machine 3: P1/5, P2/5
Version 1 Machine 4: R1/7
Version 1 Machine 6: Q1/2, R1/5
Version 2 Machine 1: P3/6, P2/4, P3/3
Version 2 Machine 2: Q1/1, Q2/2, P2/4
Version 2 Machine 3: P1/5, P2/5
Version 2 Machine 4: R1/7, Q2/1
Version 2 Machine 6: Q1/2, R1/6

Fig. E.14 Fragment of calculation of two versions



Schedule on version 1	
Machine 1:	0 (P3/6) 16 (P2/4) 25 (P3/3) 31
Machine 2:	2 (Q1/1) 5.5 (Q2/3) 18.5 (P2/4) 30.5
Machine 3:	1 (P1/5) 17 (P2/5) 32
Machine 4:	0 (R1/7) 28
Machine 6:	1 (Q1/2) 10 (R1/5) 30

Fig. E.15 Sequence of jobs on parallel machines



Stock dynamics on choiced version

Product P1: 0 (3) 4 (1.8/3.3) 7 (2.4/3.9) 10 (3/4.5) 13 (3.6/5.1) 16 (4.2/5.7)

Product P2: 0 (5/0.5) 18 (0/2/9.2) 20 (0/2/8.7) 20 (0/1.5/6.7) 21.5 (0/1.5/6.3) 22 (0/2/5.2) 23 (0/1.5/4)

24 (0/2/3.3) 24.5 (0/1.5/1.7) 26 (0/1.5/1.3) 27.5 (0/1.5/0.9) 29 (0/1.5/0.5) 30.5 (0/1.5/0.1) 32 (0.3/1.8)

Product Q1: 0 (0.3) 1 (0.1/1.1) 1 (1.1/2.1) 2 (1.9/3.4) 5 (2.8/4.3) 5 (4.3/5.3) 9 (4.5/5.5)

Product R1: 0 (6/1) 4 (2.5/4) 8 (1.5/3) 12 (0.5/2) 14 (0.8/1.8) 16 (0.5/2) 18 (0.8/1.8) 20 (0.5/2)

22 (0.8/1.8) 24 (0.5/2) 26 (0.8/1.8) 28 (0.5/2) 30 (0.8/1.8)

Product Q2: 0 (0.5) 9.5 (0/1.5/0.9) 13.5 (0/1.5) 17.5 (0.9/2.4)

Product P3: 0 (4/0.4) 2 (2.3/4.3) 4 (3.1/5.1) 6 (3.8/5.8) 8 (4.6/6.6) 10 (5.3/7.3) 12 (6.1/8.1)

27 (0/2/1.3) 29 (0/2/0.5) 31 (0.2/2.2)

Fig. E.16 Stock dynamics

The schedule quality for parallel machines is significantly determined by uniform loading of all machines. For this assessment at the end of the schedule row for each machine, the end time point of the task is specified.

Below the schedule, the total calculation of stock dynamics for each product is presented for the selected schedule version (Fig. E.16). The figure without brackets, as above, corresponds to the release time point of the new batch of product in working hours.

In brackets, in general, three numbers separated by "/" symbols are given. The first number indicates the amount of stock in thousands of litres by the time of new receipt; the second number corresponds to the stock after delivery; the third number reflects the backlog in delivery at the time of product receipt. If the debt is zero, then it is not given in the text.

In preparation for the new plan, in the region describing the state of the machines specify the date for which new planning is performed and put the marks of machine activation. Then indicate the job type for which each machine will be set up by the beginning of the task start and the expected time of machine release. In the region of the working calendar, specify the date of the task start and put marks of the working days after the date of task start. In the region of the stock, correct the values of the available stock, backlog in deliveries, and terms of raw material receipt. An example of a new task is shown in sheet 2 in File4.xls.

E.6 Description of Work with File5.xls

Please see http://extras.springer.com/2016/978-3-319-27522-2

File5.xls is intended for scheduling the operation of a production site consisting of machines of different purpose and using a "make-to-order" strategy. It is assumed that the uniform machines have similar technological capabilities and can be considered as one group of machines. The book contains two task sheets, a software module, and four user forms.

E.6.1 Worksheet

Sheet 1 table $A$5:$F$24 contains a task consisting of 20 jobs of six different types. Each row (number of job) of the task describes the job parameters: job number, required due calendar day after start of schedule execution, expected calendar day of arrival after start, job type, weighting coefficient (importance) of the job, and completion mark. The job type can mean a specific part or a group of similar parts manufactured according to the same process using the same tools. The sequence of operations of each process is fixed and is different for different processes. It is advisable to enter the jobs in ascending order of their required due time points.

Table $H$5:$L$84 is a list of operations for all the jobs in table $A$5:$F$24. Each row of this table contains the number of the operation, group of machines on which the operation must be performed, processing time in hours, and expected end time point in hours after the start of schedule execution. The operations of each job should be sorted in the order of their execution. If the value of the expected due time point equals 0, this means the relevant operation is already executed. For an operation that has not yet commenced, the expected due time point is 1.

Table $O$17:$S$22 shows time standards in hours for setup of each machine from one type of job to another. Table $O$5:$Q$9 presents the characteristics of the machine groups in the production area: quantity of machines in the group, cost of setup per hour, and cost of idleness per hour.

Region $U$5:$Y$12 contains data on each machine condition at the time of planning start: inventory number of the machine; group number, which the machine refers to; mark of machine activation; setup for the job type; and expected moment of release in hours after the start of schedule execution. Region $Q$21:$S$27 shows the working calendar for 10 days starting from the first day of planning. For the working day and machine utilization records, values 1 or 0 are used, and the operation value is 1.

E.6.2 How to Use the Program

At the beginning, the program offers the user two options: go to preparation planning for a new date or perform planning. This paragraph considers the second option. Upon selection of the latter, a “Data input” form opens similarly to the previous paragraphs.

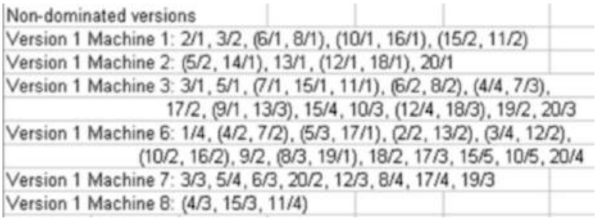
When you click “Start”, the program starts operation. It is assumed that the schedule execution starts from the set estimated hour corresponding to the start of the first working day as on the date of planning. In particular, if the date of planning is the working day, then the start of jobs can occur at 8 o’clock on this day. Values of the required due moments and expected arrival are set as the end time of the relevant calendar days. Within the program, the required due moments and expected arrival in accordance with the working calendar are automatically converted to the number of working hours after the start of schedule execution.

The working time essentially depends on the number of jobs in the task and the number of machines involved as well as various parameters of the system. For example, with 20 jobs scheduled for execution for nine machines, the estimated time does not exceed 2 min. At the end of calculation of possible non-dominated versions, these versions are entered in the worksheet (Fig. E.17).

For each machine, the result of planning is represented by the set of executed operations. Each operation is described by the combination of job number and (slash) operation number. In each version, jobs are grouped by types and the groups of operations of one type are joined by brackets.

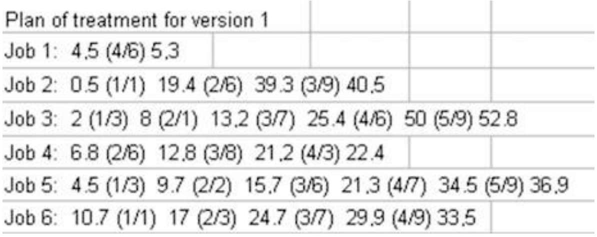
To determine the recommended solutions, the decision-making methods of Savage and Hurwitz are used in the program similarly to the previous paragraphs. For all non-dominated versions, the data on values *U* and *V* are displayed for the specified estimated horizon. Also, the value of the load coefficient and the number of the version with the least duration for the total fulfilment of the task is recorded.

As a recommended version, the version with the least total duration is often used. When scheduling the time points of job arrival, their duration and time standards for setups are considered. The figure without brackets corresponds to the job start time point, and the number of the job slash the number of operations are entered in the brackets. Below the schedule of machine loading, the schedule of job execution is shown (Fig. E.18) according to the corresponding version. At the end of the schedule row for each machine and each job, the end time of the task execution is specified.



Non-dominated versions
Version 1 Machine 1: 2/1, 3/2, (6/1, 8/1), (10/1, 16/1), (15/2, 11/2)
Version 1 Machine 2: (5/2, 14/1), 13/1, (12/1, 18/1), 20/1
Version 1 Machine 3: 3/1, 5/1, (7/1, 15/1, 11/1), (6/2, 8/2), (4/4, 7/3),
17/2, (9/1, 13/3), 15/4, 10/3, (12/4, 18/3), 19/2, 20/3
Version 1 Machine 6: 1/4, (4/2, 7/2), (5/3, 17/1), (2/2, 13/2), (3/4, 12/2),
(10/2, 16/2), 9/2, (8/3, 19/1), 18/2, 17/3, 15/5, 10/5, 20/4
Version 1 Machine 7: 3/3, 5/4, 6/3, 20/2, 12/3, 8/4, 17/4, 19/3
Version 1 Machine 8: (4/3, 15/3, 11/4)

Fig. E.17 Sequence of operations in version 1



Plan of treatment for version 1
Job 1: 4,5 (4/6) 5,3
Job 2: 0.5 (1/1) 19.4 (2/6) 39.3 (3/9) 40,5
Job 3: 2 (1/3) 8 (2/1) 13,2 (3/7) 25.4 (4/6) 50 (5/9) 52.8
Job 4: 6.8 (2/6) 12,8 (3/8) 21,2 (4/3) 22.4
Job 5: 4.5 (1/3) 9.7 (2/2) 15,7 (3/6) 21,3 (4/7) 34.5 (5/9) 36,9
Job 6: 10.7 (1/1) 17 (2/3) 24.7 (3/7) 29,9 (4/9) 33,5

Fig. E.18 Fragment of job execution schedule

In preparation for the new plan, put the completion mark (number 1) in the rows of the last task. In the region describing the state of the machines, specify the date for which new planning is performed and to put the marks of machine activation. Then indicate the job type for which each machine will be set up by the beginning of the task start and the expected time when each machine will be free. In the region of the working calendar, specify the date of the new task and put marks of the working days after the date of task start. An example of a new task is shown in sheet 2 in File5.xls.

When selecting the option of going to plan for a new date, the new dates of planning and cell names of the transferred old plan task are entered into the “Preparation of planning” form. With the “Recalculate” command, the system automatically processes the remains of the old task, deleting the executed rows and bringing the required due dates to the new time of planning.

E.7 Description of Work with File6.xls

Please see http://extras.springer.com/2016/978-3-319-27522-2

File6.xls is intended for scheduling the operation of a production site consisting of machines of different purpose. It is assumed that the uniform machines have similar technological capabilities and can be considered as one group of machines. The production area produces kits of parts using a “make-to-stock” strategy. The book contains two task sheets, a software module, and five user forms.

E.7.1 Worksheet

Sheet 1 table $A$5:$F$14 contains a task consisting of three kits that include six different types of parts. Each row describes the number of the kit, part type, and the quantity of these parts in one kit.

Table $H$5:$K$31 is a list of operations for all parts. The row of this table contains the number of the operation, group of machines on which the operation must be performed, processing time in hours, and expected end time point in hours after the start of schedule execution. The operations of each job should be sorted in the order of their execution.

Table $Z$15:$AE$24 contains data on the parts stock: part designation, the smallest transfer batch, current stock, backlogs in deliveries, safety stock, expected day of raw material readiness, number of the last delivered batch, and the weighting factor. In this case, the value of the technical batch, described in Sect. E.5.1, coincides with the value of the smallest transfer batch.

Region$Z$26:$AB$29 contains data on the demand for the produced kits: number of the kit, consumption per working day, and the last delivered number.

Table $Z$31:$ADE$36 has data on the condition of the parts batch in process: part number, batch number, quantity in a batch, last performed (fulfilled) operation, and the expected due time point in hours.

The data on the machine condition, time standards, and working calendar are entered in the worksheet similarly to the previous program.

E.7.2 How to Use the Program

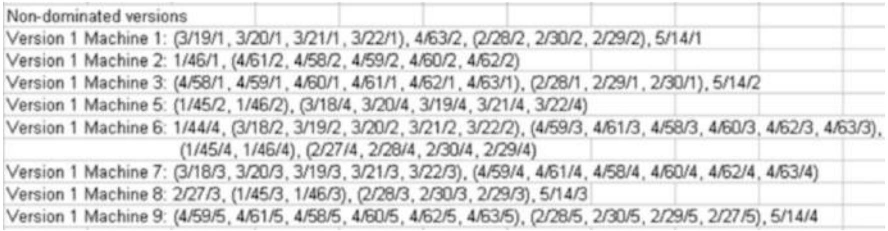
At the beginning, the program offers the user two options: go to preparation planning for a new date or perform planning. This paragraph considers the second option, which in turn offers two actions: initial calculation and simulation.

If simulation is not performed (initial calculation mode), then a "Data input" form opens. It enables inputting of necessary tables.

When you click "Start", the program starts operation, and the calculation result is entered in the sheet (Fig. E.19).

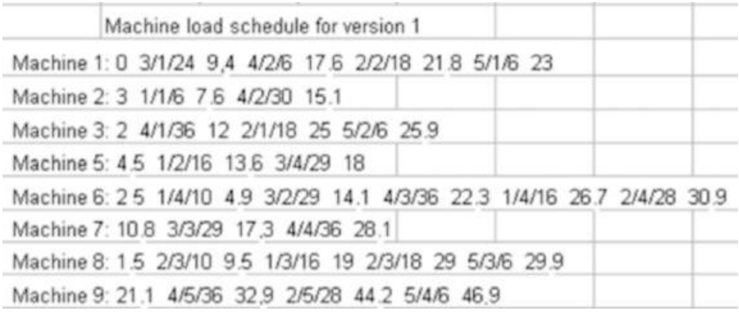
For each machine, the scheduling result is a set of operations. Each operation is described by a combination of the part number, (with slash sign) quantity of transfer batch, and (with slash sign) operation number. In each version, several transfer batches (in parentheses) are automatically combined into one technological batch. For example, on machine 1 for part 3 transfer batches 19, 20, 21, and 22 are combined into a single group to perform operation 1.

The relevant schedule of machine load is presented in Fig. E.20. For each machine, the sequence of start time points is recorded. After each time point is a combination of the part number, (with slash sign) operation number, and (with slash sign) the quantity of parts in a technological batch.



```
Non-dominated versions
Version 1 Machine 1: (3/19/1, 3/20/1, 3/21/1, 3/22/1), 4/63/2, (2/28/2, 2/30/2, 2/29/2), 5/14/1
Version 1 Machine 2: 1/46/1, (4/61/2, 4/58/2, 4/59/2, 4/60/2, 4/62/2)
Version 1 Machine 3: (4/58/1, 4/59/1, 4/60/1, 4/61/1, 4/62/1, 4/63/1), (2/28/1, 2/29/1, 2/30/1), 5/14/2
Version 1 Machine 5: (1/45/2, 1/46/2), (3/18/4, 3/20/4, 3/19/4, 3/21/4, 3/22/4)
Version 1 Machine 6: 1/44/4, (3/18/2, 3/19/2, 3/20/2, 3/21/2, 3/22/2), (4/59/3, 4/61/3, 4/58/3, 4/60/3, 4/62/3, 4/63/3),
                     (1/45/4, 1/46/4), (2/27/4, 2/28/4, 2/30/4, 2/29/4)
Version 1 Machine 7: (3/18/3, 3/20/3, 3/19/3, 3/21/3, 3/22/3), (4/59/4, 4/61/4, 4/58/4, 4/60/4, 4/62/4, 4/63/4)
Version 1 Machine 8: 2/27/3, (1/45/3, 1/46/3), (2/28/3, 2/30/3, 2/29/3), 5/14/3
Version 1 Machine 9: (4/59/5, 4/61/5, 4/58/5, 4/60/5, 4/62/5, 4/63/5), (2/28/5, 2/30/5, 2/29/5, 2/27/5), 5/14/4
```

Fig. E.19 Sequence of operations in version 1



Machine load schedule for version 1

Machine 1: 0 3/1/24 9,4 4/2/6 17.6 2/2/18 21.8 5/1/6 23

Machine 2: 3 1/1/6 7.6 4/2/30 15.1

Machine 3: 2 4/1/36 12 2/1/18 25 5/2/6 25.9

Machine 5: 4.5 1/2/16 13.6 3/4/29 18

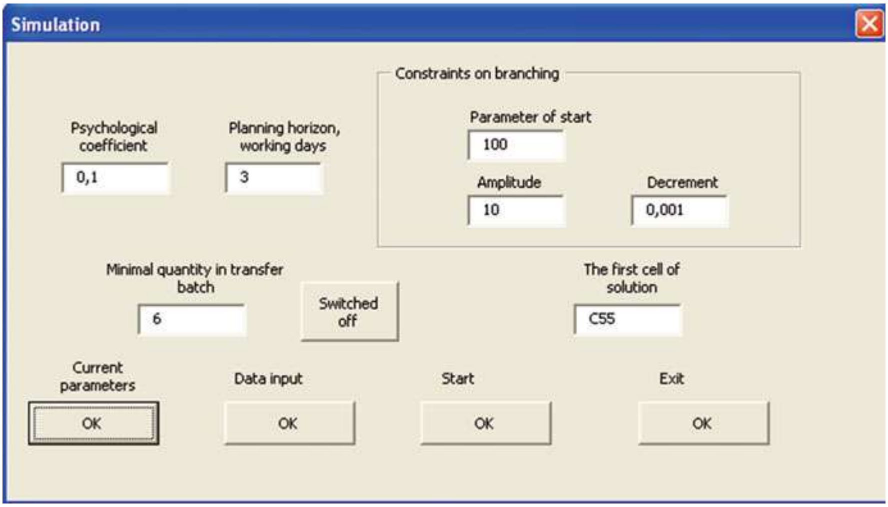
Machine 6: 2.5 1/4/10 4.9 3/2/29 14.1 4/3/36 22.3 1/4/16 26.7 2/4/28 30.9

Machine 7: 10.8 3/3/29 17,3 4/4/36 28.1

Machine 8: 1.5 2/3/10 9.5 1/3/16 19 2/3/18 29 5/3/6 29.9

Machine 9: 21.1 4/5/36 32.9 2/5/28 44.2 5/4/6 46.9

Fig. E.20 Schedule of machine load



Psychological coefficient: 0,1

Planning horizon, working days: 3

Parameter of start: 100

Amplitude: 10

Decrement: 0,001

Minimal quantity in transfer batch: 6

The first cell of solution: C55

Fig. E.21 Simulation control form

When working in simulation mode, the user can change various parameters of the system: the psychological coefficient, plan horizon, and batch size (Fig. E.21). When enabling the flag of transfer batches, they are accepted as the same for parts of all types. When you turn off this flag, the transfer batches for the parts are defined by the table of stocks.

In addition to these parameters, the user can change the values of the branching limiters, thus changing the number of possible viewed versions at each step of the calculation. Each calculation result obtained in simulation can be put into its place in the Excel worksheet for subsequent comparison.

The system allows performing calculations of several versions of the schedule, placing the obtained results in different cells. To complete the calculation, the "Finish" button is used. At the initial start of the program, the windows may be empty. To display the data from the sheet on the screen, use the "Current values" button.

Below the schedule of machine load, the lists of output batches, kits, and release batches for the selected version are shown. For each part, first the numbers of output batches and (with slash symbol) quantity of parts in the batch and then after a space the completion moment are specified. In the launch list for each part, first the moment of batch launch is entered, and then after a space the number of the launch batch in this working day and (with slash) the quantity of parts in the launch batch are recorded. In the list of kits, the number of produced kits is shown by days and (with slash) the quantity of such kits in that day.

A new task is set up in a separate sheet, a new sheet or corrected sheet. One of the available sheets may be used. To prepare a new task, calculate the production situation for the new date. The calculation is made on the same sheet on which the previous planning was performed. Without affecting the values of the original data, delete all the planning results obtained earlier in this sheet.

Then one should run the macro and select the mode of planning preparation for the new date and mode of calculation of the production situation. The system provides a form in which the date for which the calculation should be performed must be specified. The calculation is performed automatically, after which the plan of equipment load is displayed according to a preliminary selected version, a list of scheduled output batches, the list of the last numbers of output batches, as well as a list of work-in-progress. After preparation, the planning worksheet is entirely copied into a new sheet, and all required data are corrected.

E.8 Description of Work with File7.xls

Please see http://extras.springer.com/2016/978-3-319-27522-2

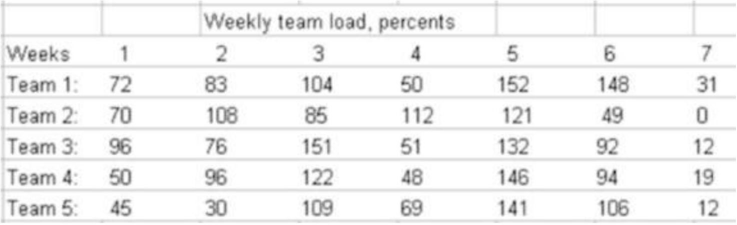
File7.xls is intended for scheduling the operation of a production team. The book contains one task sheet, a software module, and two user forms.

E.8.1 Worksheet

Sheet 1 table $A$5:$O$79 contains a task—a list of jobs subject to fulfilment. For each operation, its designation, required run time, the first possible day of the start, and the start and end of the unavailability interval are indicated. All these values are specified in calendar days after the beginning of the planning.

Furthermore, the task contains data on the total processing time in hours, the number of leading job types, normative fulfilment duration in working days, and weighting coefficient of the job priority. If the work has already begun, but is not complete, 0.5 is entered in the “Completion mark” window.

In the example shown in the sheet, the number of job types is equal to 5. Therefore, the task for each of these types indicates what percentage of the total processing time accounts for each type of relevant job.



		Weekly team load, percents					
Weeks	1	2	3	4	5	6	7
Team 1:	72	83	104	50	152	148	31
Team 2:	70	108	85	112	121	49	0
Team 3:	96	76	151	51	132	92	12
Team 4:	50	96	122	48	146	94	19
Team 5:	45	30	109	69	141	106	12

Fig. E.22 Weekly team load

Table $S$6:$W$10 contains the largest possible output per hour/day for each type of job for the relevant team. Table $S$20:$Y$35 indicates the percentage of completion of five started jobs for each job type as of 01.04.14. Region $S$64:$AD$75 in the working calendar as of 01.04.14 contains the tentative interests of planned performance for all types of jobs for the next 10 weeks.

E.8.2 How to Use the Program

When you click “Start”, the program starts operation. The result of the calculation is an execution schedule of jobs, in which for each job the calendar start and end dates are indicated. In parentheses, the numbers of production teams performing these jobs are recorded, and the first team is the team that performs the leading type of jobs.

In addition to the schedule, the program displays three tables of team load in the worksheet. In the top table (Fig. E.22), the weekly load factor of each team as a percentage of the maximum possible performance is recorded. The middle table shows the list of jobs executed by the team during each week. The lower table shows weekly team load for each job type in hours.

The schedule of jobs prepared automatically is the first approximation to the optimal one and can be improved. Correction of schedules is performed using a “Correction” form. In the form, it is possible to set the values of the end time points of jobs and the duration of five jobs. When the program is in “Correction” mode, the system automatically takes into account the specified changes and moves the start and end points of jobs to more appropriate times. With enough skills in simulation, it is possible to achieve significant improvement in schedules for both timely execution of jobs and more uniform utilization of production teams.

E.9 Description of Work with File8.xls

Please see http://extras.springer.com/2016/978-3-319-27522-2

File8.xls is intended for scheduling the operation of a production site consisting of machines of different purposes and using a “make-to-order” strategy. Compared to File5.xls in this program, all the jobs are executed according to the same technology. It is assumed that for each operation of the process, the area has several uniform machines, but some technical characteristics of these machines are different. The book contains one task sheet, a software module, and three user forms.

E.9.1 Worksheet

Sheet 1 table $A$5:$P$79 contains a task—a list of jobs subject to fulfilment. For each job, its designation, required due time point, and the first possible day of the start are indicated. All these values are specified in calendar days after the beginning of the planning. The job type can be represented by a specific part or a group of similar parts manufactured according to the same process using the same tooling. It is advisable to enter the jobs in ascending order of execution.

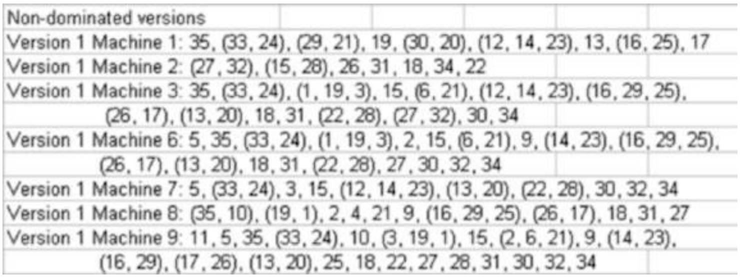
Table $V$20:$BW$25 shows time standards in hours for setups of each machine from one type of job to another; the values of standards can depend on the sequence of jobs.

Region $U$4:$AC$14 contains data on each machine condition at the time of planning start: inventory number of the machine; group number, which the machine refers to; mark of machine activation; setup for the job type; and percentage of execution of current operation.

E.9.2 How to Use the Program

When preparing the schedule, a “Data input” form opens. This form is similar to the form in the previous paragraphs. When you click “Start”, the program starts operation, and at the end of calculation of possible non-dominated versions, these versions are entered in the worksheet (Fig. E.23).

In the example, 35 works are executed, including five operations belonging to six types. Machines 1 and 2 are designed for operation 1. Machines 3, 4, and 5 can perform operation 2, but machines 4 and 5 are not activated. Machine 6 performs



```
Non-dominated versions
Version 1 Machine 1: 35, (33, 24), (29, 21), 19, (30, 20), (12, 14, 23), 13, (16, 25), 17
Version 1 Machine 2: (27, 32), (15, 28), 26, 31, 18, 34, 22
Version 1 Machine 3: 35, (33, 24), (1, 19, 3), 15, (6, 21), (12, 14, 23), (16, 29, 25),
                 (26, 17), (13, 20), 18, 31, (22, 28), (27, 32), 30, 34
Version 1 Machine 6: 5, 35, (33, 24), (1, 19, 3), 2, 15, (6, 21), 9, (14, 23), (16, 29, 25),
                (26, 17), (13, 20), 18, 31, (22, 28), 27, 30, 32, 34
Version 1 Machine 7: 5, (33, 24), 3, 15, (12, 14, 23), (13, 20), (22, 28), 30, 32, 34
Version 1 Machine 8: (35, 10), (19, 1), 2, 4, 21, 9, (16, 29, 25), (26, 17), 18, 31, 27
Version 1 Machine 9: 11, 5, 35, (33, 24), 10, (3, 19, 1), 15, (2, 6, 21), 9, (14, 23),
                (16, 29), (17, 26), (13, 20), 25, 18, 22, 27, 28, 31, 30, 32, 34
```

Fig. E.23 Sequence of operations in version 1

operation 3, machines 7 and 8 are intended for operation 4, and machine 9 is necessary for operation 5.

As can be seen from the example, in the resulting schedule the jobs are grouped by type. The machine group load for each operation is in the range 0.82–1.19. The density of the load of the machines varies. The greatest density (86 %) is observed on machine 3 and the least (28 %) is observed on machine 7.

Appendix F: Using Clobbi[1]

F.1 General

This appendix gives a short description of the basic production planning capabilities in Clobbi SaaS manufacturing management system (http://clobbi.com), built on contemporary cloud technologies.

The Clobbi service is based on the IT-Enterprise ERP system (www.it.ua, www.it-enterprise.ru) platform, widely used in industrial enterprises in Eastern European countries, which includes ready solutions for automation of all services of an industrial enterprise in manufacturing and resource management (MRP II, MES, APS), sales management, supply chain management (SCM), procurement management, equipment operation and maintenance management (EAM), quality management, human resources and payroll (HR), finance planning and budgeting, accounting, and tax. Clobbi uses trusted typical solutions and the best business practice accumulating more than 20 years' experience of IT-Enterprise in manufacturing management.

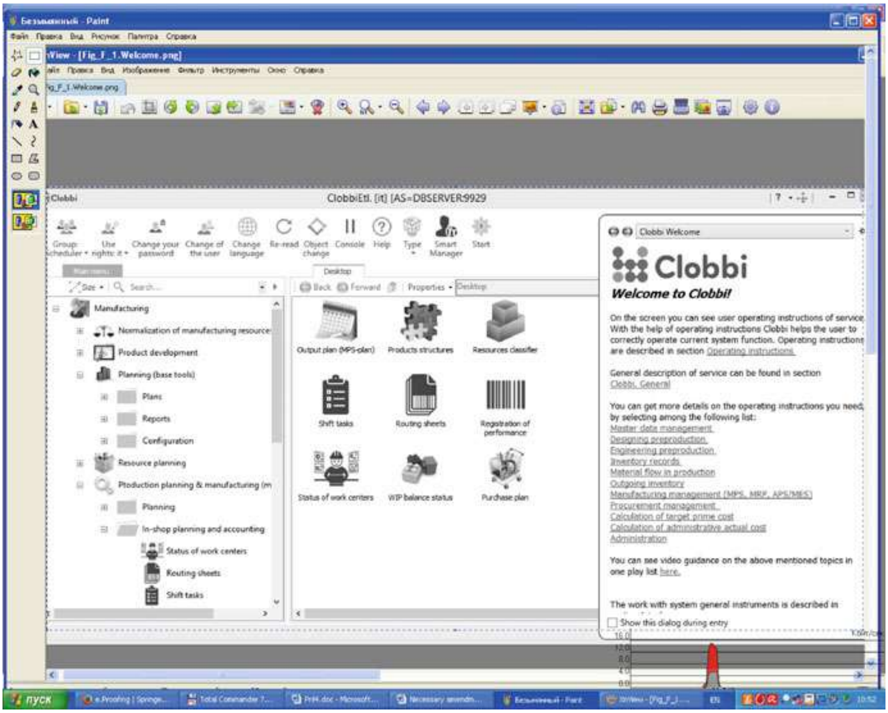
To start work with the service demo version, please go to http://clobbi.com/demo and download the demo version after the registration procedure.

After client application deploys, you will find a "Clobbi" shortcut on your desktop. When starting the shortcut, you should enter "PLAN" into the login and password fields to view the system as Chief planner.

Go to the Clobbi main menu and desktop. On the right, you will see operating instructions that will help you to learn about the system interface and understand its functionality (Fig. F.1).

[1] This material is provided by the IT-Enterprise Corporation.

Y. Mauergaz, *Advanced Planning and Scheduling in Manufacturing and Supply Chains*, DOI 10.1007/978-3-319-27523-9



Welcome to Clobbi!

On the screen you can see user operating instructions of service. With the help of operating instructions Clobbi helps the user to correctly operate current system function. Operating instructions are described in section Operating instructions.

General description of service can be found in section Clobbi. General

You can get more details on the operating instructions you need, by selecting among the following list:
Master data management
Designing preproduction
Engineering preproduction
Inventory records
Material flow in production
Outgoing inventory
Manufacturing management (MPS, MRP, APS/MES)
Procurement management
Calculation of target prime cost
Calculation of administrative actual cost
Administration

You can see video guidance on the above mentioned topics in one play list here.

The work with system general instruments is described in

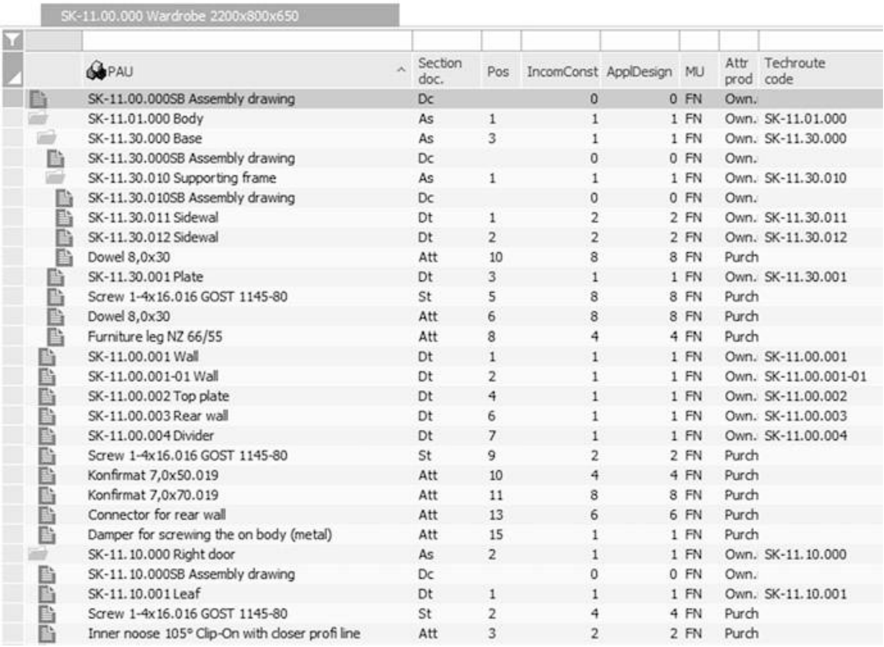
□ Show this dialog during entry

Fig. F.1 Clobbi desktop

For simplicity, a Wardrobe is selected as the basic product.

The demo version includes detailed structures of products, technological processes, and standards on Wardrobe, a simplified card file of working centers with corresponding equipment, a Main Production Schedule (MPS) example, and all reference data necessary for schedule creation. To view the initial data, just click on the shortcuts on your desktop.

The product structure for the selected wardrobe is shown below (Fig. F.2).



SK-11.00.000 Wardrobe 2200x800x650

PAU	Section doc.	Pos	IncomConst	ApplDesign	MU	Attr prod	Techroute code
SK-11.00.000SB Assembly drawing	Dc		0	0	FN	Own.	
SK-11.01.000 Body	As	1	1	1	FN	Own.	SK-11.01.000
SK-11.30.000 Base	As	3	1	1	FN	Own.	SK-11.30.000
SK-11.30.000SB Assembly drawing	Dc		0	0	FN	Own.	
SK-11.30.010 Supporting frame	As	1	1	1	FN	Own.	SK-11.30.010
SK-11.30.010SB Assembly drawing	Dc		0	0	FN	Own.	
SK-11.30.011 Sidewal	Dt	1	2	2	FN	Own.	SK-11.30.011
SK-11.30.012 Sidewal	Dt	2	2	2	FN	Own.	SK-11.30.012
Dowel 8,0x30	Att	10	8	8	FN	Purch	
SK-11.30.001 Plate	Dt	3	1	1	FN	Own.	SK-11.30.001
Screw 1-4x16.016 GOST 1145-80	St	5	8	8	FN	Purch	
Dowel 8,0x30	Att	6	8	8	FN	Purch	
Furniture leg NZ 66/55	Att	8	4	4	FN	Purch	
SK-11.00.001 Wall	Dt	1	1	1	FN	Own.	SK-11.00.001
SK-11.00.001-01 Wall	Dt	2	1	1	FN	Own.	SK-11.00.001-01
SK-11.00.002 Top plate	Dt	4	1	1	FN	Own.	SK-11.00.002
SK-11.00.003 Rear wall	Dt	6	1	1	FN	Own.	SK-11.00.003
SK-11.00.004 Divider	Dt	7	1	1	FN	Own.	SK-11.00.004
Screw 1-4x16.016 GOST 1145-80	St	9	2	2	FN	Purch	
Konfirmat 7,0x50.019	Att	10	4	4	FN	Purch	
Konfirmat 7,0x70.019	Att	11	8	8	FN	Purch	
Connector for rear wall	Att	13	6	6	FN	Purch	
Damper for screwing the on body (metal)	Att	15	1	1	FN	Purch	
SK-11.10.000 Right door	As	2	1	1	FN	Own.	SK-11.10.000
SK-11.10.000SB Assembly drawing	Dc		0	0	FN	Own.	
SK-11.10.001 Leaf	Dt	1	1	1	FN	Own.	SK-11.10.001
Screw 1-4x16.016 GOST 1145-80	St	2	4	4	FN	Purch	
Inner noose 105° Clip-On with closer profi line	Att	3	2	2	FN	Purch	

Fig. F.2 Product structure for selected wardrobe

F.2 Description of Planning Possibilities in the System

Solving manufacturing planning tasks in Clobbi is based on the following standards, concepts, and theories:

- MRPII (Manufacturing Resource Planning);
- APS (Advanced Planning and Scheduling);
- Scheduling Theory—mathematical scheduling of manufacturing;
- Theory of Constraints (TOC)—management concept used for manufacturing management.

The main aim of manufacturing planning is building of interconnected manufacturing plans for enterprises and their divisions and guaranteeing the performance of these plans by coordination of manufacturing resources. Optimization of planning in Clobbi is aimed at the formation of plans, ensuring:

- Increasing output;
- Shortening of manufacturing throughput time;
- Achieving a set level of work-in-process;
- Performing customer orders on time.

F.3 Description of Service Operation

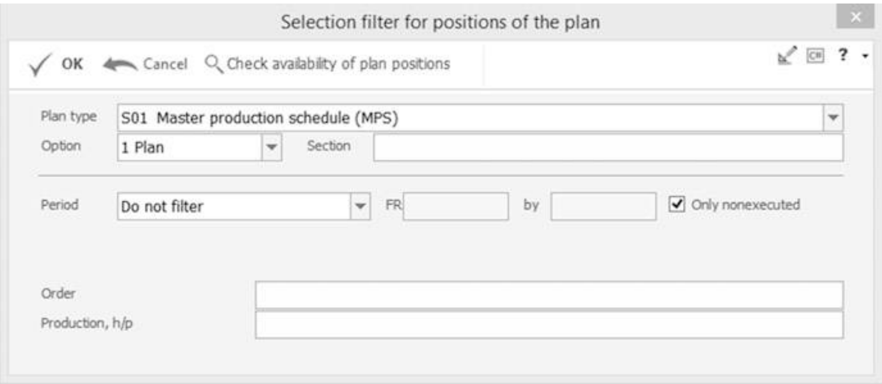
The main working area of a Chief planner in Clobbi is the Master Production Scheduling (MPS) function. It manages elaboration of the Master Production Schedule and shop floor schedules.

The system has a demonstration Master Schedule. To view it, start the shortcut “MPS” and in the field “Period” select “Do not filter” (Fig. F.3).

As a result, you get an initial MPS. Here you can add products for manufacture (“Add”, F7). In order to do this, select products and specify the amount and the due date for their manufacture (Fig. F.4).

When making changes to MPS, you should perform three calculations with the “Calculation” button or the F2 button. The first calculation is “MRP calculation by lead”, the second is “Net demand calculation”–“Complete recalculation”, and the third is “Generate MES-plan works”–“Generate works and allocate to equipment”.

Now we will focus on the capabilities of allocating earlier generated works to equipment.



Selection filter for positions of the plan

Plan type: S01 Master production schedule (MPS)

Option: 1 Plan

Section:

Period: Do not filter

FR:

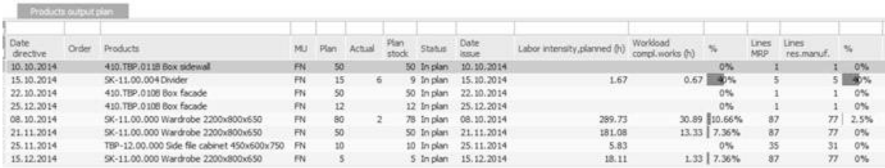
by:

☑ Only nonexecuted

Order:

Production, h/p:

Fig. F.3 Filter selection



Products output plan

Date directive	Order	Products	MU	Plan	Actual	Plan stock	Status	Date issue	Labor intensity,planned (h)	Workload compl.works (h)	%	Lines MRP	Lines res.manuf.	%
10.10.2014		410.TBP.011B Box sidewall	FN	50		50	In plan	10.10.2014			0%	1	1	0%
15.10.2014		SK-11.00.004 Divider	FN	15	6	9	In plan	15.10.2014	1.67	0.67	40%	5	5	40%
22.10.2014		410.TBP.010B Box facade	FN	50		50	In plan	22.10.2014			0%	1	1	0%
25.12.2014		410.TBP.010B Box facade	FN	12		12	In plan	25.12.2014			0%	1	1	0%
08.10.2014		SK-11.00.000 Wardrobe 2200x800x650	FN	80	2	78	In plan	08.10.2014	289.73	30.89	10.66%	87	77	2.5%
21.11.2014		SK-11.00.000 Wardrobe 2200x800x650	FN	50		50	In plan	21.11.2014	181.08	13.33	7.36%	87	77	0%
25.11.2014		TBP-12.00.000 Side file cabinet 450x600x750	FN	10		10	In plan	25.11.2014	5.83		0%	35	31	0%
15.12.2014		SK-11.00.000 Wardrobe 2200x800x650	FN	5		5	In plan	15.12.2014	18.11	1.33	7.36%	87	77	0%

Fig. F.4 Initial master production schedule

F.3.1 Generating Manufacturing Schedules

The task of operational calendar optimization manufacturing planning (APS/MES planning) is the performance of MPS and/or MRP plans of manufacturing by the optimal distribution of operations in time. As a result of optimization, manufacturing orders are produced on time or with minimum violations, capacity utilization is denser, the duration and numbers of equipment changeovers are decreased, idle time and volumes of unfinished manufacturing are reduced, the total duration of the manufacturing cycle is shortened, and, consequently, production output is increased and material and financial resources turnover is accelerated.

An APS/MES plan is generated taking into account limited capacity utilization, actual and forecast availability of materials, and other manufacturing resources. As a result, we get detailed shift operational manufacturing plans for each working center with optimal sequence of order performance.

APS/MES planning is the extension of usual calendar production planning with limited capacity. The main differences are the following:

- Absence of calendar planning intervals;
- Location of all works on a time axis with the accuracy of minutes;
- As follows from the previous point, the possibility of accounting during planning the actual sequence of works, setup time, time of waiting, and transfer between working centers;
- The plan is generated according to one or several optimization criteria.

Let’s generate a manufacturing schedule based on the demonstration example. To do this, while in MPS plan, we will use calculation F2 “Generation of MES-plan works”–“Call interface of work appointment” (Fig. F.5).

After specifying the date-time of the start point, you will get more or less the same picture of work appointment for the existing list of working centers. On the left you will see the list of working centers with the indicators of load. In the middle, a Gantt chart for visualization. If you point the cursor to any work, you will get a detailed work description. On the right you will see a slider to control the

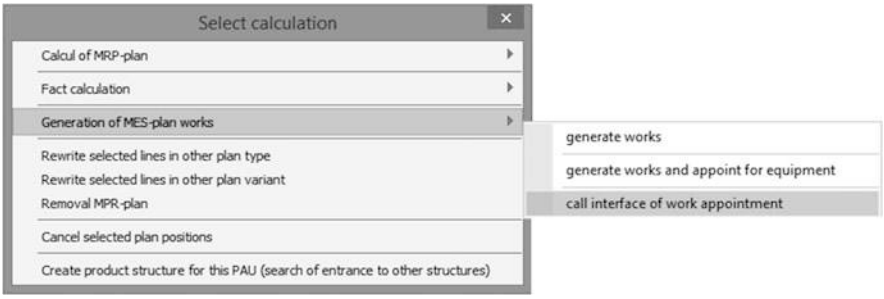
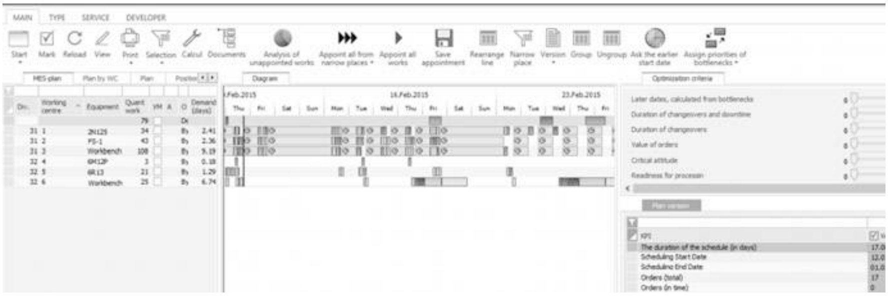

Select calculation
×
Calcul of MRP-plan
Fact calculation
Generation of MES-plan works
Rewrite selected lines in other plan type
Rewrite selected lines in other plan variant
Removal MPR-plan
Cancel selected plan positions
Create product structure for this PAU (search of entrance to other structures)
generate works
generate works and appoint for equipment
call interface of work appointment

Fig. F.5 Selection of calculation



Dn.	Working centre	Equipment	Quant work	VH	A	O	Demand (days)
			79	□		Dc	
31	1	2N125	34	□		8h	2.41
31	2	FS-1	43	□		8h	2.36
31	3	Workbench	108	□		8h	5.19
32	4	6M12P	3	□		8h	0.18
32	5	6R13	21	□		8h	1.29
32	6	Workbench	25	□		8h	6.74

Optimization criteria	
Later dates, calculated from bottlenecks	0
Duration of changeovers and downtime	0
Duration of changeovers	0
Value of orders	0
Critical attitude	0
Readiness for processin	0

KPI	V
The duration of the schedule (in days)	17.0
Scheduling Start Date	12.0
Scheduling End Date	01.0
Orders (total)	17
Orders (in time)	0

Fig. F.6 Work appointment

optimization criteria and metrics of plan quality assessment on the “Plan versions” tab. You can change the optimization criteria, use the “Appoint all works” mode, and compare the results on the “Plan versions” tab (Fig. F.6).

You can get more information on the specified function capabilities by viewing video instructions on the following channel: https://www.youtube.com/channel/UCIvwtzUhAC4WjlD0hpNYppQ

F.4 Clobbi Service Advantages

First it is worth mentioning the benefit of wide system functionality. Clobbi covers the following management tasks:

- Reference data (management of the main data);
- Design and technological manufacturing preparation;
- Manufacturing planning (MPS, MRP, APS\MES);
- Material flow accounting in manufacturing;
- Purchase management;
- Material inventory;
- Ready products inventory;
- Calculation of planned prime cost;
- Calculation of actual cost.

The general landscape of business processes and main information flows is shown below (Fig. F.7).

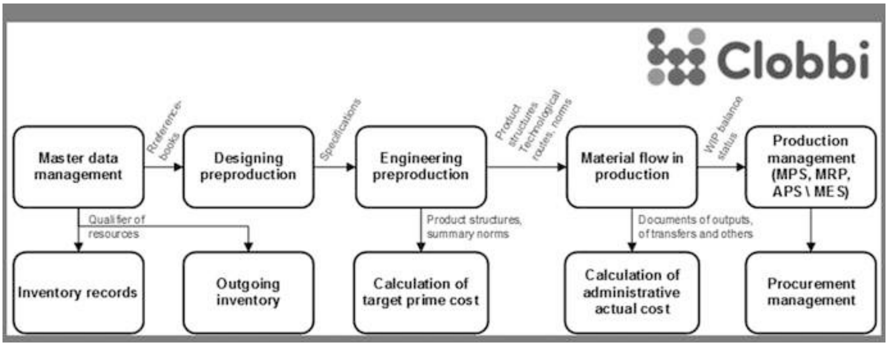

Clobbi
Master data management
Rreference books
Designing preproduction
Specifications
Engineering preproduction
Product structures Technological routes, norms
Material flow in production
WIP balance status
Production management (MPS, MRP, APS \ MES)
Qualifier of resources
Product structures, summary norms
Documents of outputs, of transfers and others
Inventory records
Outgoing inventory
Calculation of target prime cost
Calculation of administrative actual cost
Procurement management

Fig. F.7 General system landscape

Why Clobbi is recommended solution:

- Pre-installed typical system functionality for manufacturing enterprise with a discrete type of manufacture (machine and instrumentation building with multiple sub-industries).
- Possibility of using the system as a complex solution for the management of interconnected business processes as well as the combination of separate services for the solution of specific tasks of manufacturing enterprise management.
- Hot deployment of the management system. The enterprise can start working with the service in 1 day.
- Easy economic start of management in ERP. Minimization of initial investment, as there is no need to buy high production expensive server equipment.
- Guaranteed service efficiency. Scheduled maintenance and technical support of the system is provided by the solution developer. You can totally concentrate on the everyday use and development of your business processes.
- Service scalability. The enterprise can extend the functionality of its management system and you can buy additional competitive licenses if necessary.
- Availability of the management system. Clobbi provides access for users 24 h a day, 7 days a week. Users can work online from any computer.

You can learn about the product from Clobbi Youtube channel on a separate playlist https://www.youtube.com/channel/UCIvwtzUhAC4WjlD0hpNYppQ, where you can find review materials and video instructions on all approaches mentioned above. The same video instructions are available directly from Clobbi interface.

F.5 Online Registration of Manufacturing Events

For the full use of planning algorithms of levels MRP and APS\MES, you should solve the tasks of material inventory, components, and unfinished manufacture in manufacturing divisions. It is advisable to solve these tasks by using Clobbi.

The main goal of running operational inventory or material flows is to answer the following questions:

- What production orders are currently started;
- What part operation is in progress now for each production order (PO);
- What is going on now in each working center:
  - if debugging or processing is in progress—what part operation, what PO, when it was started, and when it is scheduled for finish;
  - if a working center is idle—what is the cause of idle time and how long it will be;
- Where parts or assemblies for each production order are located;
- Where are defects detected, when and how many.

Clobbi uses a documentary approach to manufacturing accounting, i.e. registering information on the manufacturing process, the system automatically provides estimation of material balance, parts, and assemblies in unfinished manufacturing, accounting of different manufacturing events, and performance of manufacturing plans.

The main manufacturing documents are:

- Manufacture act of part operations with recording lines of amortization;
- Waybill on parts and assemblies transfer;
- Documents of registration and repair of defects.

All actions on the manufacturing inventory can be performed traditionally on a computer (desktop or laptop with MS Windows) and by means of entering information from a mobile Android device.

The recommended option for the generation and processing of specified documents is the registration of corresponding events with the help of the mobile Android application Clobbi.Manufacture (to use in manufacturing divisions, you should connect to Wi-Fi and install this application from PlayMarket).

Clobbi.Manufacture is intended for online receipt of manufacturing tasks and registration of information on the manufacturing process in Clobbi. The application opens new horizons for information selection and analysis. Users of the mobile application Clobbi.Manufacture receive an easy at-hand tool that allows the following:

- Receiving manufacturing tasks online;
- Receiving information on the status of manufacturing orders and tasks;

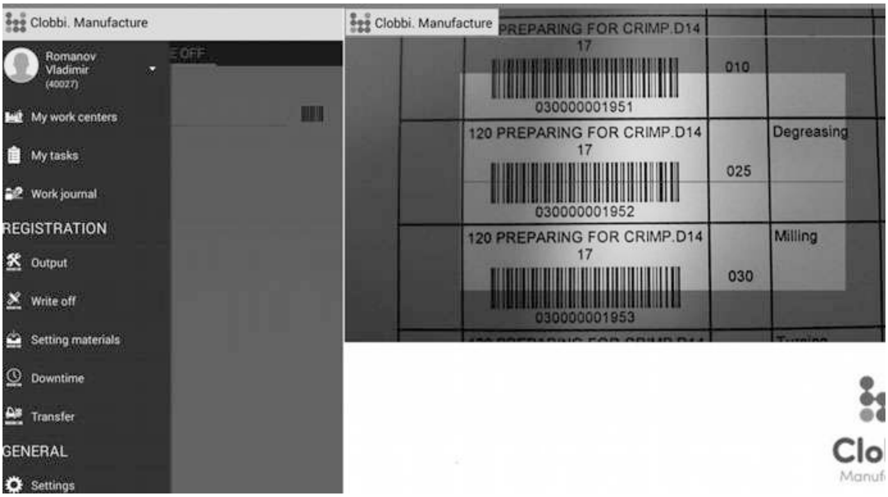


Fig. F.8 The user's mobile application

- Viewing technical documentation on parts, assemblies, and s/p from manufacturing tasks (drawings, technological processes, standards);
- Registering actual start and end of tasks online;
- Registering equipment idle time indicating the causes of idle time;
- Seeing event history by route lists;
- Tracking intrashop process of manufacturing parts, assemblies, and s/p.

The use of Clobbi.Manufacture considerably increases the mobility of employees and the efficiency of their work (Fig. F.8).

The basis for the start of part manufacturing is the shift task sent to an employee.

The start and end of each operation must be recorded in Clobbi, using one of several methods: manual recording on start/end in shift task or route list, using bar coding, etc.

The registration of the end of operation is the input of the actual volume of work—number of processed parts and assembly units (PAU), number of parts for technological needs, and correctable and final defects.

Moreover, the transfer of parts and assemblies between working centers and manufacturing divisions as well as idle time on working centers are registered.

F.6 Clobbi Commercial Use

The commercial version of Clobbi is based on the highly reliable worldwide cloud service Windows Azure (http://azure.microsoft.com).

To start the commercial use of the service, you should complete the registration form on the website http://clobbi.com/buy/. You will then receive an e-mail from the technical support service with a link for the system installation in "click once" mode.

The installation process takes several minutes. After its completion, you will have a shortcut to Clobbi on your desktop.